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HOMOGENIZATION AND COMPUTATIONAL TECHNIQUES OF ITERATED STRUCTURES

Stein Berggren², Johan Byström¹, Annette Meidell² and Peter Wall¹

¹Dept. of Math., Luleå University of Tech., S-97187 Luleå, Sweden

²Narvik University College, N-8501, Narvik, Norway

We will in this paper consider computational aspects of multiscale *iterated honeycombs*. By *honeycombs* we mean two-dimensional two-component periodic structures where the interior part consists of polygons (typically hexagons or rectangles). Iterated honeycombs are, roughly speaking, “honeycombs in honeycombs”. The theoretical basis for this work can be found in [2] and [3], where D. Lukkassen investigated the linear elastic and thermal effective properties of these types of structures. In particular, Lukkassen found simple formulae for the upper and lower bounds of the corresponding effective parameters. These bounds happen to converge to the same limit as the number of length scales turns to infinity, and, it turns out that these limits are optimal. Composite structures of this type are usually referred to as optimal. Other well known examples of optimal structures are *the Hashin ellipsoidal* structure and *the multirank laminate* structure. However, it is important to note that structures which have the same effective behavior often are different with respect to properties such as local buckling resistance, local and global strength, structural collapse etc. From this point of view iterated honeycombs seem to be very interesting, especially because they possess certain suitable symmetry properties (see [3]).

We present a numerical method for estimating the effective conductivity with good control of the accuracy. This method provides a reliable tool for comparing different types of iterated square honeycombs. Up to now only the case of widely separated length scales has been considered. From a manufacturing point of view it is interesting to investigate and compare the intermediate cases.

Let us consider the iterated square honeycomb structure defined by the characteristic function

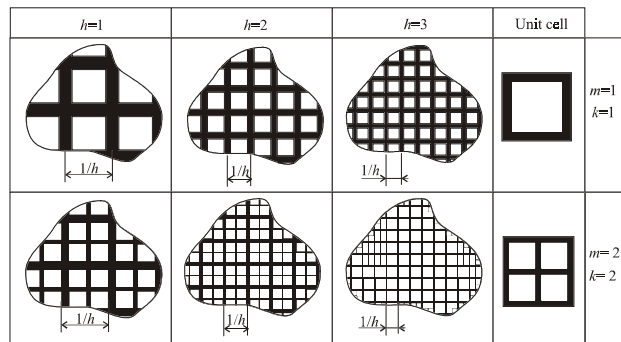


Figure 1: Iterated square honeycombs.

$\chi_{m,k,h}$ on $\mathbf{R}^2$, where k, h, m are positive integers,

$$\chi_{m,k,h}(x) = \chi^{(m)}(hx_1) \chi^{(m)}(hx_2),$$

$\chi^{(m)}$ is found iteratively by letting $\chi^{(i+1)}$ be the $[0, 1]$ -periodic extension of the function

$$\chi(\cdot) \chi^{(i)}\left(k \cdot / v^{1/(2m)} - \left(1 - v^{1/(2m)}\right) / 2\right), \chi^{(0)} = 1$$

and χ is the characteristic function of the interval $[0, v^{1/(2m)}]$. We then define the conductivity $\lambda_{m,k,h}(x)$ by

$$\lambda_{m,k,h}(x) = \lambda_{\text{in}} \chi_{m,k,h}(x) + \lambda_{\text{con}} (1 - \chi_{m,k,h}(x)),$$

where λ_{in} and λ_{con} are the conductivities of the interior and the connected part of the iterated square honeycomb-structure, respectively.

In order to be able to speak about effective conductivity for the above structure we consider the following two limit-cases:

- fix m and k and let h approach infinity. The corresponding effective conductivity is in this case denoted $\lambda_{m,k}$.

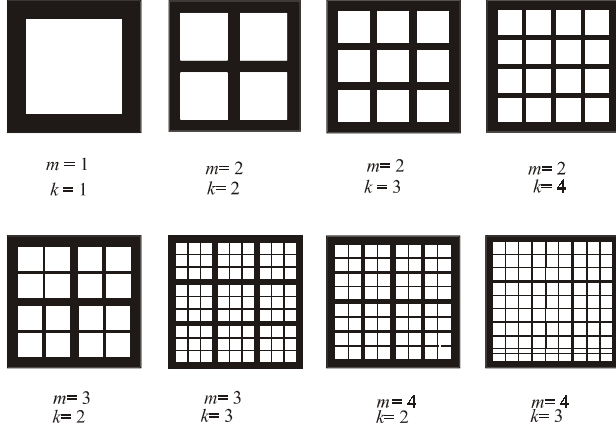


Figure 2: Examples of unit cells for $\nu = 0.52$.

- fix m and let $h = k$ approach infinity. The corresponding effective conductivity is in this case denoted $\lambda_{m,\infty}$.

To get $\lambda_{m,k}$ and $\lambda_{m,\infty}$, we need to solve certain minimization problems over a special Sobolev space $W_{\text{per}}^{1,2}(Y)$ (see [1]). For iterated square honeycombs the approximation formula of Lukkassen (see [2],[3]) reduces to

$$\frac{1}{\lambda_{m,\infty} - \lambda_{\text{con}}} = \frac{1}{\lambda_{\text{in}} - \lambda_{\text{con}}} \frac{1}{v} + \frac{r_m}{1 + v^{\frac{1}{2m}}} \frac{1 - v}{\lambda_{\text{con}} v},$$

where $v^{1/(2m)} \leq r_m \leq 1$. Note that this yields the convergence $\lambda_{m,\infty} \rightarrow \lambda_{\infty,\infty}$ as $m \rightarrow \infty$, where

$$\lambda_{\infty,\infty} = \lambda_{\text{con}} + \left(\frac{1}{\lambda_{\text{in}} - \lambda_{\text{con}}} \frac{1}{v} + \frac{1 - v}{2\lambda_{\text{con}} v} \right)^{-1},$$

which is precisely the Hashin-Shtrikman upper bound for $\lambda_{\text{in}} < \lambda_{\text{con}}$ and the Hashin-Shtrikman lower bound if $\lambda_{\text{in}} > \lambda_{\text{con}}$.

In order to get numerical estimations of $\lambda_{m,k}$ and $\lambda_{m,\infty}$, we replace $W_{\text{per}}^{1,2}(Y)$ with a finite dimensional subset of $W_{\text{per}}^{1,2}(Y)$ in the minimization problems mentioned above. In this way we obtain upper and lower numerical estimates $\lambda_{m,k}^+$ and $\lambda_{m,k}^-$, for $\lambda_{m,k}$. We note that the geometry of the problem is the same on each step. Based on this geometry we can create a fixed FEM-mesh (with finest mesh-refinement in the corners) which we can use on all the $2m$ steps. As an approximation for $\lambda_{m,k}$ we use the number $\tilde{\lambda}_{m,k}$

$$\tilde{\lambda}_{m,k} = 1/2 \left(\lambda_{m,k}^- + \lambda_{m,k}^+ \right),$$

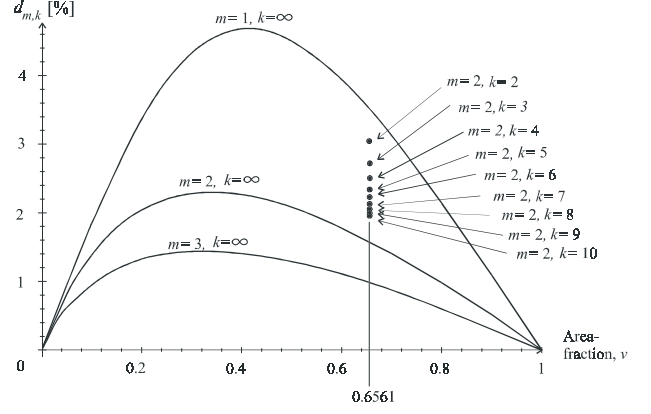


Figure 3: Difference between the effective conductivity and the Hashin-Shtrikman upper bound.

and let $d_{m,k}$ denote the deviation between $\lambda_{m,k}$ and $\lambda_{\infty,\infty}$ (in %) i.e.

$$d_{m,k} = 100 \frac{\lambda_{\infty,\infty} - \lambda_{m,k}}{\lambda_{m,k}}.$$

It seems that for a fixed value of m the conductivity increases with k . However, the convergence, as k goes to ∞ , is slower than we expected (see Figure 3). From a manufacturing point of view it is obvious that the number of white squares $k^{2(m-1)}$ in each unit cell is an important parameter (it is easier to manufacture a structure with few large squares than many small ones). We observe from Figure 3 that the conductivity may not necessarily increase with the number $k^{2(m-1)}$ (compare e.g. the case $m = 2, k = 8$ with the case $m = 3, k = 3$).

References

- [1] J. Byström, J. Helsing and A. Meidell, Some computational aspects of iterated structures, 2001 (submitted).
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- [3] D. Lukkassen, A new reiterated structure with optimal macroscopic behaviour, *SIAM J. Appl. Math.*, 59(5), 1825-1842, 1999.