

ICCE/6

SIXTH ANNUAL INTERNATIONAL CONFERENCE ON
COMPOSITES ENGINEERING

Edited by
David Hui



June 27-July 3, 1999

Orlando, Florida

Sponsored and Organized by
International Community for Composites Engineering and
College of Engineering, University of New Orleans

A COMPARISON OF DIFFERENT HOMOGENIZATION MODELS AND EXPERIMENTAL RESULTS

Johan Byström¹, Normunds Jekabsons² and Lars-Erik Persson¹

¹Dept. of Math., Luleå University of Technology, S-97187 Luleå, Sweden

²Div. of Polymer Engineering, Luleå University of Technology, S-97187 Luleå, Sweden

1 Introduction

In this paper we consider the problem of how to model the elastic properties of woven composites. More specific, we will use mathematical homogenization theory as a reference to evaluate different semi-analytical homogenization methods that are used for computation of the elastic properties.

Existing homogenization models can be divided into three different categories – semi-analytic engineering models, models based on FEM calculations and the relatively new mathematical homogenization method. We will in particular evaluate the Karayaka-Kurath Model [5] (KKM) and the modified mosaic parallel model (MMPM) (see Ishikawa and Chou [3], [4]) by using reiterated homogenization. The subject of mathematical homogenization can be found in the books by Bensoussan et al. [1] and Persson et al. [6].

2 Reiterated homogenization

A thorough description of the use of reiterated homogenization for woven composites can be found in the upcoming article [2]. We assume the composite to extend over a domain $\Omega \subset \mathbf{R}^3$ and assume that the representative unit cells of the woven composite have a periodicity of Y . Moreover, we assume the fibers in every fiber tow have a periodicity of Z . We let ε be a small parameter, put $\mathbf{y} = \mathbf{x}/\varepsilon$ and $\mathbf{z} = \mathbf{x}/\varepsilon^2$, and expand the displace-

ment field $\mathbf{u}$ in the asymptotic series $\mathbf{u}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathbf{u}_0(\mathbf{x}, \mathbf{y}, \mathbf{z}) + \varepsilon \mathbf{u}_1(\mathbf{x}, \mathbf{y}, \mathbf{z}) + \varepsilon^2 \mathbf{u}_2(\mathbf{x}, \mathbf{y}, \mathbf{z}) + \dots$, where $\mathbf{u}_i$ are Y -periodic w.r.t. $\mathbf{y}$ and Z -periodic w.r.t. $\mathbf{z}$. By the chain rule, it then follows that the strain $\underline{\varepsilon}$ (underlining indicates a second order tensor) is given by

$$\underline{\varepsilon} = \frac{1}{2} \left[\nabla_x \mathbf{u} + (\nabla_x \mathbf{u})^T + \frac{1}{\varepsilon} \left(\nabla_y \mathbf{u} + (\nabla_y \mathbf{u})^T \right) + \frac{1}{\varepsilon^2} \left(\nabla_z \mathbf{u} + (\nabla_z \mathbf{u})^T \right) \right].$$

Let us denote $\underline{\varepsilon}_\alpha(\mathbf{u}_i) = \frac{1}{2} [\nabla_\alpha \mathbf{u}_i + (\nabla_\alpha \mathbf{u}_i)^T]$, $\alpha = \mathbf{x}, \mathbf{y}, \mathbf{z}$. Then we can write $\underline{\varepsilon} = \frac{1}{\varepsilon^2} \underline{\varepsilon}_0 + \frac{1}{\varepsilon} \underline{\varepsilon}_1 + \underline{\varepsilon}_2 + \varepsilon \underline{\varepsilon}_3 + \dots$, where $\underline{\varepsilon}_0 = \underline{\varepsilon}_z(\mathbf{u}_0)$, $\underline{\varepsilon}_1 = \underline{\varepsilon}_y(\mathbf{u}_0) + \underline{\varepsilon}_z(\mathbf{u}_1)$, and $\underline{\varepsilon}_i = \underline{\varepsilon}_x(\mathbf{u}_{i-2}) + \underline{\varepsilon}_y(\mathbf{u}_{i-1}) + \underline{\varepsilon}_z(\mathbf{u}_i)$, $i = 2 \dots \infty$. Using $C(\mathbf{y}, \mathbf{z})$ as the fourth-rank constitutive tensor, the stress can be written as $\underline{\sigma} = C(\mathbf{y}, \mathbf{z}) \underline{\varepsilon} = \frac{1}{\varepsilon^2} \underline{\sigma}_0 + \frac{1}{\varepsilon} \underline{\sigma}_1 + \underline{\sigma}_2 + \varepsilon \underline{\sigma}_3 + \dots$. The equilibrium equations then give $\nabla \cdot \underline{\sigma} = \nabla_x \cdot \underline{\sigma} + \frac{1}{\varepsilon} \nabla_y \cdot \underline{\sigma} + \frac{1}{\varepsilon^2} \nabla_z \cdot \underline{\sigma} = \mathbf{0}$, or written out in full

$$\begin{aligned} & \frac{1}{\varepsilon^4} \nabla_z \cdot \underline{\sigma}_0 + \frac{1}{\varepsilon^3} [\nabla_y \cdot \underline{\sigma}_0 + \nabla_z \cdot \underline{\sigma}_1] \\ & + \frac{1}{\varepsilon^2} [\nabla_x \cdot \underline{\sigma}_0 + \nabla_y \cdot \underline{\sigma}_1 + \nabla_z \cdot \underline{\sigma}_2] \\ & + \sum_{i=1}^{\infty} \varepsilon^{i-2} [\nabla_x \cdot \underline{\sigma}_i + \nabla_y \cdot \underline{\sigma}_{i+1} + \nabla_z \cdot \underline{\sigma}_{i+2}] = \mathbf{0}. \end{aligned}$$

Since this equation must hold for every ε , we get a system of equations to solve. Solution of the five first equations then yields the homogenized tensor

A by the formula

$$A = \frac{1}{|Y|} \int_Y \hat{C}(\mathbf{y}) e_y(\eta + W_\eta) d\mathbf{y},$$

where W_η is the Y -periodic solution of the Y cell problem $\nabla_y \cdot (\hat{C}(\mathbf{y}) e_y(\eta + W_\eta)) = \mathbf{0}$, boundary conditions $\eta_i^{rs} = \delta_{ir} y_s$, and $\hat{C}(\mathbf{y})$ is the Z cell homogenized tensor. Here $\hat{C}(\mathbf{y})$ is given by

$$\hat{C}(\mathbf{y}) = \frac{1}{|Z|} \int_Z C(\mathbf{y}, \mathbf{z}) e_z(\zeta + V_\zeta) d\mathbf{z},$$

where V_ζ is the Z -periodic solution of the Z cell problem $\nabla_z \cdot (C(\mathbf{y}, \mathbf{z}) e_z(\zeta + V_\zeta)) = \mathbf{0}$ with the boundary conditions $\zeta_i^{rs} = \delta_{ir} z_s$.

3 Numerical results

We investigated plain weave glass-fiber/epoxy and carbon-fiber/epoxy composites with the following constituent properties:

Property	GF/EP	CF/EP
Fiber vol. frac./tow	0.91	0.66
Overall vol. frac.	0.60	0.41
Axial mod. fib. (GPa)	72	238
Transv. m. fib. (GPa)	72	40
Transv. Poisson's fib.	0.2	0.22
Major Poisson's fib.	0.25	0.22
In pl. shear fib. (GPa)	22	29.8
Young's mod. matrix	3	3.4
Poisson's ratio matrix	0.38	0.36

The composites had the following measured in-plane properties:

Property	GF/EP	CF/EP
Young's m. (GPa)	26.3 ± 2	46.5 ± 2
Poisson's ratio	0.14 ± 0.1	0.05 ± 0.009
Shear mod. (GPa)	4.5 ± 0.3	4.0 ± 0.3

The different semi-analytical models (KKM and MPPM) compared with reiterated homogenization (RH) gave the following results for the glass-

fiber/epoxy composite:

Property	RH	KKM	MPPM
Young's m. (GPa)	27.3	27.4	28.0
Poisson's ratio	0.15	0.12	0.11
Shear mod. (GPa)	3.9	4.1	3.6

and the following results for the carbon-fiber/epoxy composite:

Property	RH	KKM	MPPM
Young's m. (GPa)	43.1	44.3	53.7
Poisson's ratio	0.044	0.047	0.039
Shear mod. (GPa)	3.0	3.1	2.5

Also the out-of-plane properties were measured and investigated together with more models in the paper [2] by the authors.

References

- [1] A. Bensoussan, J. L. Lions, G. Papanicolau. *Asymptotic Analysis for Periodic Structures*, North-Holland, 1978.
- [2] J. Byström, N. Jekabsons and J. Varna. An evaluation of different models for prediction of elastic properties of woven composites (to appear in Composites: Part B), 1999.
- [3] T. Ishikawa, Anti-symmetric elastic properties of composite plates of satin weave cloth, *Fiber Sci. Tech.*, v. 15, 1981, pp. 127-145.
- [4] T. Ishikawa and T. W. Chou, Elastic behavior of woven hybrid materials, *J. Composite Materials*, v. 16, 1982, pp. 2-19.
- [5] M. Karayaka and P. Kurath, Deformation and failure behavior of woven composite laminates, *J. Engineering Materials and Technology*, v. 116, 1994, pp. 136-142.
- [6] L. E. Persson, L. Persson, N. Svanstedt, J. Wyller. *The Homogenization Method: An Introduction*. Studentlitteratur, Lund, 1993.