Lecture IV: Effective Interactions for Various Mass Areas

Morten Hjorth-Jensen[1,2]

[1]Department of Physics and Center of Mathematics for Applications
University of Oslo, N-0316 Oslo

[2]Department of Physics and Astronomy, Michigan State University
East Lansing, Michigan, USA

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Outline

1. Repetition and Aims
2. *sd*-shell
3. *sd* – *fp*-shells
4. *fp* – *sdg*-shells
5. ^{100}Sn and ^{132}Sn

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2. *sd*-shell
3. *sd* – *fp*-shells
4. *fp* – *sdg*-shells
5. ^{100}Sn and ^{132}Sn

Aims

Various Effective Interactions

1. Derive an effective interaction for the *sd*-shell.
2. Derive an effective interaction for the $sd - fp$-shell.
3. Derive an effective interaction for the $fp - sdg$-shell.
4. Derive an effective interaction for nuclei in the ^{100}Sn and ^{132}Sn mass regions.
5. Finally, these interactions are used as input to the shell-model code.
6. Exercises: derive one or more of these interactions using no-core and $V_{\mathrm{low}-k}$ as methods to renormalize the nucleon-nucleon interaction. G-matrix takes more time. Try to compute eigenvalues of selected nuclei for these mass regions.

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Overall Motivation and Aims

1. At the end of these lectures you should be able to run codes for the effective interaction and and perform test calculations for different mass regions.
2. Understand how nuclear spectra evolve from the underlying nuclear interactions.
3. Critical assessment of the methods, pros and cons and their limitations.

Coupled Cluster, Addendum to Yesterday's Lecture

A comparison of the binding energies per particle for ^{15}O and ^{15}N (the PR-EOMCCSD values), ^{16}O (the CCSD values), and ^{17}O and ^{17}F obtained with the N^{3}LO, CD-Bonn, and V_{18} potentials, and eight major oscillator shells. All entries are in MeV. For the CD-Bonn and N^{3}LO interactions, we used $\hbar\Omega = 11$ MeV. For V_{18}, we used $\hbar\Omega = 10$ MeV. For eight major shells $\beta_{\mathrm{CoM}} = 0.0$.

	Interaction			
Nucleus	N^{3}LO	CD-Bonn	V_{18}	Expt
^{15}O	6.643	7.584	5.246	7.464
^{15}N	6.824	7.751	5.414	7.699
^{16}O	7.406	8.327	5.897	7.976
^{17}O	7.150	8.032	5.617	7.751
^{17}F	6.987	7.879	5.462	7.542

Every NN interaction needs its own three-body force!

Coupled Cluster, Addendum to Yesterday's Lecture

A comparison of the energies of the low-lying excited states of ^{15}O, ^{15}N, ^{17}O and ^{17}F, relative to the corresponding ground-state energies.

	Interaction			
Excited state	N^{3}LO	CD-Bonn	V_{18}	Expt
^{15}O $(3/2)^-_1$	6.264	7.351	4.452	6.176
^{15}N $(3/2)^-_1$	6.318	7.443	4.499	6.323
^{17}O $(3/2)^+_1$	5.675	6.406	3.946	5.084
^{17}O $(1/2)^+_1$	-0.025	0.311	-0.390	0.870
^{17}F $(3/2)^+_1$	5.891	6.677	4.163	5.000
^{17}F $(1/2)^+_1$	0.428	0.805	0.062	0.495

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Repetition from Lecture II: $V_{\mathrm{low}-k}$

The $A-$body Hamiltonian H is defined as

$$H = \frac{1}{2m}\sum_{i=1}^{A}\mathbf{k}_i^2 + \sum_{i<j}^{A} V_{\mathrm{low}-\mathrm{k}}(i,j).$$

The spurious center of mass energy is removed by writing the internal kinetic energy as

$$T_{\mathrm{in}} = T - T_{\mathrm{c.m.}} = \left(1-\frac{1}{A}\right)\sum_{i=1}^{A}\frac{\mathbf{k}_i^2}{2m} - \sum_{i<j}^{A}\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}.$$

The introduction of an additional two-body term yields a modified two-body interaction

$$H_{\mathrm{I}} = V_{\mathrm{low}-\mathrm{k}} + V_{\mathrm{c.m.}} = \sum_{i<j}^{A}\left(V_{\mathrm{low}-\mathrm{k}}(i,j) - \frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}\right).$$

This interaction is in turn written out in terms of harmonic oscillator elements.

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Repetition from Lecture II: G-matrix

The $A-$body Hamiltonian H is defined as for the $V\mathrm{low}_k$ case

$$H = \frac{1}{2m}\sum_{i=1}^{A}\mathbf{k}_i^2 + \sum_{i<j}^{A} G(i,j).$$

The spurious center of mass energy is removed by writing the internal kinetic energy as

$$T_{\mathrm{in}} = T - T_{\mathrm{c.m.}} = \left(1-\frac{1}{A}\right)\sum_{i=1}^{A}\frac{\mathbf{k}_i^2}{2m} - \sum_{i<j}^{A}\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}.$$

The modified two-body interaction

$$H_{\mathrm{I}} = G + V_{\mathrm{c.m.}} = \sum_{i<j}^{A}\left(G(i,j) - \frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}\right).$$

This interaction is in turn written out in terms of harmonic oscillator elements.

Both the G-matrix codes and the $V_{\mathrm{low}-k}$ codes list separately G or $V_{\mathrm{low}-k}$ in addition to the term $\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}$. The last term has to be multiplied by $\hbar\omega/A$ in order to be used in derivations of the effective interaction.

etgray0 g 0 G0.5**fpsdg** 0.5 setgray0 g 0 G0.5**Sn**

Repetition from Lecture II: No-core

The total Hamiltonian is

$$H_P^\omega = \sum_{i=1}^{A} P\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\omega^2\vec{r}_i^{\,2}\right]P + \sum_{i<j}^{A} P\left[V_{ij} - \frac{m\omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right]_{\rm eff} P$$

$$-PH_{\rm CoM}P.$$

The two-body part of the center-of-mass Hamiltonian is listed separately and needs to be multiplied by $\hbar\omega/A$. Since we only give the two-body part, you need to add the Harmonic oscillator single-particle energies to this part and multiply the harmonic oscillator single-particle energies with $\hbar\omega/A$ as well.

Note that the no-core Hamiltonian depends explicitely on the mass number A. The G-matrix and $V_{\rm low-}k$ include only a mass dependence via the term $\frac{\mathbf{k}_i\cdot\mathbf{k}_j}{mA}$. The Coulomb interaction is included in all models.

Two-body Matrix Elements I

The renormalized nucleon-nucleon interaction in an arbitrary two-particle basis in the laboratory frame is given by

$$\langle ab|H_{\rm I}|cd\rangle = \langle (n_a l_a j_a t_{z_a})(n_b l_b j_b t_{z_b}) JT_z|\, H_{\rm I}\, |(n_c l_c j_c t_{z_c})(n_d l_d j_d t_{z_d}) JT_z\rangle\,.$$

Here $H_{\rm I}$ can be a G-matrix, it can be a no-core or Vlowk interaction. The two-body state $|ab\rangle$ is implicitly coupled to good angular momentum J. The labels $n_{a\ldots d}$ number all bound, resonant and discretized scattering states with orbital and angular momenta $(l_{a\ldots d}, j_{a\ldots d})$. Here these single-particle states will be the Hartree-Fock states.
In order to efficiently calculate the matrix elements, we introduce a two-particle harmonic oscillator basis completeness relation

$$\sum_{\alpha\leq\beta} |\alpha\beta\rangle\langle\alpha\beta| = \mathbf{1},$$

where the sum is not restricted in the neutron-proton case. We introduce the greek single particle labels α, β for the single-particle harmonic oscillator states in order to distinguish them from the latin single-particle labels Hartree-Fock states a, b

Two-body Matrix Elements II

The interaction can then be expressed in the complete basis is

$$H_{\mathrm{I}} = \sum_{\alpha \leq \beta} \sum_{\gamma \leq \delta} |\alpha\beta\rangle \langle \alpha\beta | H_{\mathrm{I}} | \gamma\delta \rangle \langle \gamma\delta |,$$

where the sums over two-particle harmonic oscillator states are infinite. The expansion coefficients

$$\langle \alpha\beta | H_{\mathrm{I}} | \gamma\delta \rangle = \left\langle (n_\alpha l_\alpha j_\alpha t_{z_\alpha})(n_\beta l_\beta j_\beta t_{z_\beta}) JT_z \right| H_{\mathrm{I}} \left| (n_\gamma l_\gamma j_\gamma t_{z_\gamma})(n_\delta l_\delta j_\delta t_{z_\delta}) JT_z \right\rangle,$$

represent the interaction H_{I} in an antisymmetrized two-particle harmonic oscillator basis, and may easily be calculated using the well known Moshinsky transformation coefficients.

Two-body Matrix Elements III

The matrix elements are calculated numerically up to N harmonic oscillator two-body states

$$\langle ab|H_{\rm I}|cd\rangle = \sum_{\alpha\leq\beta}^{N}\sum_{\gamma\leq\delta}^{N}\langle ab|\alpha\beta\rangle\langle\alpha\beta|H_{\rm I}|\gamma\delta\rangle\langle\gamma\delta|cd\rangle.$$

The two-particle overlap integrals $\langle ab|\alpha\beta\rangle$ read

$$\langle ab|\alpha\beta\rangle = \frac{\langle a|\alpha\rangle\langle b|\beta\rangle - (-1)^{J-j_\alpha-j_\beta}\langle a|\beta\rangle\langle b|\alpha\rangle}{\sqrt{(1+\delta_{ab})(1+\delta_{\alpha\beta})}}$$

for identical particles (proton-proton or neutron-neutron states) and

$$\langle ab|\alpha\beta\rangle = \langle a|\alpha\rangle\langle b|\beta\rangle$$

for the proton-neutron case. The single-particle overlaps are calculated in the Hartree-Fock code.

sd-shell: Using the $V_{\mathrm{low}-k}$ Part, Step 1

- The first step is to set up the input file for the renomalization of the nucleon-nucleon interaction. We should choose around six to eight major shells. We can generate a square, triangular or wing model space.
- If we want to use a Hartree-Fock basis, we should use a model space with wings so that we can perform a Hartree-Fock calculation with sufficient number of nodes for the single-particle states.
- Choose an oscillator parameter that is relevant for your closed shell core $A = 16$,

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3},$$

  which gives roughly $\hbar\omega = 14$ MeV and an oscillator length $b = 1.72$ fm.
- We choose a maximum l for the single-particle states depending on the number of shells we want.
- We choose a maximum n for the single-particle states so that we can obtain a stable Hartree-Fock result.

Note that you can replace all the $V_{\mathrm{low}-k}$ calculations with a G-matrix calculation.

Note to step 1

- Note that the code generates as output a listing of single-particle energies. This file can be changed when you want to run calculations of effective interactions.
- Basically you need to fix the model space by fixing $l_{\max}$ and $n_{\max}$ so that $l_{\max} + 2n_{\max} \leq M$, with M a chosen number.
- You need also to fix momenta for the cutoff in momentum space and the maximum momentum. This interaction depends on the choice of cutoff The number of nucleons is not important, you can change that when you perform the calculation of the effective interactions.
- You must also specify the cutoff for the partial waves in the construction of the interaction, set that to $J = 6$.
- You must also define which type of model-space operator you wish to use, square, triangular or with wings.
- Now we are ready for our first calculation. As an example we generate an interaction for six shells with a square model space.

Step 1, Input File for the $V_{\mathrm{low}-k}$ NN Interaction

```
!     File for the similarity transformed V-matrix
!     Used later in HF or effective interaction caclulations
vintlowq52shellsn3lo_14hbar.dat
!     sp data used later in HF or effective interaction caclulat
spdat52shellsn3lo_14hbar.dat
!    Read in # nucleons A and oscillator energy
 16 14
!    The min and maximum value of the partial wave angular momen
 0 6
!    number of mesh points for P and Q k-spaces: nk1, nk2
50 50
!    Read in the limit in fm^-1 on max lab and rel momenta for P
2.1 20
!   type of NN potential, e.g., Idaho-A, Idaho-B, CD-bonn, argon
n3lo
```

Input File for Vlowk Effective Interaction

```
!    read in type of Pauli operator, square, wings or triangular
!    operator you should equal the shells in the model space to
square
!    Read in number of model space max l and n in lab frame
5 2
!    Read in number of hole states for protons defined by
!    strict harmonic oscillator ordering, 1 means 0s, 2: 0s0p3/2
!    4: 0s0p3/20p1/20d5/2  5: 0s0p3/20p1/20d5/20d3/2   etc
!    Examples for 4He use 1, for 16O use 3, 40(48)Ca use 6, 56Ni
  3
!    same for neutrons
  3
```

sd-shell: Using the $V_{\mathrm{low}-k}$ Part, Step 2

Go to the directory *selfcons* and compile the code.

- Now we need to define the model space for the shell-model code.
- Enter the file called *spdat52shellsn3lo_14hbar.dat* and set the single-particle status of the orbits in the *sd*-shell to *inside*.
- Make sure that you have the right hole states and the right mass number A, in our case we generate this interaction for $A = 16$ as closed shell core.
- The term $\frac{\mathbf{k}_i \cdot \mathbf{k}_j}{mA}$ is small then, but there will be a weak dependence.
- Note that the codes follow a nuclear physics convention for isospin projections, $t_z^p = -1/2$ and $t_z^n = 1/2$. Opposite if you like particle physics notation, but then you make changes in the codes. This leads to $T_Z = -1$ for $|pp\rangle$, $T_Z = 0$ for $|pn\rangle$ and $T_Z = 1$ for $|nn\rangle$.

sd-shell: Step 2, the file *spdat52shellsn3lo_14hbar.dat*

Define the model space states as inside. All other single-particle states are outside.

		n	l	2j	2tz	oscillator energy		
Number:	1	0	0	1	-1	0.210000E+02	hole	outside
Number:	2	0	0	1	1	0.210000E+02	hole	outside
Number:	3	0	1	3	-1	0.350000E+02	hole	outside
Number:	4	0	1	3	1	0.350000E+02	hole	outside
Number:	5	0	1	1	-1	0.350000E+02	hole	outside
Number:	6	0	1	1	1	0.350000E+02	hole	outside
Number:	7	0	2	5	-1	0.490000E+02	particle	inside
Number:	8	0	2	5	1	0.490000E+02	particle	inside
Number:	9	0	2	3	-1	0.490000E+02	particle	inside
Number:	10	0	2	3	1	0.490000E+02	particle	inside
Number:	11	1	0	1	-1	0.490000E+02	particle	inside
Number:	12	1	0	1	1	0.490000E+02	particle	inside
Number:	13	0	3	7	-1	0.630000E+02	particle	outside
Number:	14	0	3	7	1	0.630000E+02	particle	outside

Protons have $2t_z = -1$.

sd-shell: Using the $V_{\mathrm{low}-k}$ Part, Step 3

- Now we are almost ready to use the code for the effective interaction.
- We need to set some additional parameters in the input file for the effective interaction code
- Enter the file called *hfin.dat* and specify the files for the interaction and the single-particle data
- Let us look at the input file for the effective interaction program.

sd-shell: Step 3, the file *hfin.dat*

```
!THIS PART SPECIFIES DATA DETERMINING THE EFFECTIVE INTERACTION
!     File for the g-matrix, no-core or vlowk interactions
vintlowq52shellsn3lo_15hbar.dat
!     single-particle data file
spdat52shellsn3lo_15hbar.dat
!    Effective interaction file for 2body after the HF calculation
HFg52n3lo_15hbar.dat
!    HF single-particle data
HF52n3lo_15hbar.dat
!     Read in if you want to do a spherichal
!     HF calculation, 0 no HF, 30-50 is a good
!     Note that your HF calculation includes only first order terms.
100
```

Here you must specify if you want a Hartree-Fock basis or not. The option 0 leaves you with a harmonic oscillator basis.

sd-shell: Step 3, the file *hfin.dat*

Then you must specify that you want a valence-space effective interaction by choosing the option *open-diagrams*. The last step is to define the order of the interaction and how many energies you would like to have when you interpolate the interaction around a given starting energy. To understand the last point we need to go back to lecture III.

```
!     you can choose if you want to compute closed-shells diagrams,
!     valence space effective interaction or coupled cluster effective
!     interactions: core-diagrams, open-diagrams, coupled-cluster
open-diagrams
!     choose the order, first, second or third, HF only first order
second
!    Number of starting energies for interpolation of g-matrix for computing di
11
!    starting energy at which you want to evaluate a given diagram, specifies m
-50.
```

sd-shell: Using the no-core Part, Step 1

- The first step is to set up the input file for the renomalization of the nucleon-nucleon interaction. We should choose again around six to eight major shells. We can generate only a triangular model space.
- If we want to use a Hartree-Fock basis, we should use a model space which is large, that is choose a high value of $n_{\max}$.
- Choose an oscillator parameter that is relevant for your closed shell core $A = 16$,

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3},$$

  which gives roughly $\hbar\omega = 14$ MeV and an oscillator length $b = 1.72$ fm.
- We choose a maximum l for the single-particle states depending on the number of shells we want.
- We choose a maximum n for the single-particle states so that we can obtain a stable Hartree-Fock result.

Step 1, Input File for the no-core NN Interaction

```
!     File for the similarity transformed V-matrix, output for a
vintnocore52shellsn3lo_14hbarA16.dat
!     File for the single-particle data
spdatnocore52shellsn3lo_14hbarA16.dat
!    Read in number of particles of closed shell nucleus and osc
 16  14
!   The min and maximum value of the partial wave j_max (use 0 6
 0  6
!  if 0, no bare interaction, if 1 computes only bare interactio
0
!    type of NN potential, e.g., Idaho-A, Idaho-B, CD-bonn, argo
n3lo
!    Read in max l and n for model space and 2n+l for max space,
 5 2 200
```

sd-shell: Using the no-core Part, Step 2

- Now we need to define the model space for the shell-model code.
- Enter the file called *spdat52shellsn3lo_14hbarA16.dat* and set the single-particle status of the orbits in the *sd*-shell to *inside*.
- Make sure that you have the right hole states.
- The interaction depends explicitely on the mass number A. If you want to change it you need to compute a new interaction.

When you have changed the input file to define what is the model space, repeat step 3 as you did with the $V_{\mathrm{low}-k}$ interaction.

$sd - fp$-shell: Using the $V_{\mathrm{low}-k}$ Part, Step 1

- Again the first step is to set up the input file for the renomalization of the nucleon-nucleon interaction. We should choose at least six to eight major shells. We can generate a square, triangular or wing model space.
- If we want to use a Hartree-Fock basis, we should use a model space with wings so that we can perform a Hartree-Fock calculation with sufficient number of nodes for the single-particle states.
- Choose an oscillator parameter that is relevant for your closed shell core, as an example $A = 16$,

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$$

, which gives roughly $\hbar\omega = 14$ MeV and an oscillator length $b = 1.72$ fm.

- We choose a maximum l for the single-particle states depending on the number of shells we want.
- We choose a maximum n for the single-particle states so that we can obtain a stable Hartree-Fock result.

Note that you can replace all the $V_{\mathrm{low}-k}$ with a G-matrix calculation.

sd – *fp*-shell: Step 2, the file *spdat52shellsn3lo_14hbar.dat*

Define the model space states as inside. All other single-particle states are outside.

```
Number:   1     0     0     1    -1     0.210000E+02  hole        outside
Number:   2     0     0     1     1     0.210000E+02  hole        outside
Number:   3     0     1     3    -1     0.350000E+02  hole        outside
Number:   4     0     1     3     1     0.350000E+02  hole        outside
Number:   5     0     1     1    -1     0.350000E+02  hole        outside
Number:   6     0     1     1     1     0.350000E+02  hole        outside
Number:   7     0     2     5    -1     0.490000E+02  particle    inside
Number:   8     0     2     5     1     0.490000E+02  particle    inside
Number:   9     0     2     3    -1     0.490000E+02  particle    inside
Number:  10     0     2     3     1     0.490000E+02  particle    inside
Number:  11     1     0     1    -1     0.490000E+02  particle    inside
Number:  12     1     0     1     1     0.490000E+02  particle    inside
Number:  13     0     3     7    -1     0.630000E+02  particle    inside
Number:  14     0     3     7     1     0.630000E+02  particle    inside
Number:  15     0     3     5    -1     0.630000E+02  particle    inside
Number:  16     0     3     5     1     0.630000E+02  particle    inside
Number:  17     1     1     3    -1     0.630000E+02  particle    inside
Number:  18     1     1     3     1     0.630000E+02  particle    inside
Number:  19     1     1     1    -1     0.630000E+02  particle    inside
Number:  20     1     1     1     1     0.630000E+02  particle    inside
Number:  21     0     4     9    -1     0.770000E+02  particle    outside
```

sd − *fp*-shell: Using the $V_{\mathrm{low}-k}$ Part, further steps

You can repeat all the steps we did for the *sd* shell in order to obtain an effective interaction. Repeat these steps for the no-core interaction as well.

$fp - sdg$-shell: Using the $V_{\mathrm{low}-k}$ Part, Step 1

- Again the first step is to set up the input file for the renomalization of the nucleon-nucleon interaction. Now you should use at least eight major shells. We can generate a square, triangular or wing model space.
- If we want to use a Hartree-Fock basis, we should use a model space with wings so that we can perform a Hartree-Fock calculation with sufficient number of nodes for the single-particle states.
- Choose an oscillator parameter that is relevant for your closed shell core $A = 40$,

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3},$$

  which gives roughly $\hbar\omega = 10$ MeV.
- We choose a maximum $l = 7$ for the single-particle states depending on the number of shells we want. That is the maximum l for eight shells.
- We choose a maximum n for the single-particle states so that we can obtain a stable Hartree-Fock result. For a square model space we set now max $n = 3$.

Note again that you can replace all the $V_{\mathrm{low}-k}$ with a G-matrix calculation.

fp – *sdg*-shell: Step 2

Number:	12	1	0	1	1	0.490000E+02	hole	outside
Number:	13	0	3	7	-1	0.630000E+02	particle	inside
Number:	14	0	3	7	1	0.630000E+02	particle	inside
Number:	15	0	3	5	-1	0.630000E+02	particle	inside
Number:	16	0	3	5	1	0.630000E+02	particle	inside
Number:	17	1	1	3	-1	0.630000E+02	particle	inside
Number:	18	1	1	3	1	0.630000E+02	particle	inside
Number:	19	1	1	1	-1	0.630000E+02	particle	inside
Number:	20	1	1	1	1	0.630000E+02	particle	inside
Number:	21	0	4	9	-1	0.770000E+02	particle	inside
Number:	22	0	4	9	1	0.770000E+02	particle	inside
Number:	23	0	4	7	-1	0.770000E+02	particle	inside
Number:	24	0	4	7	1	0.770000E+02	particle	inside
Number:	25	1	2	5	-1	0.770000E+02	particle	inside
Number:	26	1	2	5	1	0.770000E+02	particle	inside
Number:	27	1	2	3	-1	0.770000E+02	particle	inside
Number:	28	1	2	3	1	0.770000E+02	particle	inside
Number:	29	2	0	1	-1	0.770000E+02	particle	inside
Number:	30	2	0	1	1	0.770000E+02	particle	inside
Number:	31	0	5	11	-1	0.910000E+02	particle	outside

$fp - sdg$-shell: Using the $V_{\mathrm{low}-k}$ Part, further steps

You can repeat all the steps we did for the sd or $sd - fp$-shells in order to obtain an effective interaction. The same applies to the no-core part. Repeat these steps for the no-core part also.

Shell-Model Studiesof Nuclei around $A = 100$

You should have at least eight major shells. You can repeat all the steps we did either for the no-core interaction or the $V_{\mathrm{low}-k}$ interaction. Same for the G-matrix. Possible model spaces

1. $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{Z-50}$ for proton particles
2. $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{N-50}$ for neutron particles.

Shell-Model Studies of Nuclei around $A = 132$

You should have at least eight major shells. You can repeat all the steps we did either for the no-core interaction or the $V_{\mathrm{low}-k}$ interaction. Possible model spaces

1. $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{Z-50}$ for proton particles
2. $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{N-82}$ for neutron holes.
3. For $N \geq 82$ we need also neutron particle with $(0h_{9/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 0i_{13/2})^{N-82}$

Space for ^{132}Sn, neutron holes and proton particles

Here we want a space which only contains neutron-holes and proton particles

```
Number:   21     0     4     9    -1     0.770000E+02  hole        outside
Number:   22     0     4     9     1     0.770000E+02  hole        outside
Number:   23     0     4     7    -1     0.770000E+02  particle    inside
Number:   24     0     4     7     1     0.770000E+02  hole        inside
Number:   25     1     2     5    -1     0.770000E+02  particle    inside
Number:   26     1     2     5     1     0.770000E+02  hole        inside
Number:   27     1     2     3    -1     0.770000E+02  particle    inside
Number:   28     1     2     3     1     0.770000E+02  hole        inside
Number:   29     2     0     1    -1     0.770000E+02  particle    inside
Number:   30     2     0     1     1     0.770000E+02  hole        inside
Number:   31     0     5    11    -1     0.910000E+02  particle    inside
Number:   32     0     5    11     1     0.910000E+02  hole        inside
Number:   33     0     5     9    -1     0.910000E+02  particle    outside
```

Corresponding Shell-Model Input File

```
\* Shell model calculation of identical particle system -- title
\* The particle number: 8
\* Total angular momentum J is (even, odd): even
\* Twice total projection of angular momentum: 20
\* Total parity (+, -): +
\* The number of particle j-orbits: 5
     <n  l  2*j   min_part  max_part energy:>
\* Orbit:     1  2   5        0          6       1.655
\* Orbit:     2  0   1        0          2       0.332
\* Orbit:     0  4   7        0          8       2.434
\* Orbit:     1  2   3        0          4       0.000
\* Orbit:     0  5  11        0         12       0.065
```

Corresponding Shell-Model Input File

```
<***** Type of interaction ******  value ****>
    <** Two-part   veffJ interaction:            1 **>
    <** Two-part   veffM interaction:            2 **>
    <** Real three-part veffM interaction:       3 **>
    <** Simulated three-part veffM interaction:  4 **>
\* Type of effective interaction: 1
\* Input V-effective matrix element: nn132-3order.dat
\* Center_of_Mass matrix elements included(yes = 1, no = 0): 0
\* If(yes): Filename Center_of_Mass matrix elements:
```

Corresponding Shell-Model Input File, contn.

```
<***************** Type of calculation ****************     Symbo
<** Calculation to find the shell model dimension    --> dimensi
**>
<** Lanczos iteration based on random initial vector --> random-
\* Type of calculation process: random-start
<** Temporary files for storage of nondiag matrix elements and L
<** SMALL CALCULATIONS: appr dimension: |SD_basis_states>  < 500
<** LARGE CALCULATIONS: Specify max file dimension for nondiag <
<**                     and max file dimension for the Lanczos v
\* Max. file size for nondiag <SD|OP|SD> in Gb (small 0 - large
\* Max. file size for lanczos vectors in Gb    (small 0 - large
<** Energy eigenvalue parameters **>
\* Maximum Lanczos iterations (<1000): 100
\* Wanted number of converged eigenstates: 10
```

Exercise: Spectra and BE2 of ^{124}Sn

The aim is to compute the 3-4 lowest lying states with positive parity for ^{124}Sn. With the spectra we compute the relevant $BE2(2_1^+ \rightarrow 0_1^+)$ transition.

- Go to the directory shell_model/identical_particles/shellmodel_code which contains the example input file *sn-Hmodel.dat* and ditto interaction files.
- Compile the codes.
- Change the input file so that you can run for the lowest-lying states by using *lanczo sn-Hmodel.dat*.
- Save the output files. These will be used on the last day to study BE2 transitions.
- Transfer the files to the directory transition code directory and edit the file *sn-trans-input.dat* accordingly. Run thereafter the transition code.