

15.13

$$C = \int_0^{2\pi} d\varphi \sqrt{g_{\varphi\varphi}}$$

$$g_{\varphi\varphi} = \left(r^2 + a^2 + \frac{2Mr a^2 \sin^2 \vartheta}{r^2} \right) \sin^2 \vartheta$$

equatorial: $\sin \vartheta = 1, \cos \vartheta = 0$; $g^2 = r^2 + a^2 \cos^2 \vartheta = r^2$

$$g_{\varphi\varphi} = r^2 \left(1 + \frac{a^2}{r^2} + \frac{2Ma^2}{r^3} \right) = r^2 \left[1 + \frac{a^2}{r^2} \left(1 + \frac{2M}{r} \right) \right]$$

$$\Rightarrow C = 2\pi r \left[1 + \frac{a^2}{r^2} \left(1 + \frac{2M}{r} \right) \right]^{1/2}$$

$$\frac{a}{M} = 0$$

$$r/M = 6$$

$$\frac{C}{2\pi M} = 6M$$

⋮

Fig 15.3

⋮

$$\frac{a}{M} = 1$$

$$r/M = 1$$

$$\begin{aligned} \frac{C}{2\pi} &= 2M \left[1 + 1(1+2) \right]^{1/2} \\ &= 2M \end{aligned}$$

