

12.5

have to find $\frac{dr}{dz}$ along the geodesics

[determined by l and e ; $l=0$ radial geodesics]

energy measured by stationary observer at $\tilde{r}=10$:

$$E = -\underline{p} \cdot \underline{u} \stackrel{!}{=} m$$

$$\underline{u} = \left[\left(1 - \frac{2M}{\tilde{r}}\right)^{-1/2}, 0, 0, 0 \right]_{\tilde{r}=10}$$

$$\underline{p} = m \underline{u} \Rightarrow u^t$$

$$m = -m \underline{u} \cdot \underline{u} \quad \text{or} \quad -1 = \underline{u} \cdot \underline{u} = u^t \underbrace{\left(1 - \frac{2M}{\tilde{r}}\right)^{-1/2}}_{U^t} g_{tt}$$

$$\Rightarrow u^t = U^t = \left(1 - \frac{2M}{\tilde{r}}\right)^{-1/2} =$$

$$= \left(1 - \frac{2}{10}\right)^{-1/2} = \sqrt{5}/2$$

$$\text{conserved energy: } e = -\underline{J} \cdot \underline{u} = \left(1 - \frac{2M}{\tilde{r}}\right) \frac{dt}{dz} \quad (3.21)$$

$$= \left(1 - \frac{2M}{\tilde{r}}\right) u^t = \frac{2}{\sqrt{5}}$$