

98/ a) (9.46) gives $\Omega = \left(\frac{M}{r^3}\right)^{1/2} = \frac{d\phi}{dt}$

t is proper time at infinity, $\Rightarrow \Omega$ is angular velocity as measured by observer at ∞

$$P_{\infty} = \frac{2\pi}{\Omega} = 2\pi \left(\frac{r^3}{M}\right)^{1/2} = 2\pi \cdot 7^{3/2} M = 14\pi \sqrt{7} M$$

$\nwarrow$
 $r=7M$

b) we need $\frac{d\phi}{d\tau} = \frac{d\phi}{dt} \frac{dt}{d\tau} = \Omega \frac{dt}{d\tau}$

circular orbit is equidistant place:

$$\underline{u} \cdot \underline{u} = -1 = -\left(1 - \frac{2M}{r}\right) \left(\frac{dt}{d\tau}\right)^2 + \underbrace{r^2 \left(\frac{d\phi}{d\tau}\right)^2}_{r^2 \Omega^2 \left(\frac{dt}{d\tau}\right)^2}$$

$$-1 = -\left(1 - \frac{2M}{r} - \underbrace{r^2 \Omega^2}_{M/r}\right) \left(\frac{dt}{d\tau}\right)^2 \Rightarrow \frac{dt}{d\tau} = \sqrt{\frac{7}{4}}$$

$$1 - \frac{3M}{r} = 1 - \frac{3}{7} = \frac{4}{7}$$

$$\Rightarrow P_{\text{space}} = \sqrt{\frac{4}{7}} P_{\infty} = \sqrt{\frac{4}{7} \cdot 7} \cdot 14\pi M = 28\pi M$$