

ICCE/4

FOURTH INTERNATIONAL CONFERENCE ON COMPOSITES ENGINEERING

Edited by
David Hui



July 6-12, 1997

Big Island of Hawaii

Sponsored and Organized by
International Community for Composites Engineering and
College of Engineering, University of New Orleans

Computing effective conductivities by bounds and numerical methods

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It is in general impossible to compute mechanical properties of composite materials by standard numerical methods, since the coefficients of the governing partial differential equations are rapidly oscillating. However, a common feature for composite materials is that they act like a homogeneous medium when the characteristic size of the microstructure is much smaller than the whole sample. Therefore, we make some type of averaging and try to find an effective, homogeneous material that behaves like the composite material.

We use the homogenization method to compute the effective thermal conductivity of a composite material. Consider a composite material $\Omega \in \mathbf{R}^n$ that is made up by a continuous phase of material 1 and periodically distributed fibers of material 2. The periodicity assumption means that we can divide the material in identical cells Y . We let a_{ij} be the

thermal conductivity tensor of the composite material and consider the boundary value problem

$$\begin{aligned} -\frac{\partial}{\partial x_i} \left(a_{ij} \left(\frac{x}{\varepsilon} \right) \frac{\partial u^\varepsilon}{\partial x_j} \right) &= f \text{ in } \Omega, \\ u^\varepsilon &= 0 \text{ on } \partial\Omega, \end{aligned}$$

where ε is a parameter that measures the fineness of the structure. By letting $\varepsilon = (\varepsilon_n)$, $n = 1, 2, \dots$, be a sequence such that $\varepsilon_n \rightarrow 0$ as $n \rightarrow \infty$, we obtain a whole class of boundary value problems. Homogenization then yields the existence of corresponding homogenized problem on the form

$$\begin{aligned} -\frac{\partial}{\partial x_i} \left(a_{ij}^0 \frac{\partial u}{\partial x_j} \right) &= f \text{ in } \Omega, \\ u &= 0 \text{ on } \partial\Omega, \end{aligned}$$

where $a^0 = (a_{ij}^0)$ is a constant tensor that can be expressed in terms of a solution of a cell problem.

In this paper we estimate a^0 both with numerical and theoretical bounds. Moreover, we show that these methods fit very well together.

Consider a unit cell in $\mathbf{R}^2$ consisting of two materials with the thermal conductivities $a_1 (= 1)$ and $a_2 (> 1)$, respectively. Let material 2 occupy a square with size $l \times l$ ($= 0.5 * 0.5$). We used a FEM-program to solve the cell problem and compute a^0 . By also considering the complementary problem, see Jikov et. al. [2], we got upper and lower numerical bounds, a_{num}^+ and a_{num}^- , respectively.

We have also computed the bounds q^- and q^+ obtained by Lukkassen et. al. [3], [4]. These bounds depend on the cell geometry and are in this case very easy to compute. Finally, we compare the computed bounds with the conjectured value of a^0 by Mortola and Steffe [5], which we here call a_{MS} . We list the results in the table below:

a_2	a_{MS}	a_{num}^-	a_{num}^+	q^-	q^+
10^1	1.5442	1.5435	1.5449	1.4091	1.6923
10^2	1.7095	1.7079	1.7111	1.4901	1.9612
10^3	1.7297	1.7280	1.7315	1.4990	1.9960
10^4	1.7318	1.7300	1.7336	1.4999	1.9996

Conclusions: It turns out that the computed values are good approximations of a^0 . Moreover, we see that the bounds obtained by Lukkassen et. al. [4] remain tight as a_2 increases and thus give a very good estimate of a^0 . It should be noted that the Hashin-Shtrikman upper bound tends to infinity as a_2 increases, and therefore gives no informa-

tion. More information concerning this paper can be found in Byström [1] and Pavval [6].

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