

18.8

metric

$$ds^2 = -dt^2 + a^2(t) dl^2$$

translation invariance $\Rightarrow$ Killing vectors $\xi = (0, 1, 0, 0) \dots$

$\Rightarrow$ if p is four-momentum of a photon, $\xi \cdot p = \text{const.}$

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$$\xi \cdot p = g_{ab} \xi^a p^b = g_{xx} p^x = a^2 p^x = \text{const} \quad \Rightarrow \quad p^x \propto \frac{1}{a^2}$$

p is null vector, $p \cdot p = 0$

$$\Rightarrow - (p^t)^2 + a^2 (p^x)^2 = 0$$

$$\Rightarrow p^t \propto a p^x \propto \frac{1}{a}$$

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observer at rest, $\underline{u} = (1, 0, 0, 0)$, measures

$$v = - p \cdot \underline{u} = p^t \propto \frac{1}{a}$$