

PAPER

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CAN EXPERIENCES FROM USE OF COMPASS INFLUENCE GEOMETRICAL THINKING?

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Norwegian mathematics and science teaching have a boring image and are experienced to be of little relevance to young Norwegians (UFD, 2002). Thus Norwegian Government has made a strategy plan to improve teachers' and pupils' motivation to mathematics and science. Interdisciplinary activities are given prominence in the Norwegian curriculum (KUF, 1995), and curriculum opens for new ways of mathematics teaching, too:

Children learn with all their senses and by actively using their bodies.

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At the lower secondary stage, greater emphasis is given to the formal and abstract aspects of the subject and to the use of mathematics in society. Practical situations and pupils' own experience remain important, however.

(ibid., Chapters of Physical Education and Mathematics, Respectively)

According to curriculum from 1997, pupils' own experience is important to mathematics teaching during primary school. Government's Strategy plan in 2002, however, states that mathematics teaching is experienced to be of little relevance to young Norwegians. This could be due to it is difficult to perform mathematics teaching with close connection to pupils' reality.

Based on this we have used compass as tool for both teacher students' and primary school pupils' work with geometry. Compass is a naturally used tool at school trips. When you ask Norwegian grown ups what they remember from their school time, school trips are usual answers. These answers are natural, due to that Norwegian curriculum says pupils shall have outdoor experience, like a night in tent for example, during their school time. Our idea was to use an outdoor school trip as frame for teaching. At intermediate stage, the Norwegian curriculum opens for use of compass in geometry: At grade 5 pupils shall acquire basic knowledge about and practise the use of compass. At grade 6 pupils shall experience angles as rotation about a point and as two rays radiating from a point, and become acquainted with the measurement of angles. This leads to the use of compass at intermediate stage, mathematics outdoor interdisciplinary lessons.

As we see it, content of this paper is much the same as what "Kvalitetsutvalget" (Utdanningsforbundet, 2003) points out to be essential for future Norwegian teaching; basic knowledge, use of competency for change and physical activities. Isberg (1991) says, to really have something in your hands and then reflect about in your head, is necessary to develop knowledge about it. This yields especially lower and intermediate level of graduate school. As pupils become elder, mathematics thinking to an increasing extent becomes abstract; thus many pupils then do not need to view a geometrical figure to be able to talk about its

properties. This work connecting geometry to pupils' own reality is part of a web framing pupils' geometrical understanding.

Connecting theoretical issues to physical activities, can give teachers opportunities to combine important teaching time to both practical activities and theoretical issues.

THREE DIFFERENT SPACES

The space representation produced by children's usual out-of-school experiences is quite different from elementary geometry (Berthelot & Salin, 1998). One of the main sources of learning difficulties in geometry among students from 12 years age and older, is probably the previous treatment of geometrical figures on paper during elementary school.

We suppose it is necessary to get conscious about the size and dimensions of geometry in our teaching:

...natural knowledge of space is strongly structured into three main representations: *microspace* (corresponding to the usual prehension relations), *mesospace* (corresponding to the usual domestic spatial interactions) and *macrospace* (corresponding to unknown city, maritime or rural spaces...) In consequence, the space representation produced by the usual out-of-school experiences is not naturally homogenous, and is quite different from elementary geometry. (Berthelot and Salin, 1998, p 72)

Our Norwegian geometrical teaching traditionally does not include physical activity. Pupils observe geometrical figures by primary use of their eyes. Pupils' usual leisure interactions take place in mesospace, however. Our assumption is that the gap between passive observation and active physical experience leads to problems with connecting geometrical teaching to physical experiences. Curriculum's Chapter of Physical Activity says children learn by actively using their bodies, curriculum does not say anything about whether this yields geometry learning as well.

Children's experiences are three-dimensional and from mesospace. School geometry often is two-dimensional and takes place in micro space. Many teachers think geometry is concrete because it consists of drawn figures and forget that these figures are abstractions both in size and dimension. *"To obtain success in leading a person somewhere, you have to search where the person is located at that moment and then start walking from there."* (S. Kierkegaard). Three-dimensional models can be experienced as concrete for some pupils. To others they can be experienced as diminished versions of concrete figures, and diminishing is one kind of abstraction.

When the athlete Plato was looking for a proper place to situate his Academy, he was looking for a sports arena with a building nearby. Above the entrance of this Academy, Plato wrote in solid rock: "Let no one ignorant of Geometry enter this door." We think it could be a good idea to pick up these ideas of Plato and continue focusing on physical activities and geometry.

GEOMETRICAL CONCEPTS

Vygotskij (Vygotsky, 1999) distinguishes between spontaneous and scientific concepts. Spontaneous concepts are unfinished and based on children's own sensory material. Scientific concepts, however, are based on words. Both sensory material and words are indispensable parts of concept formation. If the word is studied separately, focus is only at the verbal plane. That is not characteristic of the child's thinking; the child's total experiences influences its thinking.

The development of scientific concepts is closely connected to the development of spontaneous concepts: *"The child becomes conscious of his spontaneous concepts relatively late; the ability to define them in words, to operate with them at will, appears long after he*

has acquired the concepts. The child has the concept (i.e., knows the object to which the concept refers), but is not conscious of his own act of thought." (ibid. p. 192).

Vygotskij further claims (ibid., p. 146) "*To devise successful methods of instructing the school child in systematic knowledge, it is necessary to understand the development of scientific concepts in the child's mind.*"

To large extent children's experiences are body experiences, not verbal experiences. Most of these experiences become automated moves; one of the body's ways of releasing brain from unnecessary work. Small children normally do not speak about their experiences in terms of geometrical concepts. They have a fundament for spontaneous concepts about rotation, a fundament for relating the concept to reality; they know how to rotate their body with their spinal chord as an axis. They have a fundament for spontaneous concepts about angles, too; half a turn is less than a full turn. Kinder garden personnel need consciousness about importance of language using connected to children's physical activities and experiences.

School geometry often introduces the angle concept by showing pupils visual examples of angles or by presenting them for angles drawn on the blackboard. The students' perception of angle then is reduced to a passive visual glance at figures. Afterwards pupils measure and draw angles themselves at paper. School geometry does not seem to have in mind the pupils' total perceptuated experience. Angles are normally taught as static phenomena (Johnsen, 1996); the Norwegian Curriculum of 1997 introduced the idea of angles as dynamic phenomena. According to Brekke et al (1998) focus on 180° as half a turn could help pupils getting a better angle concept. It has been problematic to Norwegian pupils to think of 180° as an angle.

Could it be that the children's development of scientific concepts in geometry would increase if their experiences from work with compass were connected to their performing figures and patterns on paper? Our Norwegian mathematics teaching tradition is not known for using children's total knowledge and experiences as a fundament. Berthelot & Salin (1998) supports our present curriculum, however, by pointing out that children's own experiences are essential in mathematics teaching. Could it be that the pupils' expectations to their own learning ability in mathematics would increase, if the teaching were based on outdoor activities familiar to the pupils? This kind of teaching agrees with the curriculum.

However, what about teachers? Some of them want to learn how to let their mathematics teaching follow the curriculum, but most teachers just practice their teaching in ways they are convinced is the best. Probably they will not change their way of teaching just because some researcher say it could be a smart thing to do.

VISIBLE AND HIDDEN GEOMETRY

To most public mathematics is largely invisible and unrecognised (Niss, 1994). Even some mathematicians and mathematics educators do not have a clear picture of the role of mathematics in society. The key to explaining this could lie in the fact that mathematics is not found on the surface of the matters to which attention is paid. "*The mathematics is invisible because it is hidden, not because it is absent*" (ibid. p 372). If teacher students experience they are able to discover some hidden mathematics, then maybe they think there could be some more mathematics for them to discover. It is necessary for teachers to see the mathematics invisible to them before they are able to include this mathematics in their teaching. It is necessary for teachers to have in mind that much mathematics visible to them is invisible to their pupils; many connections obvious to teachers are not that obvious to pupils.

One possible way of changing mathematics teaching tradition, is to give teacher students and teachers some good experiences; to make them able to discover some of their

own hidden geometrical knowledge. Knowledge related to their experience from outdoor physical activities.

Our experience is that teacher students increase their understanding of didactics by performing outdoor physical activities during mathematics lessons (Fyhn and Frenning, 2003).

TEACHER STUDENTS AT TROMSØ UNIVERSITY COLLEGE

Teacher students at our university college have performed outdoor mathematics work where sports and movements were closely connected to mathematics. This work was led by assistant professors from both these subjects. The compass was essential in this program due to the idea that compass is a useful tool for working with angles as dynamic topics. Using the compass is an activity that is worked out more easy if the users co-operate with others than if they work individually. Teacher students then wrote down their teaching ideas and ideas intentions before they had done any teaching on their own. The idea was to give teacher students some time to reflect upon how to teach mathematics as early as possible during their teacher study.

Performing physical activities during mathematics lessons gives teacher and students possibilities of non-verbal communication. The non-verbal communication can support the verbal words and the verbal words can support the non-verbal communication, too. Thus teaching cares about both students' words and their sensory material. Combining verbal and non-verbal communication can make it easier to the teacher to avoid using a language at a too high level for the students.

One task was the compass. Students were split into groups of four or five persons. Each group had one compass and one spray box containing paint. As a start, each group made several circles with common centre. One student was located at the centre. The others were holding hands, performing a straight line rotating around this centre. The last group member was painting the peripheries marking the others' steps. Then they marked up north – south – east and west, according to the compass. Then groups were given geometrical tasks including use of compass in different ways: "Where is north?", "Which direction is 270°?", "How many degrees is needed to rotate to turn from east to north?", and so on. Finally we used this blink for playing, by arranging a "ski shooting" competition; Ready – go! Nobody doubts where centre is.

Another task is "Star orientation": Choose an open area with free view in most directions. Some labelled posts are placed in the surroundings, at given degrees of direction and circa number of steps. Alternatively, students can pose the posts as a start. (In darkness, use reflects and lamps and no number of given steps). When posts are placed, everyone returns to centre for new tasks. The total amount of labels makes a rebus, a letter code or nothing. What is to be done with compass to get back to centre from each of the posts?

This outdoor teaching was followed by a written task to make students' do some reflection afterwards. Elsewhere we could risk some outdoor activity on its own could end up only as a nice happening.

TEACHER STUDENTS' TEACHING

One of these groups chose a teaching program for their period of practice that included outdoor and indoor geometry in 7th grade. Their teaching program included use of compass, too. According to these aspects, after half of their practice period the group was given a closer follow up. Rest of the group and researcher in mathematics education evaluated their mathematics lessons afterwards. Having finished their period of practice, students answered an open question: *"How do you meet with primary school mathematics? What thoughts do*

you have?" Students answered the question by writing letters, "the letter method" (Berg, 1999).

The group of students counted five persons with a variety of experiences from learning mathematics. One of them enjoyed mathematics at school, some disliked it and one was fond of the subject even though she had some learning difficulties. In the beginning, one of them did not even believe in this kind of teaching at all, and she was not fond of outdoor activities, either. She made up her mind to give this project a try; after all she was part of a group. When students told teachers at their practice school about their mathematics teaching-program, they were met with scepticism and laughter.

Only two of the students were familiar with the compass before they got started, the three others did not know anything about how to use a compass. Thus they asked the experienced ones to be responsible for the part of the teaching concerning the compass. The complete group of students then spent some hours in the gym working with the compass and helping each other to understand how to use it. To be able to take an active part in the teaching, all five students needed to see the mathematics "hidden" in the compass before they could be able to include this mathematics in their teaching.

The teaching program started with introduction of compass – how it worked and how to use it. The children were asked what mathematics they could find in a compass. According to students, the pupils' answers were circle, angles, degrees, radius, diameter and rotation. By giving pupils such a question, students gave pupils opportunity to start analysing the mathematics content of the forthcoming activity. Several of the pupils' scientific concepts were challenged, too. Students let the children use their own words about the compass' mathematics. It is not obvious to children that use of a compass is use of mathematics. That is not obvious to all mathematics teachers either.

Then the complete group continued working outside school. All outdoor activities took part in the three dimensional mesospace (Berthelot & Salin, 1998). As introduction, pupils were placed in two rods. The student who performed this lesson asked pupils to turn 90° to the right, then 180° to the left and so on. By performing these turns, students focused both on pupils' spontaneous and scientific concepts about turning. The students argued they did it this way to see if pupils had understood the explanation given in the classroom. I think these actions gave all pupils possibility to start understanding what to do and how to do it. It is important to remember, however, that pupils are not a homogenous group; some of them are supposed to have quite a well-developed angle concept already. It was natural and not forbidden to pupils to watch each other and to copy other students' moves. The teacher student asked the pupils to turn and turn, to the left and to the right, 270° , 180° and so on, until the pupils acted as a synchronised group. By doing this, after a while the students could see that all the pupils then understood how to perform these turns. At that moment the entire class was able to carry out some mathematics exercises in a correct way. All pupils then had some knowledge about turning and degrees in one specific context. They had some common mathematics experience that could be used as a fundament for classroom teaching.

Next, pupils were given "Star orientation" with different written geometrical tasks placed in the environment.

TEACHER STUDENTS' REFLECTIONS

Students wrote in their report that pupils' attitudes towards mathematics seemed to be better from the outdoor working; the pupils were really engaged in the tasks and they enjoyed the work. Afterwards, in the classroom, students were given lots of possibilities to talk and discuss geometry with pupils. During classroom work afterwards, all of students observed pupils making use of outdoor experiences while working with mathematics. All students

agreed that it was necessary for them as mathematics teachers to take an active part in the outdoor activities.

Every one of the students told they had given hints to pupils when pupils did not at first glance see the connections between outdoor and indoor exercises. Students did not tell pupils about these connections, by giving hints they led pupils to find the connections themselves.

In their final report students wrote: “*After having observed pupils for these two lessons, we were confirmed that pupils had learned something from the outdoor day. It was exiting to see how outdoor activities were used in connection with mathematical activities in the classroom.*” (Solhaug, Skau, Sørbye, Valanes og Kind, 2003, p 9, our translation). The student who did not believe in the project wrote afterwards: “*As I see it, this was a success among pupils. It is my opinion that all of them had learned something from this teaching program. It was real nice to watch pupils’ engagement and to which extent they used outdoor experience while working with mathematics afterwards. I guarantee I will make use of this kind of teaching when I become a teacher.*”

ROUND OFF

Is it so that performing outdoor physical activities in the mathematics lessons can give rise to better geometrical understanding? Maybe it is that way. But there are several difficulties, both in how to perform the out-door activities, and in how teachers connect the following classroom work to pupils’ experiences from out-door activities. Some teacher students – and teachers as well – neither arrange nor take part in out-door activities. Thus there is reason to believe they do not engage in this kind of teaching. Or- some of them may think these outdoor activities consume important time that could be spent doing more serious schoolwork.

The living body is three-dimensional, dynamic, and relates to both micro-, meso- and macrospace. Learning has to take this seriously, to let the children’s concepts development relate to reality. Learning is a life long project; concept learning goes on in kinder garden as well as in school and university. Finally, we must not forget that most learning goes on outside institutions.

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