

Numbers We Live By

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The motivation for this paper is a hunch we have concerning the concept of numbers: Is it possible to say anything about numbers and the concept of numbers without presupposing a kind of pre-understanding of numbers and counting? Our hunch is that an analysis of the concept of numbers has to be related to our activities involving numbers. When one wants to explain the concept of numbers, it seems that one cannot ignore or disregard the intuitive ideas we have about numbers, counting and related concepts and activities.

What do we need in order to understand what a number is? What do we need in order to understand the question “how many Fs?” or even, what makes us ask such questions? It seems to be many aspects involved when we employ the concept of numbers and related concepts. Take for instance the following situation:

If your father asks: “How many sandwiches do you want for lunch?” he presupposes that you have a certain knowledge and experience that enables and motivates you to answer this question. For instance, he will expect you to have experienced eating sandwiches; to have experienced that you feel fuller the more sandwiches you eat; that you can conceive of sandwiches as units, and; he will expect that you know and can see that and why three sandwiches are more than two sandwiches. In this way, it seems that talk about numbers is closely connected to our activities and to how we perceive and organize the world. It is in this very context that it makes sense to talk about numbers. This means that the active, perceptive and organizing subject must be part of an investigation of the concept of numbers.

Counting is an activity that concerns our attempts to grasp the world in systematic and ordered ways, and that is closely related to numbers. Do the numbers even make sense unless they are related to our activity of counting? It seems that whether we follow a recipe for punch, or whether we try to calculate things as how I can afford these expensive shoes, how many square meters my walk-in closet has to be, how many chocolate hearts we get each when we have thirteen or the circumference of my hat size: what we actually do is

counting. To analyze the concept of numbers without taking into account the process of counting seems then rather problematic and hardly fruitful.

It seems that in the logicist tradition, one does not focus on this sort of mathematical activity and the process of counting, but instead one tries to go underneath the mathematical terms and analyze them down to a level where they no longer appear to make any appeal to intuition. So for instance, instead of saying that two numbers are the same, one can analyze this as saying something like:

For any concepts F and G , the number of F -things is equal to the number of G -things if and only if there is a one-one correspondence between the F -things and the G -things.¹

This definition of equinumerosity is known as Hume's Principle, and it is considered as sufficient for proving the basic propositions of arithmetic (the Dedekind/Peano axioms) in second-order logic.² The question is however whether this principle is in fact purely logical or whether some intuition about numbers is presupposed. It seems that one cannot talk about such things as "one-one correspondence" and "equally many" without presupposing intuition about numbers. In *Die Grundlagen der Arithmetik* Frege tries to give an informal explanation of equinumerosity:

If a waiter wishes to be certain of laying exactly as many knives on a table as plates, he has no need to count either of them; all he has to do is to lay immediately to the right of every plate a knife, taking care that every knife on the table lies immediately to the right of a plate.³

But when one uses such expressions as *a* plate and *a* knife, does one not have to presuppose that we can perceive knives and plates as units? One can for instance imagine that a plate can be understood as a stable of plates or some bundle of atoms, so that the waiter lays a knife immediately to the right of a stable of plates. To see something as units is to already have a concept of the number 1 and the difference between one and many.

Since the concept "one-one correspondence" seems to appeal to intuition about numbers, one can try to analyze this as something like a relation R such that: i) every object falling

¹ Zalta (2003), p. 14.

² Cf. Zalta (2003).

³ Frege (1968), § 70.

under F is R -related to a unique object falling under G and ii) every object falling under G is such that there is a unique object falling under F that is R -related to it.⁴ The relation R is then reflexive, symmetric and transitive.

But what is actually done here? If confronted with the definition of the relation R *per se*, would we then be able to see that the topic in question is equinumerosity? Probably not. The Logician aims at finding the bedrock of the concept of numbers. The problem is that when one analyzes equinumerosity as a purely logical relation R , one seems to have gone from a level where the concept of numbers intuitively makes sense to us, to a level where it does no longer have sense unless we somehow interpret the intuitive concept of numbers back into the relation R . In this way, when the focus is taken from our intuitive concept of numbers, one risks that the analysis ends up on a logical level where one apparently loses the overall intuitive perspective of what a number is, while simultaneously pretending to be giving the (logical) essence of the concept of numbers.

An analogous situation can be the following: In Mort Walker's comic *Beetle Bailey*, Sarge is in his office with his dog Otto. Sarge, who is about to drink something from a bottle that seems to contain medicine of some kind, looks at the bottle and says: "I like to know what I put in my stomach." He then starts reading the list of ingredients: "Deionized water, natriumbicarbonate, natriumhydrogencarbonate, emulsifier, E471, whey concentrate..." When finished, Sarge is in shock and bewilderment. Otto, apparently unaffected, thinks to himself: "Now he knows."⁵ So even though Sarge now is supposed to know all the ingredients of the bottle, he is not in a position to find out what these ingredients amount to. Clearly, Sarge would not get any clue about the content of his bottle if he were not told that it for instance is cough syrup, penicillin or ginger beer.

It seems problematic to start out with purely logical terms (as the relation R that is reflexive, transitive and symmetric) and then express the basic concepts of mathematics by means of these (as the concept of equinumerosity). Imagine being confronted with a pile of mechanical items and some tools. In order to be able to put the parts together to a motor, one has to know that it is supposed to be a motor. The focus must be on the final product when putting together the parts.

⁴ Zalta (2003), p. 12.

⁵ Mort Walker: *Beetle Bailey*, King Features Syndicate Inc. 2003.

In the Logicist treatment of the concept of numbers, the very point is to explain what a number really is by means of its detailed logical properties, as in Hume's Principle. However, according to Frege, this principle is not sufficient for giving an explicit definition of numbers. This is because of what is called the Julius Caesar problem, namely that Hume's Principle does not describe completely the conditions under which an arbitrary object is to be identified as a (cardinal) number or not:

...we can never – to take a crude example – decide by means of our definitions whether any concept has the number JULIUS CAESAR belonging to it, or whether that same familiar conqueror of Gaul is a number or is not.⁶

Hume's Principle seems then to offer identity conditions only when x is an object that we already know is a (cardinal) number, and hence, it is a circle definition.⁷ The relation R and the other definitions that the logicist analysis results in does in this way not enable us to decide whether Julius Caesar is a number, and in that case, what number it is. This demonstrates that the logical analysis is insufficient for defining the concept of numbers *qua* numbers.

In the game of Ludo, the pieces get their function in light of a set of playing rules. In Ludo, then, it is exactly this function that gives the pieces a place in the game. For instance, the weight of the pieces is irrelevant for the game itself, even though it might be interesting in some other context. But then one loses sight of the concept of the Ludo pieces *qua* Ludo pieces, and hence the focus of the game. Let us take this example one step further and imagine that we have purely logical formulations of the rules; will someone that is not familiar with Ludo be able to understand the rules and thus the game? Once we move to a logical level, where one's only concern is relations between different combinations of truth-values, one has left the content of the rules out of account. As long as we deal with a purely syntactic, extensional level, as in logic, we are excluded from gaining insight into the content. It is like reading a novel with focus on analyzing every sentence with respect to subject and verbal. In short, one loses focus of the story.

⁶ Frege (1968), § 56.

⁷ In *Grundgesetze der Arithmetik* Frege derived Hume's Principle from his Basic Law V, an axiom that was proven inconsistent with Russell's Paradox. Some logicians claim however that Hume's Principle does not require Basic Law V. Cf. Zalta (2003).

We relate to our world through numbers. Our world, in being spatial and temporal, implies the idea of measurement, and hence of counting. All spatial and temporal expressions such as “long”, “deep”, “often”, “few” and “earlier” contain the potential of numbers. In this way the activity of counting is a way to constitute our world. Accordingly, one cannot abstract a treatment of the concept of numbers from our human agency. The Logician relates to the numbers as autonomous entities. However, the numbers are tools for relating to the world, which means that when we for instance say that we want two rhinoceroses, it is not the number 2 that is in focus, but the rhinos. With a logical approach to mathematics, the focus is not on the mathematical insights, but on truth-values and combinations of these. Once we deal with logical formulas and definitions, we run the risk of losing focus on the numbers we live by, but also on our world.

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