

11 Early universe

11.1 Time-line of important dates

Different energy form today. Let us summarize the relative importance of the various energy forms today. The critical density $\rho_{\text{cr}} = 3H_0^2/(8\pi G)$ has with $h = 0.7$ today the numerical value $\rho_{\text{cr}} \approx 7.3 \times 10^{-6} \text{ GeV/cm}^3$. This would corresponds to roughly 8 protons per cubic meter. However, main player today is the cosmological constant with $\Omega_\Lambda \approx 0.73$. Next comes (pressure-less) matter with $\Omega_m \approx 0.27$ that consists mostly of non-baryonic dark matter, while only $\Omega_b = 4\%$ of the total energy density of the universe consists of matter that we know. The energy density of cosmic microwave background (CMB) photons with temperature $T = 2.7 \text{ K} = 2.3 \times 10^{-4} \text{ eV}$ is $\rho_\gamma = aT^4 = 0.4 \text{ eV/cm}^3$ or $\Omega_\gamma \approx 5 \times 10^{-5}$.

The contribution of the three neutrino flavors to the energy density depends on the unknown absolute neutrino mass scale, $5 \times 10^{-5} \lesssim \Omega_\nu \lesssim 0.05$. The lower bound corresponds to three (effectively) massless neutrinos, the upper to one massive neutrino flavor with $m_\nu \sim 0.3 \text{ eV}$.

Different energy forms as function of time The scaling of Ω_i with redshift z , $1 + z = R_0/R(t)$ is given by

$$H^2(z)/H_0^2 = \Omega_{\text{m},0}(1+z)^3 + \Omega_{\text{rad},0}(1+z)^4 + \Omega_\Lambda - \underbrace{(\Omega_{\text{tot},0} - 1)(1+z)^2}_{\approx 0}. \quad (11.1)$$

Thus the relative importance of the different energy forms changes: Going back in time, one enters first the matter-dominated and then the radiation-dominated epoch.

The cosmic triangle shown in Fig. 11.1 illustrates the evolution in time of the various energy components and the resulting coincidence problem: Any universe with a non-zero positive cosmological constant will be driven with time to a fix-point with $\Omega_m, \Omega_k \rightarrow 0$. The only other non-evolving state is a flat universe containing only matter—however, this solution is unstable. Hence, the question arises why we live in an epoch where all energy components have comparable size.

Temperature increase as $T \sim 1/R$ has three main effects: Firstly, bound states like atoms and nuclei are dissolved when the temperature reaches their binding energy, $T \gtrsim E_b$. Secondly, particles with mass m_X can be produced, when $T \gtrsim 2m_X$, in reactions like $\gamma\gamma \rightarrow \bar{X}X$. Thus the early Universe consists of a plasma containing more and more heavier particles that are in thermal equilibrium. Finally, most reaction rates $\Gamma = n\sigma v$ increase faster than the expansion rate of the universe for $t \rightarrow 0$, since $n \propto T^3$ for relativistic particles, while $H \propto \rho_{\text{rad}}^{1/2} \propto T^2$. Therefore, reactions that have become ineffective today were important in the early Universe.

Matter-radiation equilibrium z_{eq} : The density of matter decreases slower than the energy density of radiation. Going backward in time, there will be therefore a time when the density

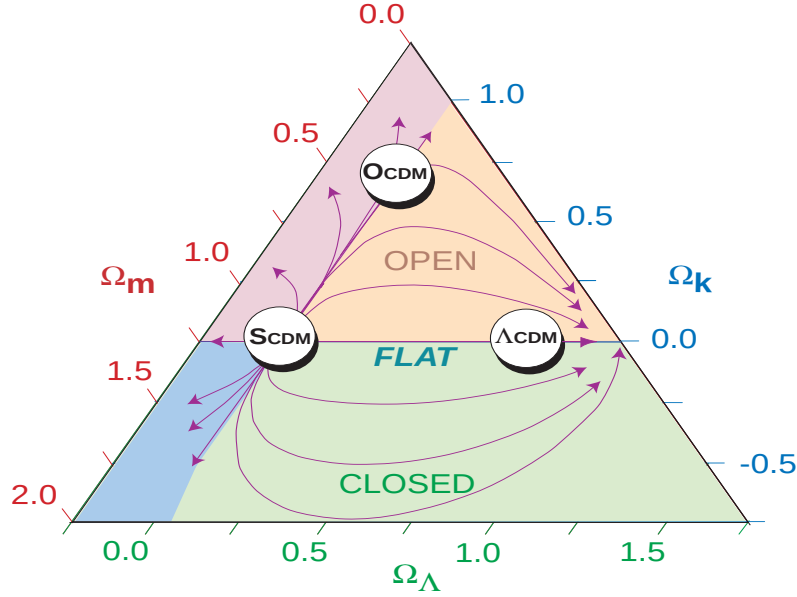


Figure 11.1: The cosmic triangle showing the time evolution of the various energy components.

of matter and radiation were equal. Before that time with redshift z_{eq} , the universe was *radiation-dominated*,

$$\Omega_{\text{rad},0}(1+z_{\text{eq}})^4 = \Omega_{\text{m},0}(1+z_{\text{eq}})^3 \quad (11.2)$$

or

$$z_{\text{eq}} = \frac{\Omega_{\text{m},0}}{\Omega_{\text{rad},0}} - 1 \approx 5400. \quad (11.3)$$

This time is important, because i) the time-dependence of the scale factor changes from $R \propto t^{2/3}$ for a matter to $R \propto t^{1/2}$ for a radiation dominated universe, ii) the E.o.S. and thus the speed of sound changed from $w \approx 1/3$, $v_s^2 = (\partial P / \partial \rho)_S = c^2/3$ to $w \approx 0$, $v_s^2 = 5kT/(3m) \ll c^2$. The latter quantity determines the Jeans length and thus which structures in the Universe can collapse.

Recombination z_{rec} : Today, hydrogen and helium in the interstellar and intergalactic medium is neutral. Increasing the temperature, the fraction of ions and free electron increases, i.e. the reaction $H + \gamma \leftrightarrow H^+ + e^-$ that is mainly controlled by the factor $\exp(-E_b/kT)$ will be shifted to the right. By definition, we call recombination the time when 50% of all atoms are ionized. A naive estimate gives $kT \sim E_b \approx 13.6 \text{ eV} \approx 160,000 \text{ K}$ or $z_{\text{rec}} = 60,000$. However, there are many more photons than hydrogen atoms, and therefore recombination happens later: A more detailed calculation gives $z_{\text{rec}} \sim 1000$.

Since the interaction probability of photons with neutral hydrogen is much smaller than with electrons and protons, recombination marks the time when the Universe became transparent to light.

Big Bang Nucleosynthesis At $T_{ns} \sim \Delta \equiv m_n - m_p \approx 1.3$ MeV or $t \sim 1$ s, part of protons and neutrons forms nuclei, mainly ${}^4\text{He}$. As in the case of recombination, the large number of photons delays nucleosynthesis relative to the estimate $T_{ns} \approx \Delta$ to $T_{ns} \approx 0.1$ MeV.

Quark-hadron or QCD transition Above $T \sim m_\pi \sim 100$ MeV, hadrons like protons, neutrons or pions dissolve into their fundamental constituents, quarks q and gluons g .

Baryogenesis All the matter observed in the Universe consists of matter (protons and electrons), and not of anti-matter (anti-protons and positrons). Thus the baryon-to-photon ratio is

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} = \frac{n_b}{n_\gamma} = \frac{\Omega_b \rho_{\text{cr}} / m_N}{2\zeta(3)T_\gamma^3 / \pi^2} \approx 7 \times 10^{-10}. \quad (11.4)$$

The early plasma of quarks q and anti-quarks $\bar{q}$ contained a tiny surplus of quarks. After all anti-matter annihilated with matter, only the small surplus of matter remained. The tiny asymmetry can be explained by interactions in the early Universe that were not completely symmetric with respect to an exchange of matter-antimatter.

11.2 Equilibrium statistical physics in a nut-shell

The distribution function $f(p)$ of a free gas of fermions or bosons in *kinetic equilibrium* are

$$f(p) = \frac{1}{\exp[\beta(E - \mu)] \pm 1} \quad (11.5)$$

where $\beta = 1/T$ denotes the inverse temperature, $E = \sqrt{m^2 + p^2}$, and $+1$ refers to fermions and -1 to bosons, respectively. As we will see later, photons as massless particles stay also in an expanding universe in equilibrium and may serve therefore as a thermal bath for other particles. A species X stays in kinetic equilibrium, if e.g. in the reaction $X + \gamma \rightarrow X + \gamma$ the energy exchange with photons is fast enough.

The chemical potential μ is the average energy needed, if an additional particle is added, $dU = \sum_i \mu dN_i$. If μ is zero, If the species X is also in *chemical equilibrium* with other species, e.g. via the reaction $X + \bar{X} \leftrightarrow \gamma + \gamma$ with photons, then their chemical potentials are related by $\mu_X + \mu_{\bar{X}} = 2\mu_\gamma = 0$.

The number density n , energy density ρ and pressure P of a species X follows as

$$n = \frac{g}{(2\pi)^3} \int d^3p f(p), \quad (11.6)$$

$$\rho = \frac{g}{(2\pi)^3} \int d^3p E f(p), \quad (11.7)$$

$$P = \frac{g}{(2\pi)^3} \int d^3p \frac{p^2}{3E} f(p). \quad (11.8)$$

The factor g takes into account the internal degrees of freedom like spin or color. Thus for a photon, a massless spin-1 particle $g = 2$, for an electron $g = 4$, etc.

Derivation of the pressure integral for free quantum gas:)

Comparing the 1. law of thermodynamics, $dU = TdS - PdV$, with the total differential $dU = (\partial U/\partial S)_V dS + (\partial U/\partial V)_S dV$ gives $P = -(\partial U/\partial V)_S$.

Since $U = V \int E f(p)$ and $S \propto \ln(V f(p))$, differentiating U keeping S constant means $P = -V \int (\partial E/\partial V) f(p)$.

We write $\partial E/\partial V = (\partial E/\partial p)(\partial p/\partial L)(\partial L/\partial V)$. To evaluate this we note that $\partial E/\partial p = p/E$, that from $V = L^3$ it follows $\partial L/\partial V = 1/3L^2$ and that finally the quantization conditions of free particles, $p_k = 2\pi k/L$ implies $\partial p/\partial L = -p/L$. Combined this gives $\partial E/\partial V = -p^2/(3EV)$.

In the non-relativistic limit $T \ll m$, $e^{\beta(m-\mu)} \gg 1$ and thus differences between bosons and fermions disappear,

$$n = \frac{g}{2\pi^2} e^{-\beta(m-\mu)} \int_0^\infty dp p^2 e^{-\beta \frac{p^2}{2m}} = g \left(\frac{mT}{2\pi} \right)^{3/2} \exp[-\beta(m-\mu)], \quad (11.9)$$

$$\rho = mn, \quad (11.10)$$

$$P = nT \ll \rho. \quad (11.11)$$

These expressions correspond to the classical Maxwell-Boltzmann statistics¹. The number of non-relativistic particles is exponentially suppressed, if their chemical potential is small. Since the number of protons per photons is indeed very small in the universe, cf. Eq. (11.4), and therefore also the number of electron (the universe should be neutral), the chemical potential μ can be neglected in cosmology at least for protons and electron.

In the relativistic limit $T \gg m$ with $T \gg \mu$ all properties of a gas are determined by its temperature T ,

$$n = \frac{g T^3}{2\pi^2} \int_0^\infty dx \frac{x^2}{e^x \pm 1} = \varepsilon_1 \frac{\zeta(3)}{\pi^2} g T^3, \quad (11.12)$$

$$\rho = \frac{g T^4}{2\pi^2} \int_0^\infty dx \frac{x^3}{e^x \pm 1} = \varepsilon_2 \frac{\pi^2}{30} g T^4, \quad (11.13)$$

$$P = \rho/3, \quad (11.14)$$

where for bosons $\varepsilon_1 = \varepsilon_2 = 1$ and for fermions $\varepsilon_1 = 3/4$ and $\varepsilon_2 = 7/8$, respectively.

Since the energy density and the pressure of non-relativistic species is exponentially suppressed, the total energy density and the pressure of all species present in the universe can be well-approximated including only relativistic ones,

$$\rho_{\text{rad}} = \frac{\pi^2}{30} g_* T^4, \quad (11.15)$$

$$P_{\text{rad}} = \rho_{\text{rad}}/3 = \frac{\pi^2}{90} g_* T^4, \quad (11.16)$$

where

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T} \right)^4. \quad (11.17)$$

Here we took into account that the temperature of different particle species can differ.

¹Integrals of the type $\int_0^\infty dx x^{2n} e^{-ax^2}$ can be reduced to a Gaussian integral by differentiating with respect to the parameter a .

Entropy Rewriting the first law of thermodynamics, $dU = TdS - PdV$, as

$$dS = \frac{dU}{T} + \frac{P}{T} dV = \frac{d(V\rho)}{T} + \frac{p}{T} dV = \frac{V}{T} \frac{d\rho}{dT} dT + \frac{\rho + P}{T} dV \quad (11.18)$$

and comparing this expression with the total differential $dS(T, V)$, one obtains

$$\frac{\partial S}{\partial V_T} = \frac{\rho + P}{T}. \quad (11.19)$$

Since the RHS is independent of V for constant T , we can integrate and obtain

$$S = \frac{\rho + P}{T} V + f(T). \quad (11.20)$$

The integration constant $f(T)$ has to vanish to ensure that S is an extensive variable, $S \propto V$.

The total entropy density $s \equiv S/V$ of the universe can again be approximated by the relativistic species,

$$s = \frac{2\pi^2}{45} g_{*S} T^3, \quad (11.21)$$

where now

$$g_{*,S} = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T} \right)^3. \quad (11.22)$$

The entropy S is an important quantity because it is conserved during the evolution of the universe. Conservation of S implies that $S \propto g_{*,S} R^3 T^3 = \text{const.}$ and thus the temperature of the Universe evolves as

$$\boxed{T \propto g_{*,S}^{-1/3} R^{-1}}. \quad (11.23)$$

When g_* is constant, the temperature $T \propto 1/R$. Consider now the case that a particle species, e.g. electrons, becomes non-relativistic at $T \sim m_e$. Then the particles annihilate, $e^+ + e^- \rightarrow \gamma\gamma$, and its entropy is transferred to photons. Formally, $g_{*,S}$ decreases and therefore the temperature decreases for a short period less slowly than $T \propto 1/R$.

Since $s \propto R^{-3}$ and also the net number of particles with a conserved charge, e.g. $n_B \equiv n_B - n_{\bar{B}} \propto R^{-3}$ if baryon number B is conserved, the ratio n_B/s remains constant.

Relativistic degrees of freedom. To obtain the number of relativistic degrees of freedom g_* in the universe as function of T , we have to know the degrees of freedom of the various particle species:

- The spin degrees of freedom of massive particles with spin s are $2s + 1$, and of neutrinos 1, where we count particles and anti-particles separately. Massless bosons like photons and gravitons are their own anti-particle and have 2 spin states.
- Below $T_{\text{QCD}} \sim 250$ MeV strongly interacting particles are bound in hadrons, while above T_{QCD} free quarks and gluons exist.
- Quarks have as additional label 3 colors, there are eight gluons.
- We assume that all species have the same temperature and approximate their contribution to g_* by a step function $\vartheta(T - m)$.

Using the “Particle Data Book” to find the masses of the various particles, we can construct g_* as function of T as shown in table 11.1.

Temperature	new particles	$4\Delta g_*$	$4g_*$
$T < m_e$	$\gamma + \nu_i$	$4 \times (2 + 3 \times 2 \times 7/8)$	29
$m_e < T < m_\mu$	$e^\pm$	14	43
$m_\mu < T < m_\pi$	$\mu^\pm$	14	57
$m_\pi < T < T_c$	$\pi^\pm, \pi^0$	12	69
$T_c < T < m_s$	$u, \bar{u}, d, \bar{d}, g$	$6 \times 14 + 4 \times 8 \times 2 - 12$	205
$m_s < T < m_c$	$s, \bar{s}$	$3 \times 14 = 42$	247
$m_c < T < m_\tau$	$c, \bar{c}$	42	289
$m_\tau < T < m_b$	$\tau^\pm$	14	303
$m_b < T < m_{W,Z}$	$b, \bar{b}$	42	345
$m_{W,Z} < T < ?$	$W^\pm, Z$	$4 \times 3 \times 3 = 36$	381

Table 11.1: The number of relativistic degrees of freedom g_* present in the universe as function of its temperature.

11.3 Big Bang Nucleosynthesis

Nuclear reactions in stars are supposed to produce all the observed heavier elements. However, stellar reaction can explain at most a fraction of 5% of ${}^4\text{He}$, while the production of the weakly bound deuterium and Lithium-7 in stars is impossible. Thus the light elements up to Li-7 are primordial: $Y(D) = \text{few} \times 10^{-5}$, $Y({}^3\text{H}) = \text{few} \times 10^{-5}$, $Y({}^4\text{He}) \approx 0.25$, $Y({}^7\text{Li}) \approx (1-2) \times 10^{-7}$. Observational challenge is to find as "old" stars/gas clouds as possible and then to extrapolate back to primordial values.

Estimate of ${}^4\text{He}$ production by stars:

The binding energy of ${}^4\text{He}$ is $E_b = 28.3$ MeV. If 1/4 of all nucleons were fused into ${}^4\text{He}$ during $t \sim 10$ Gyr, the luminosity-mass ratio would be

$$\frac{L}{M_b} = \frac{1}{4} \frac{E_b}{4m_p t} \approx 5 \frac{\text{erg}}{gs} \approx 2.5 \frac{L_\odot}{M_\odot}$$

The observed luminosity-mass ratio is however only $\frac{L}{M_b} \leq 0.05 L_\odot / M_\odot$. Assuming a roughly constant luminosity of stars, they can produce only $0.05/2.5 \approx 2\%$ of the observed ${}^4\text{He}$.

Big Bang Nucleosynthesis (BBN) is controlled by two parameters: the mass difference between protons and neutrons, $\Delta \equiv m_n - m_p \approx 1.3$ MeV and the freeze-out temperature T_f of reaction converting protons into neutrons and vice versa.

11.3.1 Equilibrium distributions

In the non-relativistic limit $T \ll m$, the number density of the nuclear species with mass number A and charge Z is

$$n_A = g_A \left(\frac{m_A T}{2\pi} \right)^{3/2} \exp[\beta(\mu_A - m_A)]. \quad (11.24)$$

In chemical equilibrium, $\mu_A = Z\mu_p + (A - Z)\mu_n$ and we can eliminate μ_A by inserting the equivalent expression of (11.24) for protons and neutrons,

$$\exp(\beta\mu_A) = \exp[\beta(Z\mu_p + (A - Z)\mu_n)] = \frac{n_p^Z n_n^{A-Z}}{2^A} \left(\frac{2\pi}{m_N T} \right)^{3A/2} \exp[\beta(Zm_p + (A - Z)m_n)]. \quad (11.25)$$

Here and in the following we can set in the pre-factors $m_p \approx m_n \approx m_N$ and $m_A \approx Am_N$, keeping the exact masses only in the exponentials. Inserting this expression for $\exp(\beta\mu_A)$ together with the definition of the binding energy of a nucleus, $B_A = Zm_p + (A - Z)m_n - m_A$, we obtain

$$n_A = g_A \left(\frac{2\pi}{m_N T} \right)^{3(A-1)/2} \frac{A^{3/2}}{2^A} n_p^Z n_n^{A-Z} \exp(\beta B_A). \quad (11.26)$$

The mass fraction X_A contributed by a nuclear species is

$$X_A = \frac{An_A}{n_B} \quad \text{with} \quad n_B = n_p + n_n + \sum_i A_i n_{A_i} \quad \text{and} \quad \sum_i X_i = 1. \quad (11.27)$$

With $n_p^Z n_n^{A-Z}/n_N = X_p^Z X_n^{A-Z} n_N^{A-1}$ and $\eta \propto T^3$ and thus $n_B^{A-1} \propto \eta^{A-1} T^{3(A-1)}$, we have

$$X_A \propto \left(\frac{T}{m_N} \right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} \exp(\beta B_A). \quad (11.28)$$

The fact that $\eta \ll 1$, i.e. that the number of photons per baryon is extremely large, means that nuclei with $A > 1$ are much less abundant and that nucleosynthesis takes place later than naively expected. Let us consider the particular case of deuterium in Eq. (11.28),

$$\frac{X_D}{X_p X_n} = \frac{24\zeta(3)}{\sqrt{\pi}} \left(\frac{T}{m_N} \right)^{3/2} \eta \exp(\beta B_D) \quad (11.29)$$

with $B_D = 2.23$ MeV. The start of nucleosynthesis could be defined approximately by the condition $X_D/(X_p X_n) = 1$, or $T \approx 0.1$ MeV according to the left panel in Fig. 11.2. The right panel of the same figure shows the results, if the equations (11.28) together with $\sum_i X_i = 1$ are solved for the lightest and stablest nuclei. Now it becomes clear that in thermal equilibrium between $0.1 \lesssim T \ll 0.2$ MeV essentially all free neutrons will bind to ${}^4\text{He}$. Clearly, for low temperatures one cannot expect that the true abundance follows the equilibrium abundance, Eq. (11.28), shown in Fig. 11.2. First, in the expanding universe the weak reactions that convert protons and nucleons will freeze out as soon as their rate drops below the expansion rate of the universe. This effect will be discussed in the following in more detail. Second, the Coulomb barrier will prevent the production of nuclei with $Z \gg 1$. Third, neutrons are not stable and decay.

11.3.2 Proton-neutron ratio

Gamov criterion The interaction depth $\tau = nl\sigma$ gives the probability that a test particle interacts with cross section σ in a slab of length l filled with targets of density n . If $\tau \gg 1$, interactions are efficient and the test particle is in thermal equilibrium with the surrounding. We can apply the same criteria to the Universe: We say a particle species A is in thermal equilibrium, as long as $\tau = nl\sigma = n\sigma vt \gg 1$. The time t corresponds to the typical time-scale

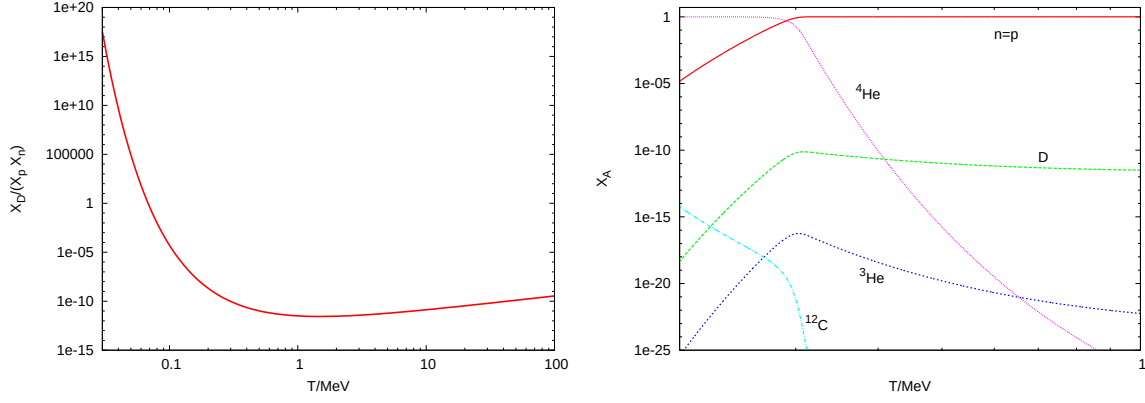


Figure 11.2: Relative equilibrium abundance $X_D/(X_p X_n)$ of deuterium as function of temperature T (left) and equilibrium mass fractions of nucleons, D, ^3He , ^4He and ^{12}C (right).

for the expansion of the universe, $\tau = (\dot{R}/R)^{-1} = H^{-1}$. Note that this is also the typical time-scale for changes in the temperature T . Thus we can rewrite this condition as

$$\Gamma \equiv n\sigma v \gg H. \quad (11.30)$$

A particle species "goes out of equilibrium" when its interaction rate Γ becomes smaller than the expansion rate H of the universe.

Decoupling of neutrinos The interaction rates of neutrinos in processes like $n \leftrightarrow p + e^- + \nu_e$ or $e^+e^- \leftrightarrow \bar{\nu}\nu$ is $\sigma \sim G_F^2 E^2$. If we approximate the energy of all particle species by their temperature T , their velocity by c and their density by $n \sim T^3$, then the interaction rate of weak processes is

$$\Gamma \approx \langle v\sigma n_\nu \rangle \approx G_F^2 T^5 \quad (11.31)$$

The early universe is radiation-dominated with $\rho_{\text{rad}} \propto 1/R^4$, $H = 1/(2t)$ and negligible curvature k/R^2 . Thus the Friedmann equation simplifies to $H^2 = (8\pi/3)G\rho$ with $\rho = g_*\pi^2/30T^4$, or

$$\frac{1}{2t} = H = 1.66\sqrt{g_*} \frac{T^2}{M_{\text{Pl}}}. \quad (11.32)$$

Here, we introduced also the Planck mass $M_{\text{Pl}} = 1/\sqrt{G_N} \approx 1.2 \times 10^{19}$ GeV. Requiring $\Gamma(T_{\text{fr}}) = H(T_{\text{fr}})$ gives as freeze-out temperature T_{fr} of weak processes

$$T_{\text{fr}} \approx \left(\frac{1.66\sqrt{g_*}}{G_F^2 M_{\text{Pl}}} \right)^{1/3} \approx 1\text{MeV} \quad (11.33)$$

with $g_* = 10.75$. The relation between time and temperature follows as

$$\frac{t}{\text{s}} = \frac{2.4}{\sqrt{g_*}} \left(\frac{\text{MeV}}{T} \right)^2. \quad (11.34)$$

Thus the time-sequence is as follows

- at $T_{\text{fr}} \approx 1$ MeV: the neutron-proton ratio freezes-in and can be approximated by the ratio of their equilibrium distribution in the non-relativistic limit.
- as the universe cools down from T_{fr} to T_{ns} , neutrons decay with half-live $\tau_n \approx 886$ s.
- at T_{ns} , practically all neutrons are bound to ${}^4\text{He}$, with only small admixture of other elements.

Proton-neutron ratio Above T_f , reactions like $\nu_e + n \leftrightarrow p + e^-$ keep nucleons in thermal equilibrium. As we have seen, $T_f \sim 1$ MeV and thus we can treat nucleons in the non-relativistic limit. Then their relative abundance is given by the Boltzmann factor $\exp(-\Delta/T)$ for $T \gtrsim T_f$ with $\Delta = m_N - m_P = 1.29$ MeV for the mass difference of neutrons and protons. Hence for T_f ,

$$\left. \frac{n_n}{n_p} \right|_{t=t_{\text{fr}}} = \exp\left(-\frac{\Delta}{T_f}\right) \approx \frac{1}{6}. \quad (11.35)$$

As the universe cools down to T_{ns} , neutrons decay with half-live $\tau_n \approx 886$ s,

$$\left. \frac{n_n}{n_p} \right|_{t=t_{\text{ns}}} \approx \frac{1}{6} \exp\left(-\frac{t_{\text{ns}}}{\tau_n}\right) \approx \frac{2}{15}. \quad (11.36)$$

11.3.3 Estimate of helium abundance

The synthesis of ${}^4\text{He}$ proceeds through a chain of reactions, $pn \rightarrow d\gamma$, $dp \rightarrow {}^3\text{He}\gamma$, $d{}^3\text{He} \rightarrow {}^4\text{He}p$. Let's assume that ${}^4\text{He}$ formation takes place instantaneously. Moreover, we assume that all neutrons are bound in ${}^4\text{He}$. We need two neutrons to form one helium atom, $n({}^4\text{He}) = n_n/2$, and thus

$$Y({}^4\text{He}) \equiv \frac{M({}^4\text{He})}{M_{\text{tot}}} = \frac{4m_N \times n_n/2}{m_N(n_p + n_n)} = \frac{2n_n/n_p}{1 + n_n/n_p} = \frac{4}{17} \sim 0.235 \quad (11.37)$$

Our naive estimate not too far away from $Y \sim 0.245$.

The dependence of $Y({}^4\text{He})$ on the input physics is rather remarkable.

- The helium abundance dependence exponentially on Δ and T_f :
 - The mass difference Δ depends on both electromagnetic and strong interactions. BBN tests therefore the time-dependence of fundamental interaction expected e.g. in string theories.
 - The freeze-out temperature T_{fr} depends on number of relativistic degrees of freedom g_* and restricts thereby additional light particle.
 - a non-zero chemical potential of neutrinos.
- A weaker dependence on start of nucleosynthesis T_{ns} and thus η_b or Ω_b .

11.3.4 Results from detailed calculations

Detailed calculations predict not only the relative amount of light elements produced, but also their absolute amount as function of e.g. the baryon-photon ratio η . Requiring that the relative fraction of helium-4, deuterium and lithium-7 compared to hydrogen is consistent with observation allows one to determine η or equivalently the baryon content, $\Omega_b h^2 = 0.019 \pm 0.001$. Although the binding energy per nucleon of Carbon-12 and Oxygen-16 is higher than the of ${}^4\text{He}$, they are not produced: at time of ${}^4\text{He}$ production Coulomb barrier prevents already fusion. Also, stable element with $A = 5$ is missing.

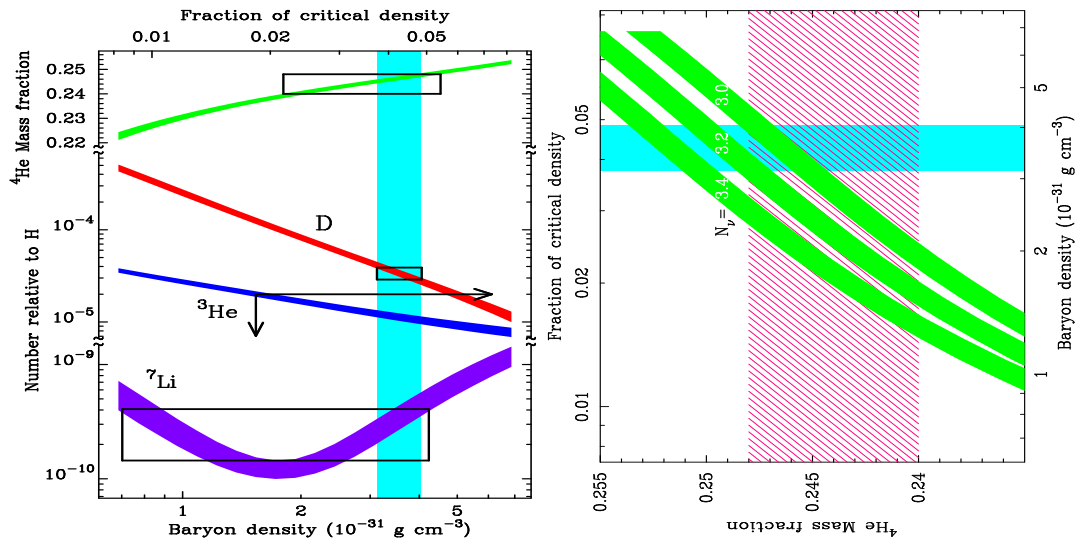


Figure 11.3: Abundances of light-elements as function of η (left) and of the number of light neutrino species (right).