

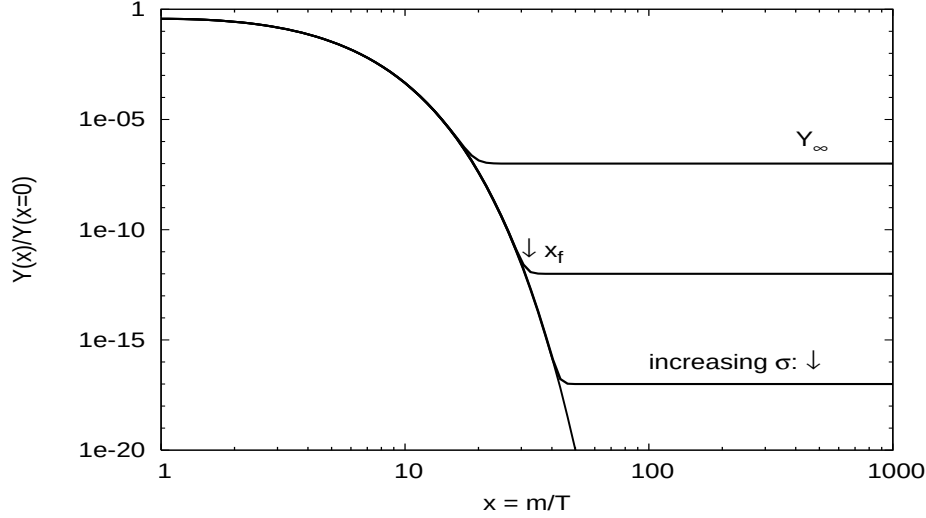


Figure 11.4: Illustration of the freeze-out process.

11.4 Dark matter

11.4.1 Freeze-out of thermal relic particles

When the number density n_X of a particle species X is not changed by interactions, then it is diluted just by the expansion of space, $n_X \propto R^{-3}$. It is convenient to account for this trivial expansion effect by dividing n_X through the entropy density $s \propto R^{-3}$, i.e. to use the quantity $Y = n/s$. We first consider again the equilibrium distribution Y_{eq} for $\mu_X = 0$,

$$Y_{\text{eq}} = \frac{n_X}{s} = \begin{cases} \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_X}{g_{*S}} x^{3/2} \exp(-x) = 0.145 \frac{g_X}{g_{*S}} x^{3/2} \exp(-x) & \text{for } x \gg 3, \\ \frac{45\zeta(3)}{2\pi^4} \frac{g_X}{g_{*S}} = 0.278 \frac{g_X}{g_{*S}} & \text{for } x \ll 3 \end{cases} \quad (11.41)$$

where $x = T/m$ and $g_{\text{eff}} = 3/4$ ($g_{\text{eff}} = 1$) for fermions (bosons). If the particle X is in chemical equilibrium, its abundance is determined for $T \gg m$ by its contribution to the total number of degrees of freedom of the plasma, while Y_{eq} is exponentially suppressed for $T \ll m$ (assuming $\mu_X = 0$). In an expanding universe, one may expect that the reaction rate Γ for processes like $\gamma\gamma \leftrightarrow \bar{X}X$ drops below the expansion rate H mainly for two reasons: i) Cross sections may depend on energy as, e.g., weak processes $\sigma \propto s \propto T^2$ for $s \lesssim m_W^2$, ii) the density n_X decreases at least as $n \propto T^3$. Around the freeze-out time x_f , the true abundance Y starts to deviate from the equilibrium abundance Y_{eq} and becomes constant, $Y(x) \approx Y_{\text{eq}}(x_f)$ for $x \gtrsim x_f$. This behavior is illustrated in Fig. 11.4.

Boltzmann equation When the number $N = nV$ of a particle species is not changed by interactions, then the expansion of the Universe dilutes their number density as $n \propto R^{-3}$. The corresponding change in time is connected with the expansion rate of the universe, the Hubble parameter $H = \dot{R}/R$, as

$$\frac{dn}{dt} = \frac{dn}{dR} \frac{dR}{dt} = -3n \frac{\dot{R}}{R} = -3Hn. \quad (11.42)$$

Additionally, there might be production and annihilation processes. While the annihilation rate $\beta n^2 = \langle \sigma_{\text{ann}} v \rangle n^2$ has to be proportional to n^2 , we allow for an arbitrary function as production rate ψ ,

$$\frac{dn}{dt} = -3Hn - \beta n^2 + \psi. \quad (11.43)$$

In a static Universe, $dn/dt = 0$ defines equilibrium distributions n_{eq} . Detailed balance requires that the number of X particles produced in reactions like $e^+e^- \rightarrow \bar{X}X$ is in equilibrium equal to the number that is destroyed in $\bar{X}X \rightarrow e^+e^-$, or $\beta n_{\text{eq}}^2 = \psi_{\text{eq}}$. Since the reaction partners (like the electrons in our example) are assumed to be in equilibrium, we can replace $\psi = \psi_{\text{eq}}$ by βn_{eq}^2 and obtain

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{ann}} v \rangle (n^2 - n_{\text{eq}}^2). \quad (11.44)$$

This equation together with the initial condition $n \approx n_{\text{eq}}$ for $T \rightarrow \infty$ determines $n(t)$ for a given annihilation cross section σ_{ann} .

Next we rewrite the evolution equation for $n(t)$ using the dimensionless variables Y and x . Changing from $n = sY$ to Y we can eliminate the $3Hn$ term,

$$\frac{dn}{dt} = -3Hn + s \frac{dY}{dt}. \quad (11.45)$$

With $(2t)^{-2} = H^2 \propto \rho \propto T^4 \propto x^{-4}$ or $t = t_* x^2$, we obtain

$$\frac{dY}{dx} = -\frac{sx}{H} \langle \sigma_{\text{ann}} v \rangle (Y^2 - Y_{\text{eq}}^2). \quad (11.46)$$

Finally we recast the Boltzmann equation in a form that makes our intuitive Gamov criterion explicit,

$$\frac{x}{Y} \frac{dY}{dx} = -\frac{\Gamma_A}{H} \left[\left(1 - \frac{Y_{\text{eq}}}{Y} \right)^2 \right] \quad (11.47)$$

with $\Gamma_A = n \langle \sigma_{\text{ann}} v \rangle$: The relative change of Y is controlled by the factor Γ_A/H times the deviation from equilibrium. For the discussion of approximate solutions to this equation, it is convenient to distinguish different dark matter particles according to their freeze-out temperature: Hot dark matter (HDM) with $x_f \ll 3$, cold dark matter (CDM) with $x_f \gg 3$ and the intermediate case of warm dark matter with $x_f \sim 3$. The evolution of $Y = n_X/s$ is shown schematically in Fig. 11.4: As the universe expands and cools down, n_X decreases at least as R^{-3} . Therefore, the annihilation rate $\propto n^2$ quenches and the abundance “freezes-out.” The reaction rates are not longer sufficient to keep the particle in equilibrium and the ratio n_X/s stays constant.

For the discussion of approximate solutions to this equation, it is convenient to distinguish according to the freeze-out temperature: hot dark matter (HDM) with $x_f \ll 3$, cold dark matter (CDM) with $x_f \gg 3$ and the intermediate case of warm dark matter with $x_f \sim 3$.

11.4.2 Hot dark matter

For $x_f \ll 3$, freeze-out occurs when the particle is still relativistic and Y_{eq} is not changing with time. The asymptotic value of Y , $Y(x \rightarrow \infty) \equiv Y_\infty$, is just the equilibrium value at freeze-out,

$$Y_\infty = Y_{\text{eq}}(x_f) = 0.278 \frac{g_{\text{eff}}}{g_{*S}}, \quad (11.48)$$

where the only temperature-dependence is contained in g_{*S} . The number density today is then

$$n_0 = s_0 Y_\infty = 2970 Y_\infty \text{cm}^{-3} = 825 \frac{g_{\text{eff}}}{g_{*S}} \text{cm}^{-3}. \quad (11.49)$$

The numerical value of s_0 used will be discussed in the next paragraph. Although a HDM particle was relativistic at freeze-out, it is today non-relativistic if its mass m is $m \gg 3K \approx 0.2\text{meV}$. In this case its energy density is simply $\rho_0 = m s_0 Y_\infty$ and its abundance $\Omega h^2 = \rho_0 / \rho_{\text{cr}}$ or

$$\Omega h^2 = 7.8 \times 10^{-2} \frac{m}{\text{eV}} \frac{g_{\text{eff}}}{g_{*S}}. \quad (11.50)$$

Hence HDM particles heavier than $O(100\text{eV})$ overclose the universe.

11.4.3 Cold dark matter

Abundance of CDM For CDM with $x_f \ll 3$, freeze-out occurs when the particles are already non-relativistic and Y_{eq} is exponentially changing with time. Thus the main problem is to find x_f , for late times we use again $Y(x \rightarrow \infty) \equiv Y_\infty \approx Y(x_f)$, i.e. the equilibrium value at freeze-out. We parametrize the temperature-dependence of cross section as $\langle \sigma_{\text{ann}} \rangle = \sigma_0 (T/m)^n = \sigma_0 / x^n$. For simplicity, we consider only the most relevant case for CDM, $n = 0$ or s-wave annihilation. Then the Gamov criterion becomes with $H = 1.66\sqrt{g_*} T^2 / M_{\text{Pl}}$ and $\Gamma_A = n_{\text{eq}} \langle \sigma_{\text{ann}} v \rangle$,

$$g \left(\frac{m T_f}{2\pi} \right)^{3/2} \exp(-m/T_f) \sigma_0 = 1.66\sqrt{g_*} \frac{T_f^2}{M_{\text{Pl}}} \quad (11.51)$$

or

$$x_f^{-1/2} \exp(x_f) = 0.038 \frac{g}{\sqrt{g_*}} M_{\text{Pl}} m \sigma_0 \equiv C. \quad (11.52)$$

To obtain an approximate solution, we neglect first in

$$\ln C = -\frac{1}{2} \ln x_f + x_f \quad (11.53)$$

the slowly varying term $\ln x_f$. Inserting next $x_f \approx \ln C$ into Eq. (11.53) to improve the approximation gives then

$$x_f = \ln C + \frac{1}{2} \ln(\ln C). \quad (11.54)$$

The relic abundance for CDM follows from $n(x_f) = 1.66\sqrt{g_*} T_f^2 / (\sigma_0 M_{\text{Pl}})$ and $n_0 = n(x_f) [R(x_f)/R_0]^3 = n(x_f) [g_{*,f}/g_{*,0}] [T_0/T(x_f)]^3$ as

$$\rho_0 = m n_0 \approx 10 \frac{x_f T_0^3}{\sqrt{g_{*,f}} \sigma_0 M_{\text{Pl}}} \quad (11.55)$$

or

$$\Omega_X h^2 = \frac{m n_0}{\rho_{\text{cr}}} \approx \frac{4 \times 10^{-39} \text{cm}^2}{\sigma_0} x_f \quad (11.56)$$

Thus the abundance of a CDM particle is inverse proportionally to its annihilation cross section, since a more strongly interacting particle stays longer in equilibrium. Note that the abundance depends only logarithmically on the mass m via Eq. (11.54) and implicitly via $g_{*,f}$ on the freeze-out temperature T_f . Typical values of x_f found numerically for weakly interacting massive particles (WIMPs) are $x_f \sim 20$. Partial-wave unitarity bounds σ_{ann} as

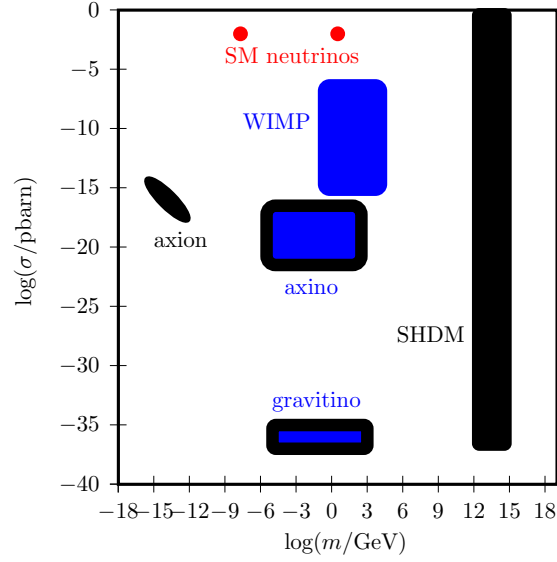


Figure 11.5: Particles proposed as DM particle with $\Omega \sim 1$, the expected size of their cross section and their mass.

$\sigma_{\text{ann}} \leq c/m^2$. Requiring $\Omega < 0.3$ leads to $m < 20 - 50$ TeV. This bounds the mass of any stable particle that was once in thermal equilibrium.

Baryon abundance from freeze-out:

We can calculate the expected baryon abundance for a zero chemical potential using the formulas derived above. Nucleon interact via pions; their annihilation cross section can be approximated as $\langle \sigma v \rangle \approx m_\pi^{-2}$. With $C \approx 2 \times 10^{19}$, it follows $x_f \approx 44$, $T_f \sim 22$ MeV and $Y_\infty = 7 \times 10^{-20}$.

The observed baryon abundance is much larger and can be not explained as a usual freeze-out process.

Cold dark matter candidates

A particle suitable as CDM candidate should interact according Eq. (11.56) with $\sigma \sim 10^{-37} \text{ cm}^2$. It is surprising that the numerical values of T_0 and M_{Pl} conspire in Eq. (11.56) to lead to numerical value of σ_0 typical for weak interactions. Cold dark matter particles with masses around the weak scale and interaction strengths around the weak scale were dubbed “WIMP”. An obvious candidate was a heavy neutrino, $m_\nu \sim 10 \text{ GeV}$, excluded early by direct DM searches, neutrino mass limits, and accelerator searches. Presently, the candidate with most supporters is the lightest supersymmetric particle (LSP). Depending on the details of the theory, it could be a neutralino (most favorable for detection) or other options. The mass range open of thermal CDM particles is rather narrow: If it is too light, it becomes a warm or hot dark matter particle. If it is too heavy, it overcloses the universe. There exists however also the possibility that DM was never in thermal equilibrium. Two examples are the axion (a particle proposed to solve the CP problem of QCD) and superheavy particle (generically produced at the end of inflation). An overview of different CDM candidates is given in Fig. 11.5.