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David Hui



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REITERATED HOMOGENIZATION WITH APPLICATIONS TO AUTOPART CONSTRUCTION

¹Nicklas Bylund, ²Johan Byström and ²Lars-Erik Persson

¹VOLVO Car Corporation, Gothenburg, Sweden

²Dept. of Math., Luleå University of Technology, S-97187 Luleå, Sweden

1 Introduction

The homogenization theory is a branch of mathematics that has been developed in the studies of partial differential equations with rapidly oscillating coefficients. This method can for instance be used in the computation of mechanical properties of composite materials, since a common feature for these materials is that they behave like a homogeneous medium when the characteristic size of the microstructure is much smaller than the whole sample. Reiterated homogenization is a generalization where we iterate the homogenization procedure on several different scales. Reiterated homogenization was introduced on the physical level by Bruggeman in the 1930's [2] and mathematically justified by Bensoussan, Lions and Papanicolaou [1] in 1978, where it was stated a result for linear operators which later has been known as the iterated homogenization theorem. This theorem has been an indispensable tool in the construction of structures with extreme effective material properties. A good introduction to this field of mathematics is given in the review article [7] by Lukkassen and Milton. As an example of the applications of reiterated homogenization we mention the paper [3] by Byström, Jekabsons and Varna where different semianalytical models for woven composite stiffness computations were evaluated.

In this paper we will present some newly proved results for reiterated homogenization of nonlinear monotone operators, see [5] and [6]. Moreover, we use reiterated homogenization to investigate woven composite reinforcements used in the engine hood and tailgate of VOLVO cars.

2 Reiterated homogenization

Let us consider the class of partial differential equations of the form

$$\begin{cases} -\operatorname{div} \left(a \left(\frac{x}{\varepsilon}, \frac{x}{\varepsilon^2}, Du_\varepsilon \right) \right) = f \text{ on } \Omega, \\ u_\varepsilon \in W_0^{1,p}(\Omega), \end{cases} \quad (1)$$

where $a \left(\frac{x}{\varepsilon}, \frac{x}{\varepsilon^2}, Du_\varepsilon \right)$ is increasingly oscillating as $\varepsilon \rightarrow 0$, Ω is an open bounded subset of $\mathbf{R}^n$, $1 < p < \infty$, $1/p + 1/q = 1$ and $f \in W^{-1,q}(\Omega)$. We let Y and Z be open bounded rectangles in $\mathbf{R}^n$ and assume that $a(\cdot, z, \xi)$ is Y -periodic, $a(y, \cdot, \xi)$ is Z -periodic and that a satisfies the following boundedness, continuity and monotonicity assumptions:

$$a(y, z, 0) = 0, \quad (2)$$

$$|a(y, z, \xi_1) - a(y, z, \xi_2)| \leq c_1 (1 + |\xi_1| + |\xi_2|)^{p-1-\alpha} |\xi_1 - \xi_2|^\alpha, \quad (3)$$

$$\begin{aligned} & (a(y, z, \xi_1) - a(y, z, \xi_2), \xi_1 - \xi_2) \\ & \geq c_2 (1 + |\xi_1| + |\xi_2|)^{p-\beta} |\xi_1 - \xi_2|^\beta, \end{aligned} \quad (4)$$

where the constants $c_1, c_2 > 0$, $0 \leq \alpha \leq \min(1, p-1)$ and $\max(p, 2) \leq \beta < \infty$. We also assume that a is of the form

$$a(y, z, \xi) = \sum_{i=1}^N \chi_{\Omega_i}(y) a_i(y, z, \xi)$$

and satisfies a continuity condition of the form

$$\begin{aligned} & |a(y_1, z, \xi) - a(y_2, z, \xi)| \\ & \leq \omega(|y_1 - y_2|) (1 + |\xi|^p), \end{aligned} \quad (5)$$

where $\omega : \mathbf{R}^n \rightarrow \mathbf{R}^n$ is continuous, increasing and $\omega(0) = 0$. Let us now consider the weak formulation of (1):

$$\begin{cases} \int_{\Omega} \left(a \left(\frac{x}{\varepsilon}, \frac{x}{\varepsilon^2}, Du_{\varepsilon} \right), D\phi \right) dx = \langle f, \phi \rangle, \\ u_{\varepsilon} \in W_0^{1,p}(\Omega), \end{cases} \quad (6)$$

for all $\phi \in W_0^{1,p}(\Omega)$. The main result from [5] and [6] then states that:

Theorem 1 *Let a satisfy (2)-(5). Moreover, let u_{ε} be the solutions of (6). Then*

$$u_{\varepsilon} \rightarrow u_0 \text{ weakly in } W_0^{1,p}(\Omega),$$

$$a \left(\frac{x}{\varepsilon}, \frac{x}{\varepsilon^2}, Du_{\varepsilon} \right) \rightarrow b(Du_0) \text{ weakly in } (L^q(\Omega))^n,$$

as $\varepsilon \rightarrow 0$, where u_0 is the unique solution of

$$\begin{cases} \int_{\Omega} (b(Du_0), D\phi) dx = \langle f, \phi \rangle, \\ u_0 \in W_0^{1,p}(\Omega), \end{cases} \quad (7)$$

for every $\phi \in W_0^{1,p}(\Omega)$. The operator $b : \mathbf{R}^n \rightarrow \mathbf{R}^n$ is defined as

$$b(\xi) = \frac{1}{|Y|} \int_Y b_1(y, \xi + Dv^{\xi}(y)) dy,$$

where v^{ξ} is the unique solution of the Y cell problem

$$\begin{cases} \int_Y (b_1(y, \xi + Dv^{\xi}(y)), D\phi) dy = 0, \\ v^{\xi} \in W_{per}^{1,p}(Y), \end{cases}$$

for every $\phi \in W_{per}^{1,p}(Y)$. The operator $b_1 : Y \times \mathbf{R}^n \rightarrow \mathbf{R}^n$ is defined as

$$b_1(y, \xi) = \frac{1}{|Z|} \int_Z a(y, z, \xi + Dv^{\xi,y}(z)) dz,$$

where $v^{\xi,y}$ is the unique solution of the Z cell problem

$$\begin{cases} \int_Z (a(y, z, \xi + Dv^{\xi,y}(z)), D\phi(z)) dz = 0, \\ v^{\xi,y} \in W_{per}^{1,p}(Z), \end{cases}$$

for every $\phi \in W_{per}^{1,p}(Z)$.

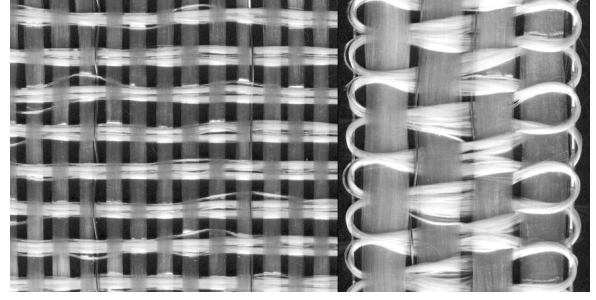


Figure 1: Weaves 1 and 2. X-direction upwards.

Remark 1 By $W_{per}^{1,p}(Y)$ we denote set of all functions $u \in W^{1,p}(Y)$ with mean value zero which have the same trace on opposite faces of Y .

It is also shown in for instance [4] that the operators b and b_1 satisfies similar structure conditions as a .

3 Woven composite reinforcements

We have evaluated two different woven composite reinforcements that are used in the construction of engine hoods and tailgates on VOLVO cars. The tailgate mentioned is currently in production in the VOLVO V70 series, whereas the engine hood still is in an experimental stadium. We evaluated the fiber weaves for a range of fiber bundle percentages, giving us the weave properties as a function of fiber bundle saturation. These curves are then used for certain composite reinforcements where the fiber bundle percentages are known. The mechanical properties of the autoparts are then computed with several different number of reinforcement layers to achieve the required mechanical properties.

The two different fiber weave reinforcements consist of glassfiber weave in epoxy resin, having the following geometries:

Property	Weave 1	Weave 2
X-Bundle width (mm)	2.08	5.0
X-Bundle thckn. (mm)	0.24	0.75
X-Bundle spacing (mm)	1.12	2.0
Y-Bundle width (mm)	1.6	3.0
Y-Bundle thckn. (mm)	0.12	0.25
Y-Bundle spacing (mm)	3.2	2.0

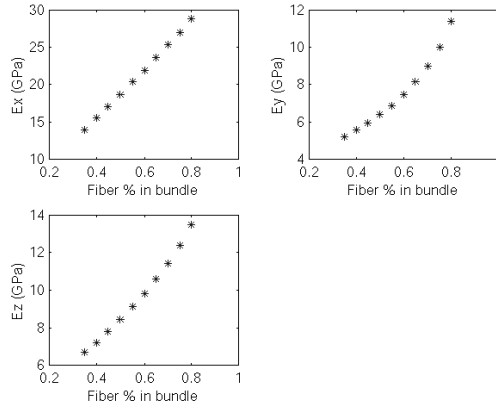


Figure 2: Weave 1 moduli.

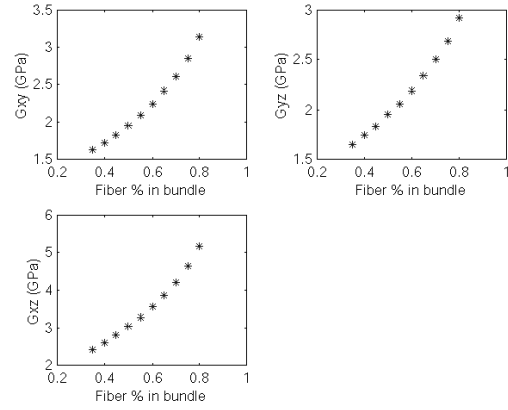


Figure 4: Weave 1 shear moduli.

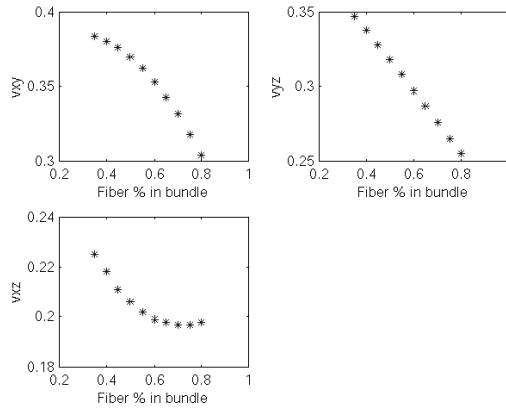


Figure 3: Weave 1 Poisson ratios.

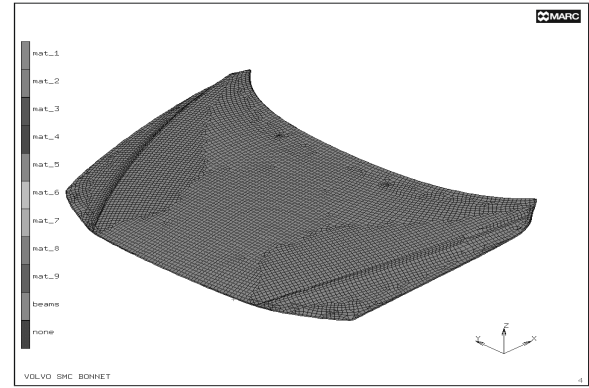


Figure 5: Engine hood, top side.

and mechanical properties:

Property	Glass fiber	Epoxy
Modulus (GPa)	72.4	3.2
Poisson's ratio	0.22	0.36

These fiber weaves were first homogenized on the fiber bundle (Z) scale for different fiber percentages, assuming that the fibers were hexagonally stacked giving isotropic effective properties. Then these fiber bundle properties were used to homogenize the two fiber weaves on the weave repeating unit cell (Y) scale for each value of the fiber bundle percentage. Some of the results of the weaves mechanical properties from the reiterated homogenization are collected in the graphs given in figures 2-4.

The values of the fiber weave mechanical prop-

erties were then used to determine the reinforcement needed for the autoparts. The meshing and material distribution of the engine hood are given in figures 5-6.

The engine hood consists mainly of SMC board, resin and steel with the following properties:

Material	E (GPa)	ν
SMC	11.7	0.34
Resin	3.0	0.49
Steel	210	0.3

The bending and torsional stiffness, dent resistance and flex resistance were then determined for several different load cases. We have shown load case B (normal position, movements while driving) in figures 7-9.

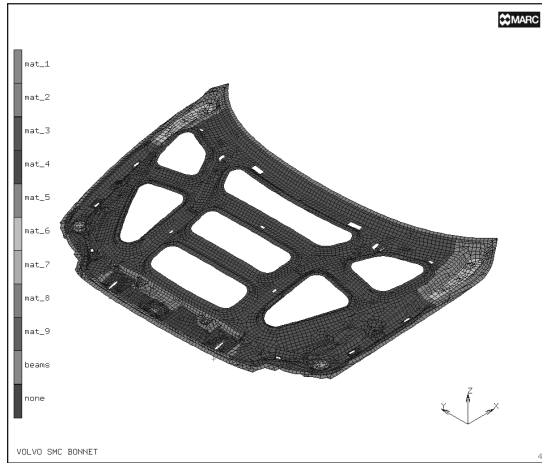


Figure 6: Engine hood, reinforcements.

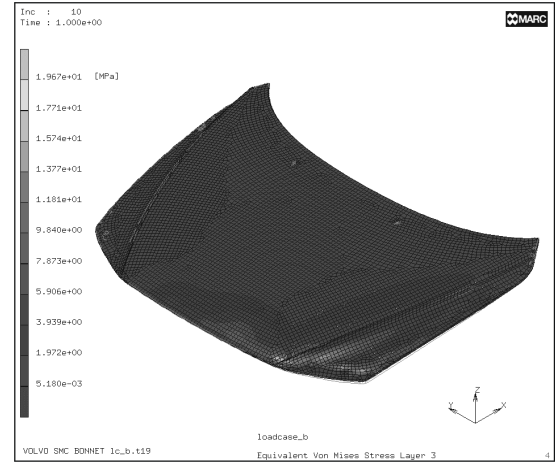


Figure 9: Load case B. Stress.

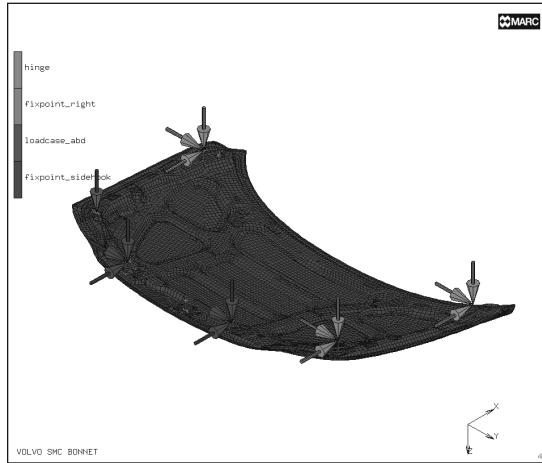


Figure 7: Load case B. Boundary conditions.

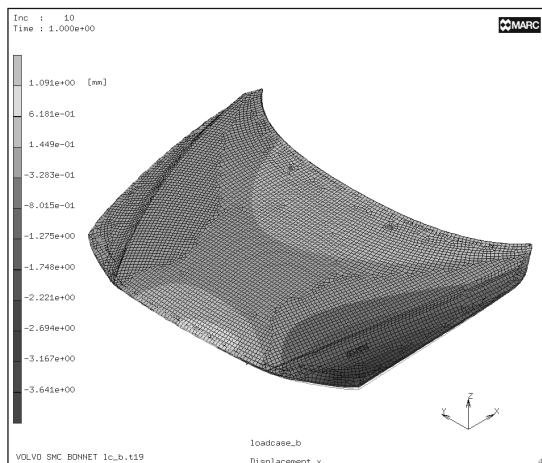


Figure 8: Load case B. Deformation in Z-direction.

References

- [1] A. Bensoussan, J. L. Lions and G. C. Papanicolaou, *Asymptotic analysis for periodic structures*, North Holland, Amsterdam, 1978.
- [2] D. A. G. Bruggeman, Berechnung verschiedener physikalischer konstanten von heterogenen substanzen, *Ann. Physik.* 24, 1935, p. 634.
- [3] J. Byström, N. Jekabsons and J. Varna, An evaluation of different models for prediction of elastic properties of woven composites, *Composites: Part B*, 31, 2000, 7-20.
- [4] J. Byström, Correctors for some nonlinear monotone operators, *J. Nonlin. Math. Physics*, 8:1, 2001, 8-30.
- [5] J. L. Lions, D. Lukkassen, L.-E. Persson and P. Wall, Reiterated homogenization of monotone operators, *C. R. Acad. Sci. Series I*, 330:8, 2000, 675-680.
- [6] J. L. Lions, D. Lukkassen, L.-E. Persson and P. Wall, Reiterated homogenization of nonlinear monotone operators, *Chin. Ann. of Math.*, 22B:1, 2001, 1-12.
- [7] D. Lukkassen and G. Milton, On hierarchical structures and reiterated homogenization, 2001, (submitted).