

Resolving distributional conflicts between generations[★]

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Summary. We describe a new approach to the problem of resolving distributional conflicts between an infinite and countable number of generations. We impose conditions on the social preferences that capture the following idea: If preference (or indifference) holds between truncated paths for infinitely many truncating times, then preference (or indifference) holds also between the untruncated infinite paths. In this framework we use such conditions to (1) characterize different versions of leximin and utilitarianism by means of equity conditions well-known from the finite setting, and (2) illustrate the problem of combining Strong Pareto and impartiality in an intergenerational setting.

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1 Introduction

The Suppes-Sen grading principle captures both a concern for equal treatment of generations and the demand for efficiency, through the conditions of Weak Anonymity and Strong Pareto. It turns out that this is all that is needed in order to justify sustainable solutions within reasonable technological frameworks, as shown by Asheim, Buchholz, and Tungodden [2].

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However, there are two problems with this approach. First, even if we accept this justification for sustainability, there exists the further problem about how to resolve distributional conflicts between generations that go beyond the sustainability question. Second, it has been argued that the Suppes-Sen grading principle cannot capture impartiality among an infinite and countable number of generations in a satisfactory manner (Liedekerke and Lauwers [20]). In the following, we consider both these problems.

In Sections 3–5, we go beyond the sustainability question by introducing conditions on (strict) preference that turn out to bring the infinite intergenerational setting into line with the framework for distributive justice in the finite setting. These conditions capture the idea that one infinite utility path should be considered strictly better than another path if the head of the former is considered strictly better than the head of the latter at infinitely many truncating times. Within this framework, we show how equity conditions well-known from the finite setting can be applied to the debate on infinite intergenerational justice. Moreover, we provide characterizations of intergenerational versions of leximin and utilitarianism.

In Section 6, we consider the possibility of extending impartiality among an infinite number of generations through the idea that one infinite utility path should be considered indifferent to another path if the head of the former is considered indifferent to the head of the latter at infinitely many truncating times. It turns out, however, that this approach does not move us beyond Weak Anonymity unless we are ready to reject Strong Pareto.

The formal framework – including the conditions of Weak Anonymity and Strong Pareto – are introduced in Section 2.

2 The framework

There is an infinite number of generations $t = 1, 2, \dots$. The utility level of generation t is given by u_t , which should be interpreted as the utility level of a representative member of this generation.

A binary relation R over paths ${}_1\mathbf{u} = (u_1, u_2, \dots)$ starting in period 1 expresses social preferences over different intergenerational utility paths. Any such binary relation R is throughout assumed to be reflexive and transitive on the countably infinite Cartesian product $\mathbb{R}^\infty$ of the set of real numbers $\mathbb{R}$. The social preferences R may be complete or incomplete, with I denoting the symmetric part, i.e. indifference, and P denoting the asymmetric part, i.e. (strict) preference. For any utility path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and any time T , ${}_1\mathbf{u}_T = (u_1, u_2, \dots, u_T)$ denotes the truncation of ${}_1\mathbf{u}$ at T , and ${}_1\tilde{\mathbf{u}}_T$ is a permutation of ${}_1\mathbf{u}_T$ having the property that ${}_1\tilde{\mathbf{u}}_T$ is non-decreasing. Refer to ${}_1\mathbf{u}_T$ as the T -head and ${}_{T+1}\mathbf{u}$ as the T -tail of ${}_1\mathbf{u}$.

A path ${}_1\mathbf{v}$ weakly Pareto-dominates another path ${}_1\mathbf{u}$ if every generation is weakly better off in ${}_1\mathbf{v}$ than in ${}_1\mathbf{u}$ and some generation is strictly better off. Assume a technology determining a set of feasible paths. A feasible path ${}_1\mathbf{v}$ is said to be efficient if there is no other feasible path that weakly Pareto-dominates it. A feasible path ${}_1\mathbf{v}$ is said to be R -maximal, if there exists no feasible path ${}_1\mathbf{u}$ such that ${}_1\mathbf{u} P {}_1\mathbf{v}$. A feasible path ${}_1\mathbf{v}$ is said to be R -optimal, if ${}_1\mathbf{v} R {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$. Any R -optimal path is R -maximal, while the converse need not hold if R is incomplete.

The following two conditions are imposed on the social preferences.

Condition SP (Strong Pareto). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if $v_t \geq u_t$ for all t and $v_s > u_s$ for some s , then ${}_1\mathbf{v} P {}_1\mathbf{u}$.

Condition WA (Weak Anonymity). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if for some finite permutation π , $v_{\pi(t)} = u_t$ for all t , then ${}_1\mathbf{v} I {}_1\mathbf{u}$.¹

The *Suppes-Sen grading principle* R^S deems two paths to be indifferent if one is obtained from the other through a finite permutation, and one utility path to be preferred to another if a finite permutation of the former weakly Pareto-dominates the other. The binary relation R^S is a subrelation² to the social preferences R if and only if R satisfies SP and WA.

3 Preference continuity

The relation generated by SP and WA – the Suppes-Sen grading principle, R^S – is incomplete. In the following three sections, we pose the problem: how to resolve distributional conflicts between generations when comparing paths that are R^S -maximal, by extending (strict) preference beyond what the Suppes-Sen grading principle entails. We will impose conditions that establish a link to the standard finite setting of distributive justice, by transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a *finite* number of generations. We may then apply well-known equity conditions from the traditional literature on distributive justice. There are two options.³

Condition WPC (Weak Preference Continuity). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$, then ${}_1\mathbf{v} P {}_1\mathbf{u}$.

Condition SPC (Strong Preference Continuity). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R {}_1\mathbf{u}$, and $\forall \hat{T} \geq 1$, $\exists T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$, then ${}_1\mathbf{v} P {}_1\mathbf{u}$.

These conditions can alternatively be formulated as follows. Write

$$\mathcal{R} := \bigcup_{\hat{T}=1}^{\infty} \bigcap_{T=\hat{T}}^{\infty} \{({}_1\mathbf{v}, {}_1\mathbf{u}) \mid ({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R {}_1\mathbf{u}\}.$$

¹ A permutation, i.e., a bijective mapping of $\{1, 2, \dots\}$ onto itself, is finite whenever there is a T such that $\pi(t) = t$ for any $t > T$.

² R' is said to be a *subrelation* to R'' if (i) ${}_1\mathbf{v} I' {}_1\mathbf{u}$ implies ${}_1\mathbf{v} I'' {}_1\mathbf{u}$ and (ii) ${}_1\mathbf{v} P' {}_1\mathbf{u}$ implies ${}_1\mathbf{v} P'' {}_1\mathbf{u}$, with I' and I'' and P' and P'' denoting the symmetric and asymmetric parts of R' and R'' , respectively.

³ This approach is in spirit related to the concepts of ‘overtaking’ (Atsumi [3]; von Weizsäcker [21]) and ‘agreeable plans’ (Hammond and Mirrlees [13]; Hammond [10]). WPC is implied by Axioms 3 and 4 of Brock [7]. WPC and SPC are different from Koopmans et al.’s [14] conditions of weak and strong time perspective, which involve the evolution of welfare differences between $({}_1\mathbf{w}_T, {}_{T+1}\mathbf{v}^T)$ and $({}_1\mathbf{w}_T, {}_{T+1}\mathbf{u}^T)$ when T increases, where for all $T, t \geq 1$, $u_{T+t}^T = u_t$ and $v_{T+t}^T = v_t$.

If R is complete for comparisons between paths having the same tail, then $\mathcal{R}$ denotes the set pairs $({}_1\mathbf{v}, {}_1\mathbf{u})$ satisfying that beyond some $\hat{T}$ there exists no T such that ${}_1\mathbf{u}$ is preferred to $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u})$. Write $\mathcal{P}_T := \{({}_1\mathbf{v}, {}_1\mathbf{u}) \in \mathcal{R} \mid ({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}\}$ and $\mathcal{P}_\infty := \{({}_1\mathbf{v}, {}_1\mathbf{u}) \mid {}_1\mathbf{v} P {}_1\mathbf{u}\}$. Then WPC means that $\liminf$ of the sequence $\mathcal{P}_T$ is included in $\mathcal{P}_\infty$,

$$\bigcup_{\hat{T}=1}^{\infty} \bigcap_{T=\hat{T}}^{\infty} \mathcal{P}_T \subseteq \mathcal{P}_\infty,$$

while SPC means that $\limsup$ of the sequence $\mathcal{P}_T$ is included in $\mathcal{P}_\infty$,

$$\bigcap_{\hat{T}=1}^{\infty} \bigcup_{T=\hat{T}}^{\infty} \mathcal{P}_T \subseteq \mathcal{P}_\infty.$$

In the following we illustrate how the conditions of WPC and SPC can be used to characterize the intergenerational versions of the Rawlsian leximin principle and the utilitarian principle. All binary relations considered (in Definitions 1–4) are still incomplete. However, it follows from Svensson's [18] Theorem 2 that there exist completions of these binary relations.⁴

4 Characterizing leximin

The Rawlsian leximin principle has been stated as follows in the infinite case (see, e.g., Asheim [1, p. 355]), where “S” indicates that R_S^L will be shown to correspond to the *Strong* Preference Continuity:

Definition 1 (S-Leximin). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} R_S^L {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, either ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ or there is a $s \in \{1, \dots, T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

Alternatively, there is a weaker formulation of leximin, R_W^L , that will be shown to correspond to the *Weak* Preference Continuity.

Definition 2 (W-Leximin). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} R_W^L {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$, and ${}_1\mathbf{v} P_W^L {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, there is a $s \in \{1, \dots, T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

We start out by characterizing R_S^L . It is well-known that the leximin principle covering finite cases can be characterized by the Suppes-Sen grading principle and the equity condition suggested by Hammond [11, 12].

Condition HE (Hammond Equity). If ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist j, k such that $u_j > v_j > v_k > u_k$ and $u_t = v_t$ for all $t \neq j, k$, then ${}_1\mathbf{v} R {}_1\mathbf{u}$.

⁴ Svensson [18] invokes Szpilrajn's [19] Lemma, which is non-constructive. It is an open question whether one can *construct* a complete binary relation satisfying SP and WA; cf. Fleurbaey and Michel [9]. Basu and Mitra [5] show that one cannot represent a complete binary relation satisfying SP and WA by a SWF.

It is not straightforward to translate this result into the infinite case.⁵ However, by applying SPC, we obtain the following characterization.

Proposition 1. R_S^L is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Strong Preference Continuity.

Lemma 1. Assume that R satisfies Weak Anonymity. For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$, then ${}_1\mathbf{v} I {}_1\mathbf{u}$.

Proof. For all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_{T+1} = {}_1\tilde{\mathbf{u}}_{T+1}$, implying $v_{T+1} = u_{T+1}$. Hence, $\hat{T}_{+1}\mathbf{v} = \hat{T}_{+1}\mathbf{u}$. By WA, ${}_1\mathbf{v} I {}_1\mathbf{u}$. $\square$

Poof of Proposition 1. (If) Assume that R satisfies SP, WA, HE, and SPC. According to the definition of a subrelation (cf. Footnote 2), we have to show that, for any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} I_S^L {}_1\mathbf{u}$ implies ${}_1\mathbf{v} I {}_1\mathbf{u}$ and ${}_1\mathbf{v} P_S^L {}_1\mathbf{u}$ implies ${}_1\mathbf{v} P {}_1\mathbf{u}$. This divides the if part of the proof into two subparts.

(1) Consider any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ such that ${}_1\mathbf{v} I_S^L {}_1\mathbf{u}$. By definition of R_S^L , $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. By WA and Lemma 1, ${}_1\mathbf{v} I {}_1\mathbf{u}$.

(2) Consider any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ such that ${}_1\mathbf{v} P_S^L {}_1\mathbf{u}$. For T satisfying ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$, it follows from WA that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}$. For T satisfying that there is a $s \in \{1, \dots, T\}$ such that $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$, we can construct a T -head ${}_1\hat{\mathbf{u}}_T$ by means of a sequence of steps involving conflicts between two generations. In particular, let

$${}_1\mathbf{u}_T^n = \begin{cases} {}_1\tilde{\mathbf{u}}_T & \text{for } n = 0 \\ ({}_1\tilde{\mathbf{u}}_{s-1}, u_s^n, {}_{s+1}\tilde{\mathbf{w}}_{s+n}, {}_{s+n+1}\tilde{\mathbf{u}}_T) & \text{for } n = 1, \dots, T-s-1 \\ ({}_1\tilde{\mathbf{u}}_{s-1}, u_s^n, {}_{s+1}\tilde{\mathbf{w}}_T) = {}_1\hat{\mathbf{u}}_T & \text{for } n = T-s, \end{cases}$$

where, for $n = 1, \dots, T-s$, $u_s^n = \tilde{u}_s + n(\tilde{v}_s - \tilde{u}_s)/(T-s+1)$ and, for $t = s+1, \dots, T$, $\tilde{w}_t = \min\{\tilde{u}_t, \tilde{v}_t\}$. Then, for $n = 1, \dots, T-s$,

$$({}_1\mathbf{u}_T^n, {}_{T+1}\mathbf{u}) R ({}_1\mathbf{u}_T^{n-1}, {}_{T+1}\mathbf{u})$$

by HE if $\tilde{v}_{s+n} < \tilde{u}_{s+n}$ (since $\tilde{u}_{s+n} > \tilde{w}_{s+n} > u_s^n > u_s^{n-1}$) and SP if $\tilde{v}_{s+n} \geq \tilde{u}_{s+n}$ (since $\tilde{u}_{s+n} = \tilde{w}_{s+n}$ and $u_s^n > u_s^{n-1}$). Hence, by transitivity,

$$({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u}) R ({}_1\tilde{\mathbf{u}}_T, {}_{T+1}\mathbf{u}).$$

Since R satisfies SP, it follows that $({}_1\tilde{\mathbf{v}}_T, {}_{T+1}\mathbf{u}) P ({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u})$, while WA implies that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) I ({}_1\tilde{\mathbf{v}}_T, {}_{T+1}\mathbf{u})$ and $({}_1\tilde{\mathbf{u}}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}$. Hence, by transitivity, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$. This shows, by the definition of R_S^L , that $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R {}_1\mathbf{u}$, and $\forall \hat{T} \geq 1$, $\exists T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$. Since R satisfies SPC, it now follows that ${}_1\mathbf{v} P {}_1\mathbf{u}$.

(Only if) Assume that R_S^L is a subrelation to R . It is trivial to establish that R satisfies SP, WA, and HE. To show that R satisfies SPC, assume that $\exists \hat{T}$

⁵ Lauwers [15] characterizes the maximin relation by a version of Hammond Equity within a framework where Strong Pareto is relaxed.

≥ 1 such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R {}_1\mathbf{u}$, and $\forall \hat{T} \geq 1$, $\exists T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$. Since R_S^L is a subrelation to R and R_S^L is complete for comparisons between paths having the same tail, this implies that $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R_S^L {}_1\mathbf{u}$, and $\forall \hat{T} \geq 1$, $\exists T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P_S^L {}_1\mathbf{u}$. By definition of R_S^L , this entails that ${}_1\mathbf{v} P_S^L {}_1\mathbf{u}$, which in turn implies ${}_1\mathbf{v} P {}_1\mathbf{u}$ since R_S^L is a subrelation to R . Thus, we have established that R satisfies SPC. $\square$

This result deals with an objection to the Rawlsian leximin position – that the leximin principle is implausible because it assigns *absolute* priority to the interests of the *worst off* generation in cases where it is in conflict with the interest of an *infinite number of future generations*. Proposition 1 tells us that our view on intergenerational justice can be determined by considering a particular set of two-generation conflicts. If we agree on assigning absolute priority to the worse off in such a conflict, then we have to assign absolute priority to the worse off in general. Hence, our result provides a defense for the leximin principle in the infinite setting since it seems less difficult to accept the two-generation claim.

An analogous result can be established for R_W^L through a trivial modification of the parts of the proof of Prop. 1 that involve SPC.

Proposition 2. R_W^L is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity.

We must impose an assumption on the technological framework in order to ensure that there exists a maximal path according to R_S^L (and thus R_W^L , since R_W^L is a subrelation to R_S^L). This provides the following complete justification for an egalitarian approach to intergenerational justice.

Proposition 3. Assume a technology determining a set of feasible paths. If there exists a feasible and efficient path ${}_1\mathbf{v}$ with constant utility, and R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity, then ${}_1\mathbf{v}$ is the unique R -optimal path.

Proof. Any alternative feasible path ${}_1\mathbf{u}$ provides at least one generation with lower utility than does ${}_1\mathbf{v}$. Hence, ${}_1\mathbf{v} P_W^L {}_1\mathbf{u}$ for any other feasible path. It follows from Prop. 2 that R_W^L is a subrelation to R . Therefore, ${}_1\mathbf{v} P {}_1\mathbf{u}$ for any other feasible path, implying that ${}_1\mathbf{v}$ is the unique R -optimal path. $\square$

It follows that the feasible and efficient path with constant utility is preferred to any other feasible path according to any binary relation to which R_W^L is a subrelation; in particular, this holds for R_S^L . Hence, the egalitarian path is the unique optimal path also under the stronger version of leximin.

5 Characterizing utilitarianism

The utilitarian overtaking criterion, introduced by Atsumi [3] and von Weizsäcker [21], represents an important alternative approach to intergenerational justice. As with leximin, there are two versions to consider.

Definition 3 (Catching Up). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} R_S^U {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $\sum_{t=1}^T v_t \geq \sum_{t=1}^T u_t$.

Definition 4 (Overtaking). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} I_W^U {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $\sum_{t=1}^T v_t = \sum_{t=1}^T u_t$, and ${}_1\mathbf{v} P_W^U {}_1\mathbf{u}$ holds if $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $\sum_{t=1}^T v_t > \sum_{t=1}^T u_t$.

As an illustration, compare ${}_1\mathbf{v} = (2, 0, 2, 0, \dots)$ and ${}_1\mathbf{u} = (1, 1, 1, 1, \dots)$. Here ${}_1\mathbf{v} P_S^U {}_1\mathbf{u}$ since ${}_1\mathbf{u}$ never catches up with ${}_1\mathbf{v}$, while the utility paths are incomparable according to R_W^U since ${}_1\mathbf{v}$ never overtakes ${}_1\mathbf{u}$. Atsumi [3, p. 128] and von Weizsäcker [21, p. 85] define optimality by catching up (i.e., ${}_1\mathbf{v}$ is optimal if ${}_1\mathbf{v} R_S^U {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$), while von Weizsäcker defines preference by overtaking (i.e., ${}_1\mathbf{v}$ is preferred to ${}_1\mathbf{u}$ if ${}_1\mathbf{v} P_W^U {}_1\mathbf{u}$).

To provide characterizations of these utilitarian criteria, we appeal to a weak two-generation version of an invariance condition.⁶

Condition 2UC (2-Generation Unit Comparability). For any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, if ${}_1\mathbf{v} R {}_1\mathbf{u}$ and there exist j, k and $(a_j, a_k) \in \mathbb{R}^2$ such that $\hat{u}_j = u_j + a_j$, $\hat{v}_j = v_j + a_j$, $\hat{u}_k = u_k + a_k$, $\hat{v}_k = v_k + a_k$, and $\hat{u}_t = u_t$ and $\hat{v}_t = v_t$ for all $t \neq j, k$, then ${}_1\hat{\mathbf{v}} R {}_1\hat{\mathbf{u}}$.

Lemma 2. Assume that R satisfies Weak Anonymity and 2-Generation Unit Comparability. If ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist j, k such that $u_j - v_j = v_k - u_k$ and $u_t = v_t$ for all $t \neq j, k$, then ${}_1\mathbf{v} I {}_1\mathbf{u}$.

Proof. Set $a_j = -v_j$ and $a_k = -u_k$ and form ${}_1\hat{\mathbf{u}}$ and ${}_1\hat{\mathbf{v}}$ as follows: $\hat{u}_j = u_j + a_j$, $\hat{v}_j = v_j + a_j$, $\hat{u}_k = u_k + a_k$, $\hat{v}_k = v_k + a_k$, and $\hat{u}_t = u_t$ and $\hat{v}_t = v_t$ for all $t \neq j, k$. Clearly, $\hat{u}_k = \hat{v}_j = 0$ and, since $u_j - v_j = v_k - u_k$, $\hat{u}_j = \hat{v}_k$, while $\hat{u}_t = \hat{v}_t$ for all $t \neq j, k$. By WA, ${}_1\hat{\mathbf{v}} I {}_1\hat{\mathbf{u}}$, and by 2UC, ${}_1\mathbf{v} I {}_1\mathbf{u}$. $\square$

By applying Lemma 2 we overcome an objection to the catching up and overtaking criteria, namely that these criteria allow a large number of smaller gains for many generations to outweigh a greater loss for a single generation. The following results show that this is only a consequence of considering two-generation conflicts where one generation's gain equals the others loss.

Proposition 4. R_S^U is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, 2-Generation Unit Comparability, and Strong Preference Continuity.

Lemma 3. Assume that R satisfies the following condition: If ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ satisfy that there exist j, k such that $u_j - v_j = v_k - u_k$ and $u_t = v_t$ for all $t \neq j, k$, then ${}_1\mathbf{v} I {}_1\mathbf{u}$. Then it holds that for any ${}_1\mathbf{u}$, $({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}$ whenever $\hat{u}_s = \sum_{t=1}^T u_t/T$ for all $1 \leq s \leq T$.

⁶ See Sen [17], d'Aspremont and Gevers [8], Roberts [16], Basu [4], and in the infinite setting, Basu and Mitra [6].

Proof. We can construct the egalitarian T -head ${}_1\hat{\mathbf{u}}_T$ by means of a sequence of steps involving conflicts between two generations. In particular, let

$${}_1\mathbf{u}_T^n = \begin{cases} {}_1\mathbf{u}_T & \text{for } n = 0 \\ ({}_1\hat{\mathbf{u}}_n, \tilde{u}_{n+1}, {}_{n+2}\mathbf{u}_T) & \text{for } n = 1, \dots, T-2 \\ ({}_1\hat{\mathbf{u}}_n, \tilde{u}_T) = {}_1\hat{\mathbf{u}}_T & \text{for } n = T-1, \end{cases}$$

where, for $n = 1, \dots, T-1$, $\tilde{u}_{n+1} = \sum_{t=1}^{n+1} u_t - \sum_{t=1}^n \hat{u}_t$. Then it follows by the premise of the lemma that $({}_1\mathbf{u}_T^n, {}_{T+1}\mathbf{u}) I ({}_1\mathbf{u}_T^{n-1}, {}_{T+1}\mathbf{u})$ for $n = 1, \dots, T-1$. Hence, by transitivity, $({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}$. $\square$

Proof of Proposition 4. (If) Assume that R satisfies SP, WA, 2UC, and SPC. According to the definition of a subrelation (cf. Footnote 2), we have to show that, for any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v} I_S^U {}_1\mathbf{u}$ implies ${}_1\mathbf{v} I {}_1\mathbf{u}$ and ${}_1\mathbf{v} P_S^U {}_1\mathbf{u}$ implies ${}_1\mathbf{v} P {}_1\mathbf{u}$. This divides the if part of the proof into two subparts.

(1) Consider any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ such that ${}_1\mathbf{v} I_S^U {}_1\mathbf{u}$. By definition of R_S^U , $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $\sum_{t=1}^T v_t = \sum_{t=1}^T u_t$ and $\sum_{t=1}^{T+1} v_t = \sum_{t=1}^{T+1} u_t$, implying $v_{T+1} = u_{T+1}$. Hence, ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$. Let, for all $1 \leq s \leq \hat{T}$, $\hat{u}_s = \hat{v}_s = \sum_{t=1}^{\hat{T}} u_t / \hat{T} = \sum_{t=1}^{\hat{T}} v_t / \hat{T}$. By WA, 2UC, and Lemmas 2 and 3,

$$({}_1\mathbf{v}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u}) I ({}_1\hat{\mathbf{v}}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u}) \text{ and } ({}_1\hat{\mathbf{u}}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u}) I {}_1\mathbf{u}.$$

Since ${}_1\hat{\mathbf{v}}_{\hat{T}} = {}_1\hat{\mathbf{u}}_{\hat{T}}$ and ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$, it follows by transitivity that ${}_1\mathbf{v} I {}_1\mathbf{u}$.

(2) Consider any ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$ such that ${}_1\mathbf{v} P_S^U {}_1\mathbf{u}$. For T satisfying $\sum_{t=1}^T v_t = \sum_{t=1}^T u_t$, it follows, by adapting subpart (1), that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}$. For T satisfying that $\sum_{t=1}^T v_t > \sum_{t=1}^T u_t$, let, for all $1 \leq s \leq T$, $\hat{u}_s = \sum_{t=1}^T u_t / T$ and $\hat{v}_s = \sum_{t=1}^T v_t / T$. By WA, 2UC, and Lemmas 2 and 3,

$$({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) I ({}_1\hat{\mathbf{v}}_T, {}_{T+1}\mathbf{u}) \text{ and } ({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u}) I {}_1\mathbf{u}.$$

Since R satisfies SP, it follows that $({}_1\hat{\mathbf{v}}_T, {}_{T+1}\mathbf{u}) P ({}_1\hat{\mathbf{u}}_T, {}_{T+1}\mathbf{u})$. Hence, by transitivity, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$. This shows, by the definition of R_S^U , that $\exists \hat{T} \geq 1$ such that $\forall T \geq \hat{T}$, $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) R {}_1\mathbf{u}$, and $\forall \hat{T} \geq 1$, $\exists T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) P {}_1\mathbf{u}$. Since R satisfies SPC, it now follows that ${}_1\mathbf{v} P {}_1\mathbf{u}$.

(Only if) Assume that R_S^U is a subrelation to R . It is trivial to establish that R satisfies SP, WA, and 2UC. Arguments similar to those used in the only-if part of the proof of Prop. 1 establish that R satisfies SPC. $\square$

An analogous result can be established for R_W^U through a trivial modification of the parts of the proof of Prop. 4 that involve SPC.

Proposition 5. R_W^U is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, 2-Generation Unit Comparability, and Weak Preference Continuity.

It is more difficult to establish conditions that guarantee that there exists an optimal (or maximal) path according to the catching up and overtaking criteria, and we leave such a task for another occasion.

6 Intergenerational impartiality

According to Liedekerke and Lauwers [20, p. 163], formal impartiality is ensured by imposing the axiom of Strong Anonymity (entailing indifference to any permutation of utilities of an *infinite* number of generations). As the following example shows, this demand cannot be combined with SP: ${}_1\mathbf{v} = (1, 0, 1, 0, 1, 0, \dots)$ can be attained from ${}_1\mathbf{u} = (0, 0, 1, 0, 1, 0, \dots)$ by a permutation where generation 2 gets the utility of generation 1, generation t the utility of generation $t+2$ when t is an odd number, and generation $t+2$ the utility of generation t when t is an even number.

Liedekerke and Lauwers suggest to establish a framework where an acceptable trade-off between the demands of impartiality and SP can be made. This implies a rejection of the Suppes-Sen grading principle, which is characterized by WA (entailing indifference to any permutation of utilities of only a *finite* number of generations) and SP. One might think that it should be possible to find some intermediate position, where impartiality is extended beyond WA (by entailing indifference to *some* – but not all – permutations of an infinite number of generations) within a framework satisfying SP, and hence, the Suppes-Sen grading principle.

However, if one – in analogy with WPC – imposes that an infinite utility path should be considered indifferent to another infinite utility path if the head of the former is considered indifferent to the latter *at every point in time* beyond a certain initial phase, then it follows from Lemma 1 that no extension of WA is obtained.

On the other hand, if one – in analogy with SPC – imposes that an infinite utility path should be considered indifferent to another infinite utility path if, at *any* point in time, there is a *future* point in time at which the head of the former is considered indifferent to the latter, then it is not possible to establish an equivalence relation without coming in conflict with SP: If ${}_1\mathbf{u} = (0, 0, 1, 0, 1, 0, \dots)$, ${}_1\mathbf{v} = (1, 0, 1, 0, 1, 0, \dots)$, and ${}_1\mathbf{w} = (0, 1, 0, 1, 0, 1, \dots)$, then such strong indifference continuity implies ${}_1\mathbf{v} I {}_1\mathbf{w}$ and ${}_1\mathbf{w} I {}_1\mathbf{u}$, and by transitivity, ${}_1\mathbf{v} I {}_1\mathbf{u}$. This contradicts SP.

These difficulties, associated with extending indifference beyond WA without coming into conflict with SP, illustrate the problem of constructing a complete binary relation satisfying SP and WA (cf. Footnote 4).

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