

$$\underline{u} \cdot \underline{u} = -1 = -\left(1 - \frac{2M}{r}\right) (u^t)^2 + \left(1 - \frac{2M}{r}\right)^{-1} (u^r)^2$$

$\parallel \left(\frac{dr}{d\tau}\right)^2$

$$\begin{aligned} \left(\frac{dr}{d\tau}\right)^2 &= -\left(1 - \frac{2M}{r}\right) + \underbrace{(u^t)^2 \left(1 - \frac{2M}{r}\right)^2}_{e^2} = \frac{4}{5} - 1 + \frac{2M}{r} \\ &= \frac{2M}{r} - \frac{1}{5} \end{aligned}$$

test:  $\frac{dr}{d\tau} = 0$  for  $r = 10M$

$$\tau = \int_0^{10M} dr \frac{dr}{\frac{dr}{d\tau}} = \int_0^{10M} dr \left(\frac{2M}{r} - \frac{1}{5}\right)^{-1/2} \quad r = 10M \}$$

$$\begin{aligned} &= 10M \int_0^1 d\zeta \left(\frac{2}{10\zeta} - \frac{1}{5}\right)^{-1/2} = 10\sqrt{5}M \underbrace{\int_0^1 d\zeta \left(\frac{1}{3} - \zeta\right)^{-1/2}}_{\frac{\pi}{2}} \\ &= 5\sqrt{5}\pi M \end{aligned}$$