UNIVERSITY OF TROMSØ
FACULTY OF SCIENCE

Exam: MAT-0001
Day: February 19th 2007
Time 09.00-13.00

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1

On a fish-farm some place in Finnmark, they are also producing containers for transporting fish in. They have capacity to produce a lot more containers than they use, the rest they sell to other fish-farms in the area. They produce two types of containers. Most of the work is done by a machine, but the finish has to be done by hand. Actually, the two types of containers only differs in the work done by hand. The machine produces 6 containers in one hour and has a capacity to do this for 1000 hours a year.

The finish, done by hand, takes 30 minutes for a type A container and 15 minutes for a type B container. Two men are doing this and they work a total of 2000 hours each year.

Let x be the number of type A containers, and let y be the number of type B containers produced. The first year of production they plan to use of 1000 of each type container and sell the rest. The profit for a type A container is 150 NOK, and for a type B container it is 100 NOK.

	Type A	Type B	Capacity
Machine	$\frac{1}{6}$ hours	$\frac{1}{6}$ hours	1000 hours
By hand	$\frac{1}{2}$ hours	$\frac{1}{4}$ hours	2000 hours
Profit in NOK	150	100	

a) Explain that x and y satisfies

$$\begin{aligned} x + y &\leq 6000 \\ 2x + y &\leq 8000 \\ x &\geq 1000 \\ y &\geq 1000 \end{aligned}$$

b) In a Cartesian coordinate system, sketch the area described by the inequations in a).

c) Find an expression for the profit.

d) Find the values of x an y that maximizes the profit. What is the maximal profit?

2

For values of x, varying by integer steps from 1 to 10, the following measurements were made on a dependent variable y.

x	1	2	3	4	5	6	7	8	9	10
y	60	36	21	13	8	5	3	1.9	1.1	0.6

The dependency could be given by an exponential function

$$y = ca^x,$$

or by a power function

$$y = cx^r$$

a) Extend the table above with a third and fourth row containing the values $X = \log(x)$ and $Y = \log(y)$.

b) Check by plotting the coordinate pairs (X, Y) for all 10 values of X and Y if the dependency could be given by a power function. Explain why it is, or why it is not.

c) Check if the dependency could be given by an exponential function. Explain why you conclude as you do.

d) You will get a positive answer in either b) or c). Find the dependency, that is compute c, and either a or r.

3

a) Minimize the function $f(x,y) = x^2 + \frac{1}{2}y^2$ subject to the condition $x^2y = 8$.

b) Find and classify all critical points on $f(x,y) = 2x^3 - 6xy + 3y^2$.

4

Do not use calculator in this exercise.

a) For the function $f(x) = \frac{1}{\pi}\sin(\pi x) + \ln(x)$ find the tangent line at $x = 1$. Use linear approximation to find an an approximate value for $f(1,1)$.

b) Compute the integral

$$\int_0^8 \frac{\cos(\sqrt{x+1}}{\sqrt{x+1}} dx$$

The answer is $2\sin(3) - 2\sin(1)$.