

Categorical Statements and General Conditionals

Rani Lill Anjum
Johan Arnt Myrstad

Frege's interpretation of general categorical statements as general material conditionals is taken as a basic assumption in Barker's proof of the equivalence between general natural (indicative) and material conditionals in his paper 'Material Implication and General Indicative Conditionals'. We will here examine this and other, internally related and commonly accepted, assumptions that Barker relies on in his proof.

Frege's Equivalence

Categorical statements such as 'All men are animals' are within Frege's truth functional logic, as it is formulated in *Begriffsschrift*, given a general conditional formulation: 'If x is a man, then x is an animal'. This again is formalized as a general material conditional $(x)(Mx \supset Ax)$. Even though there are some grammatical discrepancies between the categorical statement and the general conditional, Frege considered them to be logically equivalent, since they both can be analyzed via the general material conditional. He then claims logical equivalence between the following statements:

- (A) All men are animals.
- (B) If x is a man, then x is an animal.
- (C) $(x)(Mx \supset Ax)$

In his paper 'Material Implication and General Indicative Conditionals', Barker gives, among other things, a proof of this equivalence based on two principles. The first principle concerns the relation between categorical statements and general natural conditionals:

- P1. If a universal quantification, e.g., *Every F that is G is H* , is assertable for a speaker U then the corresponding general indicatives, i.e., in the case given, *If an F is G , it is H* and *Any/every F , if it is G , is H* , are assertable for U .

Barker defines assertability as high degree of subjective probability for U of the conventional content S . "Idealizing slightly, then, let ' S is assertable for U ' mean that U ,

given his (or her) actual beliefs, is committed to a subjective probability state B that warrants utterances of S , whether or not U is in B . I assume that if S is assertable for U in this sense then the logical equivalents of S are also assertable for U .¹

Consider now the categorical statement (A): ‘All men are animals’ and the general natural conditional (B): ‘If something is a man, then it is an animal’: Both these statements seem to have an extensional aspect, saying that the set of men is contained in the set of animals, a property that is associated with the material conditional (C): $(x)(Mx \supset Ax)$. However, the general natural conditional is normally considered to express an intentional relation between the concepts ‘man’ and ‘animal’, without asserting that these concepts have extensions. The intentional relation and conditional form are the focal qualities of the natural conditional, while the extensional aspect seems to be essential in the categorical statement. The material conditional, on the other hand, is defined extensionally only, but not in the same sense as the categorical statement. We will come back to these divergences between (A), (B) and (C) in the sequel.

Singular Conditionals

Barker’s second principle (P2) concerns the relation between general natural conditionals and the singular instantiations of these:

- P2. If a general indicative, e.g., *If an F is G , it is H* or *Any/every F , if it is G , is H* , is assertable for U , then all instances, i.e., in the cases given, sentences of the form *If t is G , t is H* where t denotes an F , are C-assertable for U .

There are good reasons for assuming this relation between general and singular conditionals. For instance, a singular instance of a general natural conditional, such as ‘If Socrates is a man, then he is an animal’ seems to follow both from our categorical statement and from our general natural conditional. And if we formalize it as a singular material conditional $Ma \supset Aa$, we get the same valid derivation of it from the categorical statement and the general material conditional. In fact, the truth of the singular (natural or material) conditional is grounded in the truth of the corresponding general expression, categorical or conditional. With natural conditionals, this entailment seems at least some

¹ Barker (1997), p. 196.

times to go both ways: The general natural expression is true on the basis that *any*, hence all, of the singular instantiations of it are true. As a matter of fact, in saying that ‘If Socrates is a man, then he is an animal’, we express the same conditional relation as we do in ‘All men are animals’ and ‘If something is a man, then it is an animal’; hence the equivalence lies in the conditional relation expressed, not in the degree of generality. With material conditionals, this same equivalence does not hold unless we understand $Ma \supset Aa$ to represent *all*, hence any, possible instantiations of the general conditional $(x)(Mx \supset Ax)$:

- (b) If Socrates is a man, then he is an animal $\leftrightarrow (B) \leftrightarrow (A)$
- (c) $Ma \supset Aa$ (with a being any instantiation of x) $\leftrightarrow (C) \leftrightarrow (A)$

A consequence of the fact that the singular natural conditionals may rely on a corresponding general categorical statement is that the natural conditional relation expressed does not change under variations in hypotheticality: If it is true that all men are animals, then it is also true that if x were a man, then x would be an animal. And it is also true that if the wood nymph had been a man, then she would have been an animal. This indicates that we here deal with an “ordinary” conditional, one that takes *modus ponens*, *modus tollens* and contraposition, as opposed to “rhetorical” conditionals that do not take contraposition, and only *modus ponens* or *modus tollens*.²

In *Begriffsschrift*, Frege explicitly acknowledges the difference between two kinds of singular conditionals: the first belonging to first-order logic, namely (c): $Ma \supset Aa$, and the second belonging to propositional logic: $p \supset q$. The first is, as already mentioned, a singular instantiation of a general conditional $(x)(Mx \supset Ax)$, while the second is not. In $p \supset q$ there is no corresponding general conditional on which the truth of the singular conditional relies. Examples of such conditionals with no corresponding general conditionals are: ‘If the Socialist Party wins the election, public schools will get more money’; ‘If you come over to my place, I’ll make you some dinner’; ‘If I don’t get a raise, I’ll quit my job’; ‘If she’s not in her office, she’s out for coffee’; ‘If Oswald didn’t kill Kennedy, someone else did it’; ‘If you want to see him tomorrow, he’ll be home after five’, and so on. This distinction is due to the fact that these singular conditionals, asserting

² These notions of ordinary and rhetorical conditionals are due to the classification of conditionals of Schapansky and Anjum.

a conditional relation between some particular facts or events that is not grounded in a corresponding general conditional or categorical statement, are existential conditionals. On the other hand, the general conditional (B) and its singular instantiation (b), asserting the same general logical relation, are both generic conditionals that ground in a corresponding categorical statement (A). In the following, when we use the term ‘singular conditional’, we will refer to these generic conditionals (b) and their material interpretations (c), unless otherwise is indicated.

Frege, although he is not explicit about the nature of this distinction, claimed that the singular natural conditional *if p then q* should not in general be considered as equivalent to the material conditional $p \supset q$, while on the other hand the singular natural conditional ‘If Socrates is a man, then he is an animal’ is accurately interpreted as the material conditional (c): $Ma \supset Pa$, that are instantiations of general ones. Frege considered the general material conditionals as different from the singular ones with respect to their ability to express the consequence relation that is evident in natural conditionals. He distinguished carefully between the material and the natural conditionals on the singular level, though he claimed that on the general level, there are equivalences between categorical, hypothetical and material statements. Furthermore, he considered the material conditional as the logically perspicuous alternative, thus abstracting from inessential grammatical differences of the conditional and categorical forms of general statements. This is why Frege claims that the distinction between categorical statements and general conditionals is of significance only for grammar and not for logic.

Barker’s Proof

The equivalence Barker wants to prove is first of all Frege’s interpretation of categorical statements and general natural conditionals as general material conditionals. In addition, he wants to prove the equivalence between singular natural and material conditionals.

Barker’s proof amounts to demonstrating that if one accepts (P1) and (P2), one must also accept that natural conditionals are in general equivalent to material conditionals, whether the natural conditional in question is general or not:

The argument for a material implication theory of indicatives using (P1) and (P2) is then as follows: if indicative *if* p, q is not treated as material implication, (P1) and (P2) cannot both be accepted.³

Barker's proof of this equivalence is accordingly a *reductio ad absurdum* proof (RAA). He starts out with the thesis that on the assumption that (i) 'Every F that is G is H ' is assertable for U , it does not follow that all conditionals of the form (ii) 'If t is G , t is H ', where t denotes an F and "if" is non-material, are assertable for U . He then gives the following RAA-proof of the contradictory of this thesis:

Proof: given that (i) [*Every F that is G is H*] is logically equivalent to (iii) '*Every F is such that $(x \text{ is } G \supset x \text{ is } H)$* ', (i) is assertable for U where (iii) is. (iii)'s assertability for U , and thus (i)'s, entails no more than that each material implication ($t \text{ is } G \supset t \text{ is } H$), where t denotes an F , is C-assertable for U . All non-material implication theories of the indicative hold that the C-assertability condition for ($t \text{ is } G \supset t \text{ is } H$) is weaker than that for '*If t is G , t is H* '. Thus it is logically possible for U to accept (i) and reject (ii), where *if* is non-material and t denotes an F , and still have consistent beliefs. But if we embrace both (P1) and (P2), U 's beliefs cannot be consistent, for in such a situation (P1) predicts that the general indicative '*If an F is G it is H* ' is assertable for U because (i) is, but (P2) predicts that it is not, because an instance of (ii) is not C-assertable for U .⁴

Barker has now allegedly proved the equivalence between the singular natural conditional and the singular material conditional. However, the *reductio ad absurdum* form of his argument and the use of the notions of assertability and C-assertability seem to obscure a clear view of the steps in the proof. Let us therefore have a closer look at the proof, but this time by means of a deductive approach, and, for the sake of perspicuity, where possible, ignore the use of the notions of assertability and C-assertability.⁵

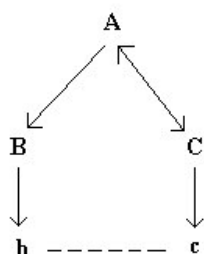
³ Barker (1997), p. 197. This clearly indicates that Barker intends this also as a proof of the equivalence between singular material and natural conditionals. After the presentation of the argument and a few illustrations, Barker declares: "In this paper I discuss whether non-material theories can extricate themselves from this difficulty. I assume from now on that (P1) is correct. Below, in §II, I show, while dealing with a range of other possible responses, that non-material theories cannot avoid contradiction by denying A-entailment, for it is, I argue, correct. I then demonstrate in §III that, although not all general indicatives need be treated in the Geachian way – some can be analysed in the manner suggested by Lewis – there is nevertheless a class of conditionals that must be so analysed. I therefore conclude that, given current accounts of generality, indicative conditionals have to be analysed as material implications. I end in §IV by defending a pure material implication analysis of indicatives against the battery of objections raised chiefly by Jackson and Edgington" (p. 198). This shows that Barker in the rest of the paper only considers how the contradiction demonstrated in the argument on p. 197 can be avoided by questioning the soundness of the premises (P2) (which follows from the Geach-Quine analysis of general conditionals and A-entailment) and (P1).

⁴ Barker (1997), p. 197.

⁵ These notions only play a part in Barker's discussion of the soundness of the premises (P1) and (P2), and not in the discussion of the validity of the proof itself. So we may safely disregard them in our discussion of the proof. Barker limits his following discussion to the purported soundness of (P2). Fintel (1998) questions

Reconstruction of the proof

According to Barker's account, we have the following logical relations: Following (P1), the categorical statement (A) entails the general natural conditional (B), and according to (P2), (B) entails the singular natural conditional (b). Barker presupposes Frege's logical equivalence between the categorical statement (A) and the general material conditional (C),⁶ which enables him to make the inference from (C) to the natural conditional (b). The general material conditional (C) entails the singular material conditional (c). However, (c) does not entail (C). This means that even if we tentatively accept all the entailments and equivalences that Barker explicitly states in his proof, this is still not sufficient for inferring the singular natural conditional (b) from the singular material conditional (c):



The singular natural conditional (b) and the singular material conditional (c) are therefore not proved equivalent within the scheme of entailments offered in Barker's proof. So what makes him think that he has proved this equivalence? Since the problem with equating 'p $\supset$ q' and 'if p then q' is a well-known problem, we will concentrate our discussion on pointing out some mistaken (implicit and explicit) assumptions in the proof on Barker's part.

The First Assumption in Barker's Proof

On the level of general statements, Barker assumes Frege's equivalence between categorical statements (A) and general material conditionals (C). And since (P1) states the

Barker's defence of the Geachian analysis of general conditionals against non-Geachian interpretations of general indicatives as adverbial generalities. However, what matters to the proof is entailment from (B) to (b), whether this entailment is supported by a Geachian analysis of general conditionals or not, and notwithstanding that a Geachian analysis would have made the alleged identity of the natural and material singular conditionals syntactically perspicuous. It is hard to see that any kind of analysis of general conditionals could obstruct an entailment from general to corresponding singular conditionals, even though Barker claims that this is the case with an adverbial analysis of general conditionals. Cf. Barker, p.207.

⁶ Barker (1997), p. 197: "given that [A] is logically equivalent to [C] Every F is such that (x is G $\supset$ x is H)".

entailment from (A) to the general natural conditional (B), Barker infers the entailment of (B) from (C). However, the assumption of Frege's equivalence of (A) and (C) is an idling wheel in the RAA: We cannot infer the equivalence of singular material and natural conditionals from the assumption of (C) any more than we could from the assumption of (A). But since the assumption of the equivalence of (A) and (C), together with the usual assumption of the entailment of (C) from (B), gives the equivalence on the level of general statements, (A), (B) and (C), Barker's assumption of the equivalence of (A) and (C) is of interest in itself. Thus, it will be clarifying to explicitly introduce Frege's equivalence as a third principle, in addition to (P1) and (P2):

- P3. The universal quantification *Every F that is G is H* is logically equivalent⁷ to the general material conditional *Every F is such that (x is G $\supset$ x is H)*.

There are however good reasons for questioning this equivalence. We will demonstrate this by discussing the different properties of categorical statements, natural conditionals and material conditionals.

Categorical, Hypothetical or Material?

The divergence between the three general statements (A), (B) and (C) is first of all a difference with respect to existence claims: The categorical statement 'All men are animals' is false if there are no men, while it is true that 'No men are animals' under such circumstances. We can thus say that a categorical statement has qualitative existential import. But when we formulate this relation as a general natural conditional 'If something is a man, then it is an animal', we only assert the hypothetical relation between being a man and being an animal, regardless of whether there are any men or animals. The general natural conditional has accordingly no existential import, making no claims of existence. The general material conditional, on the other hand, has quantitative existential import, which indicates that it cannot be an adequate interpretation of general natural conditionals.

Furthermore, when we make a singular instantiation of general natural conditionals, there seems to be a difference even between the singular and the general statement with respect

⁷ Barker is explicit only on the logical equivalence, but perhaps it should also be included in the principle that (A) and (C) are supposed to be assertably equivalent. However, it is by no means straightforward to understand what assertability could mean for the material conditional independently of its association to a corresponding non-material conditional.

to the logical property of existence: While the general natural conditional does not involve claims of existence of men, but only says that if there are any men, then these are animals, the singular natural conditional ‘If the wood nymph is a man, then she is an animal’ seems to be open on the question whether there are any wood nymphs, but not on the existence of men and animals. Here, the hypothetical question in focus is to whether x is a man, not to whether there is an x that is a man. In this way, the singular natural conditional is asserted on the acceptance of the categorical statement that all men are animals, where we do in fact have existence claims of men and animals.⁸

The reason for this discrepancy between the general and the singular natural conditional is that the singular conditional in question is closer to the corresponding categorical statement than to the general conditional. Asserting a non-causative relation between the antecedent and the consequent, the conditional is used to assign a property to a general class of objects:⁹ ‘If an x is G , then x has the property H ’, based on the categorical statement ‘All G s are H s’. The singular conditional thus connects naturally to the categorical statement, not to the general conditional.

There are, however, singular conditionals that as such are closer to the conditional rather than to the categorical form. These are the generic conditionals where the conditional relation is used, not to assign a property to a general class, but to assert a dependency between *events*: ‘If an x is in the state G , it will, as a result, be in the state H ’. This is for instance the case in physical laws, as in ‘If an object is dropped, it will fall towards the ground’. Here, the conditional is the most obvious form to use when we want to express the general physical laws of gravity. Being purely hypothetical with respect to the dropping and falling of objects, the conditional relation is what is in focus. There will then not be a difference between asserting a singular conditional ‘If I drop this pen, it will fall towards the ground’ and asserting the general conditional ‘If an object is dropped, it will fall towards the ground’ with respect to the logical property of existence. But as with the non-causative conditionals, we still have equivalence with respect to their corresponding categorical statement ‘Any x that is in the state G will as a result be in the state H ’: ‘Any

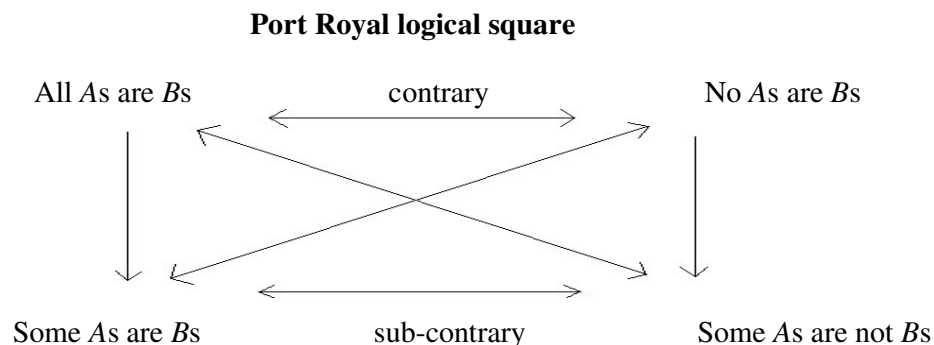
⁸ We do not want to say that it is false to assert that ‘All angels have wings’, meaning all representations of angels as we for instance find them in Christmas shops and in pictures. Existence then adjusts to the level of talk about the objects.

⁹ The notions of “non-causative”, “causative”, “generic” and particular “existential” conditionals are due to the classification of Schapansky and Anjum.

object that is dropped will fall towards the ground'. So even though there is some discrepancy between categorical statements (A), general natural conditionals (B) and singular natural conditionals (b), this does not affect their logical properties. When a categorical statement is in focus, namely in non-causative relations, we have qualitative existential import, and the general natural conditional form will be superfluous. When the general natural conditional is in focus, on the other hand, we will not have qualitative existential import, and hence the hypothetical conditional relation is the focal point.

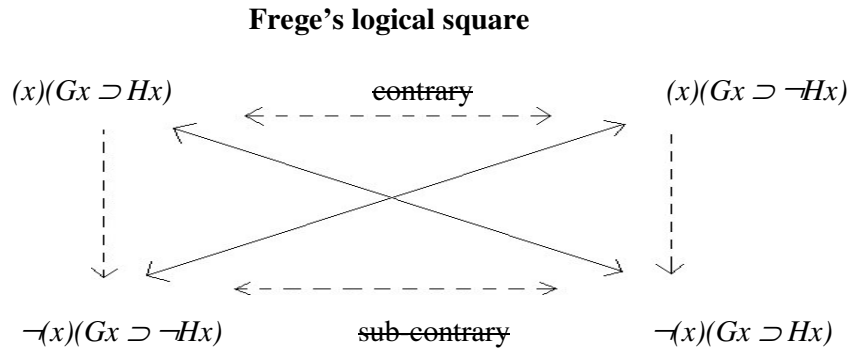
The real discrepancy is between the general natural statements on one hand and the general material conditional on the other. This is because the general material conditional has quantitative existential import, which means that both 'All men are animals' and 'No men are animals' are true if there are no men, while 'Some men are animals' and 'Some men are not animals' are both false under this condition. So even though we have claims of existence both in categorical statements and in general material conditionals, they do not claim existence in the same manner: While 'All men are animals' asserts existence about men and animals, $(x)(Mx \supset Ax)$ asserts that there does not exist an x that is a man and not an animal; $(\neg \exists x)(Mx \ \& \ \neg Ax)$. Since the material conditional is purely truth functional, it will be true if there are no x s that are men or if all x s are animals.

That the material conditional involves existence in a different respect than categorical statements is apparent when we compare the so-called logical square of the *Port Royal* logic with that of Frege's logic:



Here, the statements related through the diagonals are contradictory to each other, which means that when one is true the other is false and *vice versa*. The top statements are contrary to each other, i.e. they may both be false but cannot both be true. The bottom two

are sub-contrary to each other, that is, they may both be true but cannot both be false. Finally, the statements on the sides are sub-alternate, such that the top statement entails the bottom statement. In the corresponding logical square of Frege, however, only the relations along the diagonals are preserved:¹⁰



As we see, the general material conditional interpretation of categorical statements leaves behind many of the traditional logical relations between different quantified statements. All attempts to revise this square and still use the logical language of *Begriffsschrift* in the hope of restoring the logical relations of the *Port Royal* logic have failed. Obviously, then, the material conditional reading of categorical statements is not precise with respect to logical properties. The question is whether this difference is of any practical significance.

In our everyday and scientific reasoning we often use categorical statements, like ‘Any body unaffected by forces will move uniformly in straight lines or stay at rest’, to express general causative conditional “consequences”: ‘If a body is unaffected by forces, it will move uniformly in straight lines or stay at rest.’ In such cases, we have neither qualitative nor quantitative existential import, as mentioned above. However, contingent and non-causative categorical statements, like ‘All John’s children are asleep’, are understood to have qualitative existential import both in natural language and in traditional logic. It is thus incumbent on us to interpret grammatically indistinguishable general categorical statements as logically different. Interestingly, the translation of general law statements, like ‘Any body unaffected by forces will move uniformly in straight lines or stay at rest’, as general material conditionals seems to fail on all accounts. On one hand, it fails as an interpretation of categorical statements, since the material conditional has quantitative existential import, while the non-causative categorical statement has a qualitative

¹⁰ Frege (1972), §12.

existential import. On the other hand, the general material conditional fails as a translation of the general natural conditional, since the latter has no existential import at all, while the material conditional is true solely for the reason that the antecedent is false.

One and the same statement may therefore express different propositions, depending on whether it is considered as a non-causative general categorical or as a causative general conditional statement. For instance, Kepler's first law, 'All planetary orbits are elliptic', may be considered either as a non-causative categorical statement, stated on the basis of observations of the actual planetary orbits, or as the expression of an essential "consequence", as in causative general conditionals.¹¹ In fact, Kepler himself claimed to have deduced his law in the second sense, something that is missed by many philosophers and historians of science.

The Second Assumption in Barker's Proof

Barker seems to believe that the general categorical (A), natural (B) and material (C) statements are all equivalent, even semantically and logically identical. He therefore seems to assume that they cannot express basically different kinds of statement forms. This leads us to his reasons for the assumption of the equivalence of singular natural and material conditionals. In the discussion of possible objections against his proof, Barker argues: "Any ambiguity which *if p, q* may possess cannot be restricted merely to singular indicatives. Where there is a singular form there must be a general form; the apparatus of generality is free to apply itself in appropriate ways to any well-formed open sentence. As we are now postulating two types of singular indicative, we must envisage two types of corresponding general indicative."¹²

To Barker, this implies that (P1) is allegedly only supported by a material reading, and is invalid for a non-material reading. He objects to this: "However, there seems to be no

¹¹ The distinction between general categorical statements that express general conditionals (causative conditionals) and general categorical statements that link a certain property to a certain class (non-causative conditionals) may sometimes be marked by the choice of universal quantifier. If the conditional is causative, we will use 'any' instead of 'all' or 'every' to express the generality: 'Any pen if dropped will fall.' This is because the use of 'any' indicates that the knowledge expressed in the categorical statement is not based in observation of *all* relevant cases, but in knowledge of the essential (causative) connection between *a* case of the antecedent and *a* case of the consequent. If the conditional is non-causative, on the other hand, we will use 'all', since it is to be expected that all members of a class must have this assigned property in order to be a member of that class: 'All men are mortal.'

¹² Barker (1997), p. 201.

natural reading of the general indicative for which (P1) breaks down, that is, which allows one to assert felicitously ‘*Every F that is G is H*’ and dissent in the same breath from ‘*If an F is G, it is H*’. For, as noted in the introduction, that just sounds as if the speaker is being inconsistent. If this is correct, it would appear that the claim that singular indicatives have a non-material conditional disambiguation is false.”¹³

However, as argued above, we can only go from a singular to a general conditional when they are both generic and natural, that is, when they are both grounded in a categorical statement or in a general natural conditional. Barker seems to believe that all singular conditionals have a corresponding general conditional, but this is not the case for singular material conditionals $p \supset q$, or even for singular natural conditionals *if p then q*. However, any relevant disambiguation must be carried to the general natural level: if we have a non-causative general natural conditional, this always follows from a categorical statement, that is, ‘All men are animals’ implies ‘If something is a man, then it is an animal’. With causative general relations, we have the implication from the general natural conditional to the categorical statement. Both these general natural statements also imply the general material conditional, but we do not in general have the converse entailment from the material to the natural statements. This leads us to the third of Barker’s assumptions.

The Third Assumption in Barker’s Proof

Barker assumes in the proof that the categorical statement (A) is true or assertable for *U*. This amounts to making (A) a premise of the proof, something that is reflected in Barker’s formulation of the thesis of the RAA: “[O]n the assumption that [A] is assertable for *U*, ...it does *not* follow that all conditionals of the form [b] are C-assertable for *U*”.¹⁴ But this is actually a necessary assumption if one wants to prove the equivalence between natural and material conditionals, both on the general and singular level. Under the presupposition that the general categorical statement is true, it will certainly not be possible for the singular (or general) natural conditional to be true and the singular (or general) material conditional to be false at the same time, or *vice versa*. This is so because the passages from the categorical statement (A) to the singular material conditional (c) and the singular natural conditional (b) are valid in the sense that if (A) is true, then both (b) and (c) will be

¹³ Barker (1997), p. 202.

¹⁴ Our brackets and italics: the actual text is: “Thus, on the assumption that (i) below is assertable for *U*, (i) Every F that is G is H, it does not follow that all conditionals of the form of (ii) below, where *t* denotes an F and if is non-material, are C-assertable for *U*: (ii) If *t* is G, *t* is H.” Barker, p. 197.

true. In the simplest cases, (b) and (c) having the same truth-value, namely true, will depend on the truth of (A), and hence (B), since we will then have a general relation between antecedent and consequent, which (A) and (B) express.

If (A) is false, on the other hand, we will not have a general relation between antecedent and consequent, and both (B) and (b) will then be false while (C) and (c) might actually be true. For instance, if (A) is the false statement ‘All philosophers who are moral philosophers are logicians’, it would be false that (b): ‘If Popper is a moral philosopher, then he is a logician’, but true that (c): (Popper is a moral philosopher $\supset$ Popper is a logician), since both the antecedent and the consequent are false. Hence, (c) cannot entail (b). In the case of singular conditionals, (b) logically entails (c), since (c) is true of more possible circumstances than (b). It is thus not possible for (b) to be true and (c) false at the same time. If (b) is true, (c) is true, and if (c) is false, (b) is false. Consequently, natural conditionals entail their corresponding material conditionals, but not *vice versa*. This analysis nevertheless presupposes that the conditionals involved have a simplistic structure. Once we add a negation, as for instance in *not (if p then q)*, we no longer have that the material conditional is true of more possible circumstances than the corresponding natural conditional. This is so since a negated material conditional $\neg(p \supset q)$, being equivalent to $p \ \& \ \neg q$, is stronger than the corresponding natural conditional.¹⁵

The Fourth Assumption in Barker’s Proof

Barker operates on the level of general statements, but draws inferences about the equivalence between singular natural and material conditionals, even though this does not follow from his analysis. Since we intuitively understand the supposed equivalence of general natural and material conditionals as an expression of the identity of the conditional relation in the two statements, we naturally think that this equivalence implies that the corresponding singular conditionals are equivalent. However, the equivalence of (B) and (C) cannot *as such* guarantee the corresponding equivalence between the singular conditionals (b) and (c). The logical entailments are all extensionally “downward”, from general to singular statements. We may only move “upward” by intentionally understanding the truth conditions of the singular natural statement as given by the corresponding natural general statement. This can be done only when we understand the

¹⁵ For a more detailed account, see Edwards (1973/74).

general statements (both the natural conditional and the categorical) as expressing causative or non-causative *consequences*.

The material interpretation can however not be understood in this way. This is so since the singular material conditional $Ga \supset Ha$ is true for the sole reason that Ga is false.

Materially interpreted, the singular conditional *if a is G, a is H* may well be true even if there is no intentional relation between the extensions of G and H . We still have the entailment, though: if the natural conditional is true, so is the material conditional. But if we ask for the meaning of the sentential schemes *if a is G, a is H* and $a \text{ is } G \supset a \text{ is } H$, the singular natural conditional states a hypothetical, conditional relation between the antecedent and the consequent, and is true if there in fact is such a relation, while the material conditional is true unless the antecedent is true and the consequent is false. But what does the material conditional say? We find ourselves unable to connect any propositional meaning to this construction. The only thing the material conditional says anything about is its own truth-value as a function of the truth-values of the antecedent and the consequent. It is a Boolean function, no more and no less. But a function is no proposition. Material conditionals are thus not propositions on a par with other statements, while natural conditionals obviously are full-fledged propositions in the normal sense.

The Generality of Mathematical Conditionals

In mathematics the different kinds of general and singular statements seem to display particularly close relationships. The infinity (and finiteness!) and generality that is characteristic of mathematics cannot easily be assimilated to the generality of other statements. One interesting semantic fact of mathematics is that the difference between ‘any’ and ‘every’ is unimportant in one sense and in another sense decisive.

Frege thought that first-order logic exposed the relevant logical structure of general statements. However, if the quantifiers are read extensionally, the general material statement $(x)(Fx \supset Mx)$ asserts nothing but the conjunction of the set of corresponding singular material conditionals: $(Fa \supset Ma) \ \& \ (Fb \supset Mb) \ \& \ \dots \ (Fn \supset Mn) \ \dots$. Thus there is really no expression of a consequence relation in the general material conditional, as is pointed out by Stenlund:

The usual interpretation of generality as the general quantifier of the predicate calculus is based in this false picture. There is supposed to be one notion of generality, one sense of the expression ‘for all’ and ‘there exist’, which is common to finite extensions and infinite domains. And the paradigm for this sense of generality is the accidental generality exemplified in the above sentence about ‘all letters’. In the extensional view this notion of generality is given an ‘idealized sense’ applicable to infinite domains by disregarding the fact that we cannot literally ‘run through’ or ‘overview’ the ‘infinite list’ of instances of a general proposition about, say, all even numbers – i.e. the ‘cannot’ is misunderstood as though it signified just a factual and not a conceptual impossibility. This is one point where the method of paraphrase of formal logic is conceptually inadequate.¹⁶

The reference that the general material conditional is supposed to make to the totality of objects could perhaps secure the connection to the consequence relation. Or perhaps, as Frege seems to suggest in *Begriffsschrift*, the consequence relation is nothing but this generality, and, eventually, the implied reference to a transfinite set or “whole” of objects. But whether this ontological or semantic interpretation makes sense or not, what we actually do when we calculate with numbers (or numerals) and prove mathematical statements about them, is not even similar to reviewing the objects and claiming truth or falsity of the antecedents and consequents of the corresponding singular material conditionals. What we do is make and reproduce conditional operations, following rules that most adequately will be formulated linguistically in generic *singular* (!) conditionals that state the step we know we shall always be able to make. However, the modality of this claim is not about our physical and mental capacities, it is about the *potentialities* characteristic of the mathematical realities with which we are performing our operations.

Mathematical Induction

Take an example from elementary number theory: I state that for *any* natural number n I can prove that the sum of all the natural numbers, from 1, 2, 3 ... and on to n , is given by the expression $\frac{1}{2}n(n+1)$. Now, there are known to be infinitely many natural numbers, and this knowledge is already in essence a knowledge about a potentiality, that is, the potentiality to go on from one natural number to the next by the successor operation. But this is not to be understood as an actual infinite process that is performed in the proof:

Since the pure mathematical notion of ‘finite’ is taken as the fundamental notion, the rules governing the talk of ‘finite processes of computation’ in mathematical logic are the rules of the mathematical theory of finite sequences, and not the rules

¹⁶ Stenlund (1990), pp. 157-158.

for our notion of computation as it is performed in actual human practices (with or without the help of computers). This talk is felt to make sense against the background of the extensional picture of these ‘idealized finite processes of computation’ as being series of individual computational steps. So we may also express the misconception involved here by saying that the extensional view of the mathematical notions of a finite sequence and a finite set is as mistaken as the extensional view of infinite sets and sequences.¹⁷

This knowledge about the potentiality of numbers is basically my acquaintance with the rule for generating the natural numbers, my certainty of the mathematical possibility of following this rule and my assurance of my capacity to perform this in calculations with, and applications of, the natural numbers. These capacities are now used in the proof of the statement by mathematical induction. I ascertain that $1 = \frac{1}{2} * 1 * (1+1)$, and then assume that the formula is correct for all numbers from 1 and on to a *certain* natural number n . I will then prove the induction step, i.e. that the formula is true for the natural number $n + 1$, *given* that the formula has been shown true for the number 1 and for any number from 1 and on to n :

$$1 + 2 + \dots + n + (n + 1) = \frac{1}{2}n(n + 1) + (n + 1) = (n + 1)\frac{1}{2}n + (n + 1) = (n + 1)(\frac{1}{2}n + 1) = (n + 1)\frac{1}{2}(n + 2) = \frac{1}{2}(n + 1)(n(n + 1) + 1).$$

There are lots of conditionals involved in this proof. Seen from the perspective of the first person performing the proof, the insight itself is basically a generic singular conditional: I know that *if* n is a natural number *then* the sum of all natural numbers from 1 and on to and including n is $\frac{1}{2}n(n + 1)$. And I know this because I know how to prove it for a paradigmatic natural number n or $n + 1$. This knowledge could also be expressed in a corresponding general conditional, though the more adequate expression of the generality of the insight would perhaps be the corresponding categorical statement. However, the categorical statement is then understood to express the mathematical reality of the *potentiality* of the natural numbers, as themselves being basically the *reality* of a potentiality including a potentiality to be shown to have these sums:

The generality expressed by the variable ‘ x ’ [...] and by the signs ‘...’ and ‘etc.’ [...] is the generality of the form of a series generated by an operation. It does not express the general applicability of a function or a law to some independently given

¹⁷ Stenlund (1990), p. 204.

objects, because it is precisely how the natural numbers *are conceived as being given* in arithmetic which is expressed.¹⁸

The difference between ‘any’ and ‘every’ can then be put to some relevant use. The distinction can be understood as signaling the difference between the intentional (rule-based) relation between the categorical statement expressing a mathematical consequence relation and the paradigmatically understood singular conditional on the one hand, and the extensional relation of the general and singular material conditionals on the other. In the first case we would have that: for *any* natural number n , the sum of all natural numbers from 1 and on to and including n , is given by the expression $\frac{1}{2}n(n + 1)$. By contrast, the general material conditional would state that: for *every* object x , we have $(Nx \supset Sx)$, where the predicates N and S are respectively, “being a natural number” and “having the associated sum given by the description above”.

Frege’s Last Thoughts

In his discussion with Hilbert about the foundations of geometry, Frege dismissed the formalistic account of the relation between axioms and theorems, and argued that the axioms cannot be understood as hypotheses, but must be stated as categorically true.¹⁹ Consequently, according to Frege, if and when the axioms, theorems or their relations are naturally expressed as conditionals, these conditionals cannot be understood as combined by antecedents and consequents having sense, reference and truth-values, taken separately. The antecedent and consequent are mere pseudo-propositions, primarily having determinate meaning only in the conditional as a whole.²⁰ This conception of the mathematical conditional is in agreement with our exposition above, but blatantly in conflict with the conception upheld by the material conditional interpretation, as we have exposed it above.

This indicates that Frege eventually must have realized, if only vaguely, that the general material interpretation could not adequately express the consequence relation in natural conditionals, and especially not the consequence relation in the paradigmatic singular conditionals so central to mathematical proofs. Anyway, this gives a plausible background

¹⁸ Stenlund (1990), p. 161.

¹⁹ Cf. Frege (1971), p. 25: “Axioms do not contradict one another, since they are true; this does not stand in need of any proof.”

²⁰ Frege (1906), part II, pp. 69ff.

for his last remarks on the knowledge source of mathematics - not only of geometry but also of arithmetic (and algebra) - being intuition and not pure formal logic.²¹ With this intuition we grasp for instance the generality of the induction step as paradigmatically exposed in the generic singular conditional ‘if n is a natural number with the property S , then $n + 1$ can be shown to have the property S ’. As we have shown, the equivalences between the categorical, and the corresponding general and singular conditionals can be confirmed in mathematics. Furthermore, these logical relations can be seen to expose some of the essence of both the mathematical objects and the subject doing mathematics, and how they are internally related to one another.

Appendix: The Material Conditional Nail Soup

A Norwegian folk tale goes something like this: A wanderer arrives at a farm in some outlying district of Norway and asks the farmer’s wife if she will allow him to cook his meal on her stove. Since it won’t cost her anything, she consents. The wanderer puts a kettle filled with water on the stove and pulls a huge nail out of his pocket, cleans it thoroughly, and then sinks it carefully into the boiling water. Full of curiosity, the farmer’s wife asks how that is supposed to make a meal, to which the wanderer replies that the soup tastes really good, but that it of course would taste even better if he had a carrot to put in. Little by little he lures the farmer’s wife into giving him all the ingredients that make a good soup. Eventually she is allowed to taste it. And of course she marvels over the strong and tasty soup that has been made on a nail.

Barker tells us that conditionals are really nothing but material conditionals. But when we ask how this theory tastes, we are told that it will taste even better if we apply some ingredients of pragmatic implicatures to the soup. Defending the Gricean approach against criticism, Barker applies the wanderer’s strategy to an even greater degree. He declares that he will restate the pragmatic defense in two parts:

The first part is this: conversational maxims are defeasible. In particular, when U [the utterer] believes r and s and that r is logically stronger than s , U follows the maxim of asserting the stronger r unless there is some rational ground for doing

²¹ Cf. Frege (1924/1925a), p. 277, and Frege (1924/1925b), p. 280: “From the geometrical source of knowledge flows pure geometry. In the case of arithmetic, just as in the case of geometry, I exclude only sense perception as a source of knowledge. This is significant because it means that arithmetic and geometry, and hence the whole of mathematics flows from one and the same source of knowledge – that is the geometrical one.”

otherwise. The main reason for doing otherwise is that *U* can express a certain type of information by a compound *s* not conveyable by the stronger *r* because *s* has a range of *grounds* that *r* does not have, and *U* can convey by uttering *s* in the context that one of these grounds holds. Thus, when *U* asserts *if p, q*, *U* does not just convey the literal content of *if p, q*. *U* also expresses information related to the ground upon which *U* asserts *if p, q*. There are two main grounds for *if p, q*: the first is that the truth of *p* is nomologically or otherwise connected to that of *q*, or in symbols $p \Rightarrow q$ (I shall call this type of ground ‘*connex*’). I stress that $p \Rightarrow q$ does not entail that *p* or *q* is true. $p \Rightarrow q$ means that *p*’s coming to be true, whether it is true or not, would bring about *q*’s truth. The second ground is that *q* is true and *p*’s truth – if it is true – is irrelevant to it. On this ground (‘*irrel*’) *U* may utter *even if p, still q*. Thus, according to this conception of grounded utterances, *U* may believe $\sim p$ or *q* but assert *if p, q*, although $\sim p$ and *q* are stronger, because by uttering *if p, q* *U* can convey information about a ground *connex* or *irrel*.²²

So, what is it that makes the conditional soup tasty and nourishing: the material nail or the “grounds”? What Barker here refers to as the grounds *connex* and *irrel*, are among the most common types of the non-material (natural) conditionals. We agree that the conditions of the material conditional are fulfilled by natural conditionals of different types, i.e. that it is possible to put the nail in the soup without doing any harm. However, Barker accepts that we in many cases do not even need the nail. For though the natural conditional agrees with the material in that it cannot have the combinations of truth-values that the antecedent is true and the consequent is false at the same time, we do not express the truth or falsity of either the antecedent or the consequent in a conditional based on the ground *connex*. Why then insist that what really matters semantically is the nail, and not the meat, the vegetables, the butter and the onion of the grounds?

Jackson gives what he believes is a proof that the nail is what makes the soup work. Like Barker, he claims to prove that the natural (indicative) conditional is equivalent to the material conditional. The proof is formulated as a paradox that occurs from denying the equivalence.²³ However, while Barker defends a modified version of the Gricean “assert the stronger” account to protect the material interpretation from the paradoxes of material implication, Jackson favors an account that supplements the material conditional with a conventional (not conversational) implicature of robustness. The notion of robustness can

²² Barker (1997), pp. 205-6. An example of a *connex* ground, according to Barker: “A conditional like ‘If the sun goes out of existence in ten minutes, the earth will be plunged into darkness in eighteen minutes’ is highly assertable, but not far more so than the negation of its consequent. The first part of [Pragmatic Defence] can explain this example: *U* may believe $\sim p$ and *if p, q*, but assert the weaker *if p, q*, not because his ground for doing so is $\sim p$ but because he believes that the ground *connex* holds, i.e., $p \Rightarrow q$. In short, *U* asserts *if p, q* to convey *connex*, which *U* cannot convey by uttering $\sim p$.” Ibid. p. 207.

²³ This paradox is known as the Paradox of Indicative Conditionals. Cf. Jackson (1987), pp. 4-6.

be fully expressed by the notion of conditional probability.²⁴ One can perhaps say that Jackson makes the notion of robustness the *ground* of assertion conditions for conditionals. However, in contradistinction to Grice, Jackson insists that as a mandatory ingredient, though unnecessary for the nutritional value of the soup, the robustness should be mentioned in the list of ingredients on the soup box: “It makes sense that we should have syntactical constructions which signal the possible information relative to which we take what we are saying to be robust.”²⁵

With respect to our question whether the soup is basically made on the material nail or from the ingredients of the grounds *connex* and *irrel* (Barker), or from robustness with respect to the truth of the antecedent (Jackson), Jackson makes a somewhat curious observation concerning the logic of the conditionals. As he points out, though the material conditional validates *modus ponens*, it cannot unconditionally be used in inferences according to the rule of *modus ponens*, unless the conditional is robust with respect to the truth of the antecedent: “In sum, we must distinguish the validity of *modus ponens* from its *utility* in a situation where I know $(P \supset Q)$ but do not know P . The robustness of $(P \supset Q)$ relative to P is what is needed to ensure the utility of *modus ponens* in such situations.”²⁶

It seems pertinent, then, to ask how it can be that robustness does not belong to the truth conditions of the conditional. The nail is to be considered as the logical kernel of the conditional soup, but only as long as the soup is not eaten! The use of conditionals in inferences by means of *modus ponens* is one of the main logical uses we make of conditionals. But this use is secured by the conventional implicature of robustness, not by the material conditional as such. And *modus tollens* and contraposition are not even protected in this rather minimal sense that *modus ponens* is on Jackson’s account. He argues that it is acceptable that *modus tollens* is valid for conditionals but unprotected by the notion of robustness, with examples taken from the class of rhetorical conditionals:

It does not, though, ensure the utility of *modus tollens*. $Pr(P \supset Q/P)$ can be high when $Pr(\neg Q \supset \neg P/\neg Q)$ is low. And this is how things should be. You may properly assert $(P \rightarrow Q)$ when you would not infer $\neg P$ on learning $\neg Q$. Suppose you say ‘If he doesn’t live in Boston, then he lives somewhere in New England’ or ‘If he works, he will still fail’, you will – despite the validity of *modus tollens* –

²⁴ Jackson (1991), pp. 125ff.

²⁵ Jackson (1991), p. 120.

²⁶ Jackson (1991), p. 124.

neither infer that he lives in Boston on learning (to your surprise) that he doesn't live in New England nor infer that he didn't work on learning (to your surprise) that he passed. Rather on learning either you would abandon the original conditional as mistaken.²⁷

In both cases discussed here by Jackson, we have conditional constructions that are not ordinary in the sense discussed, and thus obey other logical rules than the ordinary conditionals. But according to Jackson, only paradigmatic examples of conditionals can be used as counter-examples against his theory. He thus rejects conditionals of the type 'If he's telling the truth, then I'm a Dutchman' as counter-examples to the validity of contraposition and *modus tollens*.²⁸ Contrary to what Jackson says with respect to contraposition and *modus tollens*, such rhetorical conditionals should not be allowed to influence the general analysis of the ordinary conditionals in the case of the signaled conventional implicatures.

Strangely enough, we then have that, on Jackson's account, the most basic logical inference rules for conditionals: *modus ponens*, *modus tollens* and contraposition, though *valid*, cannot be used in inferences with conditionals without the guarantee of robustness, which, however, does not constitute a part of the truth conditions of the conditionals. *Modus ponens* may then be used in inferences with conditionals, since conditionals signal, as a conventional but not truth conditional implicature, that the conditional is robust with respect to the truth of the antecedent. Unprotected by the notion of robustness, *modus tollens* and contraposition do not even have that much protection as rules of inference for conditionals. Without strong reasons from other sources, there is not much to recommend this account as an account of the logical properties of conditionals in general.

Acknowledgments

We have benefited from the comments of Anita Leirfall, Thor Sandmel and two anonymous referees to an earlier version of this paper. We also want to thank Kirsten Fjellheim for valuable comments on our English.

²⁷ Jackson (1991), p. 124.

²⁸ We have already classified such examples as "rhetorical".

References

- Barker, S.: 'Material Implication and General Indicative Conditionals', *The Philosophical Quarterly*, 47, 1997, pp. 195-211.
- Edwards, J.S.: 'A Confusion About If..Then', *Analysis* 34, 1973-4, pp. 84-90.
- Fintel, K. von: 'Quantifiers and 'If'-Clauses', *The Philosophical Quarterly*, 1998, pp. 209-214.
- Frege, G. (1971): *On the Foundations of Geometry and Formal Theories of Arithmetic*, New Haven, Yale University Press 1971.
- Frege, G.: *Conceptual Notations and Related Articles*, London, Oxford University Press 1972.
- Frege, G.: *Posthumous Writings*, Oxford, Blackwell 1979.
- Frege, G. (1885): 'On Formal Theories of Arithmetic' in Frege 1971, pp. 141-153.
- Frege, G. (1906): 'On the Foundation of Geometry', in Frege 1971, pp. 49-112.
- Frege, G. (1924/1925a): 'Numbers and Arithmetic', in Frege 1979, pp. 275-277.
- Frege, G. (1924/1925b): 'A New Attempt at a Foundation of Arithmetic', in Frege 1979, pp. 278-281.
- Jackson, F.: *Conditionals*, Oxford, Blackwell 1987.
- Jackson, F.: 'Assertion and Conditionals', *Conditionals*, in Jackson (ed.) 1991, pp. 111-135.
- Johnston, D.K.: 'The Paradox of Indicative Conditionals', *Philosophical Studies* 83, 1996, pp. 93-112.
- Schapansky N. and R.L. Anjum: 'A Classification of Conditionals', unpublished 2004.
- Stenlund, S.: *Language and Philosophical Problems*, London, Routledge 1990.
- Thompson, M.: 'On Aristotle's Square of Opposition', *The Philosophical Review* 62, 1953, pp. 251-265.