

MATHEMATICAL MODELLING OF COMPOSITES USING THE HOMOGENIZATION METHOD

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We define $A^h(x) = A(x/\varepsilon_h)$, where the conductivity matrix $A(x) = (a_{ij}(x))$ satisfies $a_{ij} = a_{ji}$, $a_{ij} \in L^\infty(\Omega)$ and is Y -periodic for every $i, j = 1, \dots, n$. Consider the weakly formulated stationary heat conduction problem

$$\begin{cases} \int_{\Omega} \langle A^h(x) \nabla u_h, \nabla \phi \rangle dx = \int_{\Omega} f \phi dx \text{ on } \Omega, \quad \forall \phi \in H_0^{1,2}(\Omega), \\ u_h \in H_0^{1,2}(\Omega), \end{cases}$$

where $f \in L^2(\Omega)$ is an internal heat source and (ε_h) is a sequence of positive real numbers converging to zero. The main theorem in the homogenization theory then states that $u_h \rightharpoonup u_0$ weakly in $H_0^{1,2}(\Omega)$ and $A^h(x) \nabla u_h \rightharpoonup B \nabla u_0$ weakly in $(L^2(\Omega))^n$, where u_0 is the unique solution of the so called homogenized problem. The constant matrix B depends on the solution w^k of a problem over one periodic cell Y , see for instance [2].

A common assumption in the homogenization method is to assume that the inclusions are periodically distributed in the material. However, this assumption might often not be in accordance with the situation in real-world composites, where the inclusions in most cases are randomly distributed. We will in this talk use some of the methods described in [1] and [3] to compare the effect of the geometries and the distributions of the inclusions on the effective conductivity of some composites.

References

- [1] J. Byström. Influence of the inclusions distribution on the effective properties of heterogeneous media. (*submitted*), 2002, 13 pages.
- [2] V.V. Jikov, S.M. Kozlov and O.A. Oleinik. *Homogenization of differential operators and integral functionals*. Springer-Verlag, Berlin, 1994.
- [3] S. Torquato. *Random Heterogeneous Materials*. Springer-Verlag, New York, 2002.