Interaction between superconducting films and magnetic nanostructures

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The interplay between a superconducting film and a magnetic nanostructure is studied. The generalized London equation for this system is solved, and the magnetic fields, the energy, and the interaction forces are computed. In particular, we focus on how to manipulate vortices using magnetic nanostructures.

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I. INTRODUCTION

The structure of vortices in thin films was first investigated in detail by Pearl.[1] He found that such vortices interact mainly via their stray field, which extends far into the nonsuperconducting medium. Later, this theory was extended to thin-film systems with and without anisotropy.[2–4] It has also been of considerable interest to understand Josephson vortices in thin-film junctions. To this end, a very interesting result was obtained by Kogan *et al.*,[5] who found that the field from a Josephson junction is a superposition of fields from Pearl vortices along the junction with a certain line density. This result was derived by noting that the Pearl solution is the Green's function for arbitrary sources.

The possibility of enhancing the critical currents by magnetic nanostructures has been studied by many researchers.[6,7] In particular, Bulaevskii *et al.* showed that magnetic domain structures in a magnetic film in close contact with a superconducting film may enhance pinning of vortices, since this gives an opportunity to pin the magnetic flux of the vortex rather than its core.[7] It was suggested that the pinning of vortices in superconductor/ferromagnetic multilayers can be many times greater than the pinning by columnar defects.

In general, the interaction between superconductivity and magnetism has been studied for several decades. Systems composed of magnetic and superconducting materials are of interest not only because they are model systems for the interplay of competing superconducting and magnetic order parameters, but also because of numerous possible applications. Recently, the development of magnetic thin-film technology has triggered a lot of interest in this field. Of particular importance has been the possibility of examining the interaction between superconductivity and magnetism in high-temperature superconductors.[6–15]

In a recent paper we estimated the interaction between a bulk superconductor and a Bloch wall.[16] It was shown that the Bloch wall shrinks on cooling through the superconducting transition temperature. Furthermore, we suggested that if the magnetization of the Bloch wall is strong enough, it may capture and move the vortices.

The aim of the current paper is to study the interaction between Pearl vortices and magnetic nanostructures in more detail. We solve the generalized London equation and compute the magnetic fields, the energy, and the interaction forces. In the case of a movable magnetic nanostructure (e.g., a domain wall), one may be able to manipulate the static as well as the dynamic behavior of the vortices. Thus, tunable and movable nanomagnets may be used as vortex manipulators.

II. THIN-FILM SYSTEM

Consider a thin superconducting film located at $z=0$ with thickness much smaller than the penetration depth of the superconductor; see Fig. 1. The surface is covered by a magnetic structure with thickness much smaller than that of the superconducting film. That is, we assume that the magnetic structure consists of surface charges separated from the superconductor by a very thin oxide layer (of thickness t) to avoid spin diffusion and proximity effects (which may supress the superconducting and magnetic order parameters).[17] In general, the current density is a sum of the supercurrents and magnetically induced currents, which can be expressed through the generalized London equation as

$$\nabla\times\boldsymbol{J}=-\frac{1}{\lambda^2}\mathbf{H}+\frac{1}{\lambda^2}V(\boldsymbol{\rho})\hat{\mathbf{e}}_z+\nabla\times\nabla\times\mathbf{M}_v\,, \tag{1}$$

where $\mathbf{J}$ is the current density, λ is the penetration depth, $\mathbf{H}$ is the magnetic field, and $\nabla\times\mathbf{M}_v$ is the magnetically induced current. The vortex is aligned in the z direction, and its source function $V(\boldsymbol{\rho})$ is assumed to be rotational symmetric. In the case of a Pearl vortex we may set $V(\rho)=(\Phi_0/\mu_0)\,\delta(\rho)$, where Φ_0 is the flux quantum and μ_0 the permeability of vacuum. The requirement of rotational symmetry should be relaxed when dealing with certain types of anisotropic superconductors, which are known to have azimuthal corrections to the vortex core.[18] In this case one should also use the anisotropic version of Eq. (1).

Recently, the Ginzburg-Landau theory was used to calculate the vortex states in a thin mesoscopic disk in the presence of a steplike external magnetic field.[19] Naturally, one may expect more accurate results from the Ginzburg-Landau theory, in particular when the dimensions are small. On the other hand, the generalized London theory considered here gives a significant reduction in computational efforts, and some important points could be deduced without numerical analysis. Moreover, the inherent shortcomings of the London model can be relaxed by introducing corrections to the vortex core. Thus, we believe that the generalized London model adopted in this paper is a reasonable first approach.

We solve the current London equation by adopting a generalization of the very useful approach of Refs. 5 and 20. The current can only flow in a thin layer of thickness d ($d\ll\lambda$),

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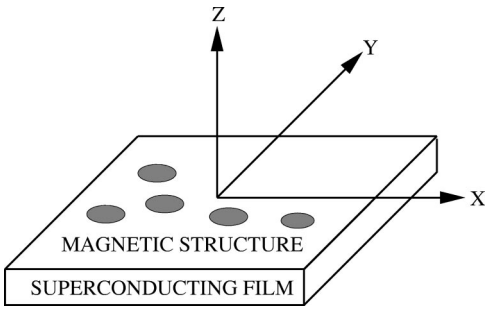


FIG. 1. The system under study. The thin oxide layer separating the superconducting film and magnetic nanostructure is not shown.

and it is therefore necessary to average over this thickness and consider the z component only

$$H_z+\lambda_e\left(\frac{\partial J_y^s}{\partial x}-\frac{\partial J_x^s}{\partial y}\right)=V(\rho)+\lambda_e(\nabla\times\nabla\times\boldsymbol{M})_z, \quad (2)$$

where $\lambda_e=\lambda^2/d$ is the effective penetration depth and $\mathbf{M}=d\mathbf{M}_v$. The sheet current flowing in the thin layer is now given by $\mathbf{J}_s=d\mathbf{J}$. The Maxwell equation $\nabla\times\mathbf{H}=\mathbf{J}$ gives

$$J_x=\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}, \quad J_y=\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}. \quad (3)$$

Since all derivatives $\partial/\partial z$ are large compared to the tangential $\partial/\partial\boldsymbol{\rho}$, we may set

$$J_x^s\approx H_y^- - H_y^+, \quad J_y^s\approx H_x^+ - H_x^-, \quad (4)$$

where H_i^+ and H_i^- $(i=x, y)$ are the components at the upper and lower surfaces, respectively. Since the environments of the upper and lower half-spaces are identical, we have the boundary condition $H_i^+=-H_i^-$, which results in

$$J_x^s\approx -2H_y^+, \quad J_y^s\approx 2H_x^+. \quad (5)$$

Here we will only consider the upper half-space. Using $\nabla\cdot\mathbf{H}=0$, Eq. (2) becomes

$$H_z-2\lambda_e\frac{\partial H_z}{\partial z}=V(\rho)+\lambda_e(\nabla\times\nabla\times\mathbf{M})_z. \quad (6)$$

We now apply the superposition principle to separate the contributions from the vortex and the magnetic nanostructure. The vortex part of Eq. (6) can be written as

$$H_{vz}-2\lambda_e\frac{\partial H_{vz}}{\partial z}=V(\rho), \quad (7)$$

whereas the magnetic part is

$$H_{mz}-2\lambda_e\frac{\partial H_{mz}}{\partial z}=\lambda_e(\nabla\times\nabla\times\mathbf{M})_z. \quad (8)$$

A. Vortex solution

In order to solve Eq. (7), it is useful to note that $\nabla\times\mathbf{H}=\nabla\cdot\mathbf{H}=0$ outside the system. Therefore, we can introduce a scalar potential ϕ which vanishes at $z\to\infty$,

$$\phi_v(\boldsymbol{\rho},z)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(\mathbf{k})\exp(i\mathbf{k}\cdot\boldsymbol{\rho}-k|z|)d^2k, \quad (9)$$

where $k=\sqrt{k_x^2+k_y^2}$ and $\mathbf{H}=\nabla\phi$. It is helpful to note that $H_z(\mathbf{k})=-k\phi(\mathbf{k})$ for the upper half-space.

Applying the two-dimensional (2D) Fourier transform to Eq. (7) we obtain

$$\phi_v(\mathbf{k})=-\frac{V(k)}{(1+2\lambda_e k)k}. \quad (10)$$

In the case that the vortex is displaced a distance ρ_0 from the origin we may set

$$\phi_v(\mathbf{k},\rho_0)=-\frac{V(k)\exp(-i\mathbf{k}\cdot\boldsymbol{\rho}_0)}{(1+2\lambda_e k)k}. \quad (11)$$

However, for the moment we assume that the vortex is located at the origin, and therefore the resulting scalar potential is

$$\phi_v(\boldsymbol{\rho},z)=-\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{V(k)\exp(i\mathbf{k}\cdot\boldsymbol{\rho}-k|z|)}{k(1+2\lambda_e k)}d^2k. \quad (12)$$

Due to the rotational symmetry, the potential is found to be

$$\phi_v(\boldsymbol{\rho},z)=-\frac{1}{2\pi}\int_0^{\infty}V(k)\frac{J_0(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk, \quad (13)$$

where we have used that

$$\int_0^{2\pi}\exp(ik\rho\cos\phi)d\phi=2\pi J_0(k\rho). \quad (14)$$

To obtain the magnetic field components (in the radial and z directions), we apply the formula

$$\frac{d}{d\rho}J_0(k\rho)=-kJ_1(k\rho) \quad (15)$$

and find

$$H_{vz}(\boldsymbol{\rho},z)=\frac{1}{2\pi}\int_0^{\infty}kV(k)\frac{J_0(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk, \quad (16)$$

$$H_{v\rho}(\boldsymbol{\rho},z)=\frac{1}{2\pi}\int_0^{\infty}kV(k)\frac{J_1(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk. \quad (17)$$

In the case of a Pearl vortex the magnetic potential and fields reduce to

$$\phi_v(\boldsymbol{\rho},z)=-\frac{\Phi_0}{2\pi\mu_0}\int_0^{\infty}\frac{J_0(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk, \quad (18)$$

$$H_{vz}(\boldsymbol{\rho},z)=\frac{\Phi_0}{2\pi\mu_0}\int_0^{\infty}k\frac{J_0(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk, \quad (19)$$

$$H_{v\rho}(\boldsymbol{\rho},z)=\frac{\Phi_0}{2\pi\mu_0}\int_0^\infty k\frac{J_1(k\rho)}{1+2\lambda_e k}\exp(-k|z|)dk. \quad (20)$$

B. Magnetic solution

In general, Eq. (8) can be solved for arbitrary magnetization distributions. For simplicity, we will here consider only magnetization distributions which are directed in the z direction and have no volume charges, $\nabla\cdot\mathbf{M}=0$. Inserted in Eq. (8) this gives

$$H_z-2\lambda_e\frac{\partial H_z}{\partial z}=-\lambda_e(\nabla^2\mathbf{M})_z. \quad (21)$$

The potential from the magnetic nanostructure can be obtained in the same way as for the vortices. We will here only work out explicitly the solution for 2D magnetic distributions, but the 1D solution is found in a similar manner. Applying the 2D Fourier transform to Eq. (9) results in

$$\phi_m(\mathbf{k})=-\lambda_e\frac{kM(\mathbf{k})}{1+2\lambda_e k}, \quad (22)$$

which gives the following magnetic potential:

$$\phi_m(\boldsymbol{\rho},z)=-\frac{\lambda_e}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{kM(k)}{1+2\lambda_e k}\exp(i\mathbf{k}\cdot\boldsymbol{\rho})\times\exp(-k|z|)d^2k. \quad (23)$$

In the case of a rotational symmetric magnetization distribution, we may write

$$\phi_m(\boldsymbol{\rho},z)=-\frac{\lambda_e}{2\pi}\int_0^\infty J_0(k\rho)\frac{k^2M(k)}{1+2\lambda_e k}\exp(-k|z|)dk. \quad (24)$$

Using that $\mathbf{H}=\nabla\phi$, we can calculate the radial and vertical field components to be

$$H_{mz}(\boldsymbol{\rho},z)=\frac{\lambda_e}{2\pi}\int_0^\infty J_0(k\rho)\frac{k^3M(k)}{1+2\lambda_e k}\exp(-k|z|)dk \quad (25)$$

and

$$H_{m\rho}(\boldsymbol{\rho},z)=\frac{\lambda_e}{2\pi}\int_0^\infty J_1(k\rho)\frac{k^3M(k)}{1+2\lambda_e k}\exp(-k|z|)dk. \quad (26)$$

So far we have neglected the thickness of the oxide layer. However, to a first approximation, it is easy to show that the only effect of this layer is to shift the z coordinate, $z\rightarrow z-t$.

C. Energies and forces

We now work out the energies and forces in the case of a circular symmetric 2D magnetization distribution, but will later also discuss briefly a 1D distribution as well. Consider a vortex located a distance ρ_0 from the origin and a magnetization distribution centered at the origin. The three contributions to the energy are the magnetic self-energy, the direct interaction energy, and the energy associated with the vortex.

The magnetic self-energy is given by

$$E_m=-\frac{1}{2}\mu_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{M}\cdot\mathbf{H_m}d^2\rho. \quad (27)$$

This expression can be transformed by using the following formula:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A(\rho)B(\rho)d^2\rho=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}A(-k)B(k)d^2k. \quad (28)$$

Thus, in the case of a circular symmetric magnetization distribution we have

$$E_m=-\frac{\mu_0\lambda_e}{4\pi}\int_0^\infty k^3\frac{M(k)M(k)}{1+2\lambda_e k}dk, \quad (29)$$

where we have used that $M(-k)=M(k)$ and also that $d^2k\rightarrow kdkd\phi$ (and integrated over the azimuthal angle).

The direct interaction energy between the vortex and the magnetization is given by

$$E_{vm}=-\mu_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{M}\cdot\mathbf{H_v}d^2\rho, \quad (30)$$

where the vortex is assumed to be displaced a distance ρ_0 from the origin. Then, in the cylindrical symmetric situation one finds

$$E_{vm}=-\frac{\mu_0}{2\pi}\int_0^\infty kV(k)\frac{M(k)J_0(k\rho_0)}{1+2\lambda_e k}dk. \quad (31)$$

The associated force can be found by taking the derivative with respect to ρ_0, resulting in

$$F_{vm}=-\frac{\mu_0}{2\pi}\int_0^\infty k^2V(k)\frac{M(k)J_1(k\rho_0)}{1+2\lambda_e k}dk. \quad (32)$$

The energy associated with vortex field is expressed as

$$E_v=\int_V\left(\frac{1}{2}\mu_0H_v^2+\frac{1}{2}n_smv_s^2\right)dV, \quad (33)$$

where n_s is the density of superconducting electrons and v_s is their velocity. This expression is most easily transformed by applying the formalism of Ref. 20. In particular, the energy associated with a Pearl vortex can be evaluated to logarithmic accuracy as

$$E_v\approx\frac{\Phi_0^2}{4\pi\mu_0\lambda_e}\ln\left(\frac{4\pi\lambda_e}{\xi}\right), \quad (34)$$

where ξ is the coherence length. More accurate expressions are obtained by introducing the appropriate vortex source function, but this will not be considered here.

III. INTERACTION WITH A BUBBLE

Consider first a magnetic bubble that can be moved, generated, or annihilated by external stress patterns or magnetic fields.[21] Let the vortex be separated a distance ρ_0 from the magnetic bubble located at the origin. The system has cylindrical symmetry, and the magnetic bubble has a magnetization vector pointing in the z direction given by

$$\mathbf{M}=\begin{cases} M_0\hat{\mathbf{e}}_z & \text{if } \rho\leqslant W, \\ 0 & \text{if } \rho>W. \end{cases}$$

The Fourier transform of this distribution is

$$M(k)=2\pi M_0 W\frac{J_1(kW)}{k}, \tag{35}$$

which generates the following magnetic field components:

$$H_z^b(\boldsymbol{\rho},z)=\lambda_e W M_0\int_0^\infty J_0(k\rho)\frac{k^2J_1(kW)}{1+2\lambda_e k}\exp(-k|z|)dk, \tag{36}$$

$$H_\rho^b(\boldsymbol{\rho},z)=\lambda_e W M_0\int_0^\infty J_1(k\rho)\frac{k^2J_1(kW)}{1+2\lambda_e k}\exp(-k|z|)dk. \tag{37}$$

These components were also found in Refs. 13 and 14 using a different method. To obtain the total field one should add the vortex contribution. The energy is found to be

$$E_b=\frac{\Phi_0^2}{4\pi\mu_0\lambda_e}\ln\left(\frac{4\pi\lambda_e}{\xi}\right)-M_0\Phi_0 W\int_0^\infty\frac{J_1(kW)J_0(k\rho_0)}{1+2\lambda_e k}dk$$
$$-\lambda_e\mu_0 M_0^2\pi W^2\int_0^\infty k\frac{J_1^2(kW)}{1+2\lambda_e k}dk, \tag{38}$$

and the direct force between the bubble and vortex is given by

$$F_{vm}=-M_0 W\Phi_0\int_0^\infty k\frac{J_1(kW)J_1(k\rho_0)}{1+2\lambda_e k}dk. \tag{39}$$

Figure 2 shows the force when $W=\lambda_e$, and we see that it falls off rather quickly. Moreover, we note that there are peaks at the maximum magnetization gradients. A vortex in the vicinity of the bubble will be either repelled or attracted to the center of the bubble, where it eventually comes to rest. The vortex-bubble system may still attract more vortices, but now vortex-vortex repulsion must be taken into account, which is outside the scope of the current study.

Consider now an annular structure as illustrated in Fig. 3 where the inner dot has magnetization M_1 and the outer annulus M_2. Then the force becomes

$$F_{vm}=-M_1W_1\Phi_0\int_0^\infty\left[J_1(kW_1)+\frac{M_2W_3}{M_1W_1}J_1(kW_3)\right.$$
$$\left.-\frac{M_2W_2}{M_1W_1}J_1(kW_2)\right]\frac{kJ_1(k\rho_0)}{1+2\lambda_e k}dk. \tag{40}$$

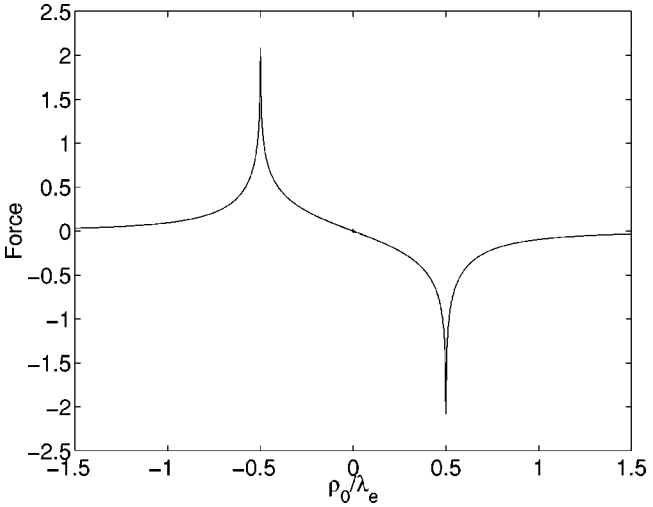


FIG. 2. The force (divided by $\Phi_0 M_0 W$) between a vortex and a bubble when $W=\lambda_e$.

As an example, Fig. 4 shows the force when $M_1=|M_2|$, $W_1=0.5\lambda_e$, $W_3=2W_1$, and $W_2=1.5W_1$. The solid line shows the case of a positive M_2, whereas the dashed line corresponds to negative M_2. A slowly moving vortex approaching the annular structure from the outside will experience attraction if $M_2>0$, and its equilibrium position is directly under the annulus. If M_2 suddenly switches sign, then the vortex is attracted to the center of the annular structure (i.e., directly under the dot) and kept there. Thus, this annular structure could be used to catch and trap vortices by switching the sign of the outer annulus. This could in principle be done by using a soft magnetic material at the outer annulus, whereas the inner dot consists of a hard magnetic material. Naturally, the magnetic forces must overcome the Lorentz forces generated by the external magnetic field. Moreover, it is probably not so easy to fabricate such a structure. Therefore, it is doubtful whether this circuit could find any practical realization, although it is a nice example of a vortex manipulation structure.

IV. INTERACTION WITH A 1D MAGNETIZATION

Most domains and walls are not cylindrical symmetric. In fact, they are more often 1D, and therefore the corresponding

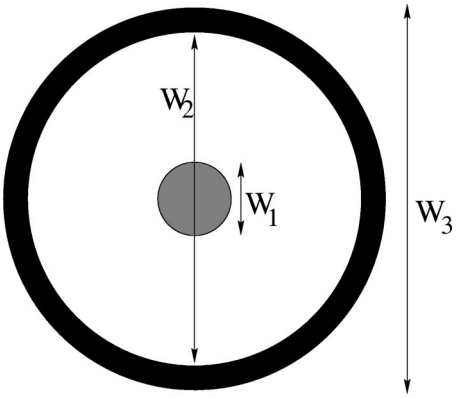


FIG. 3. An annular structure for manipulating of vortices.

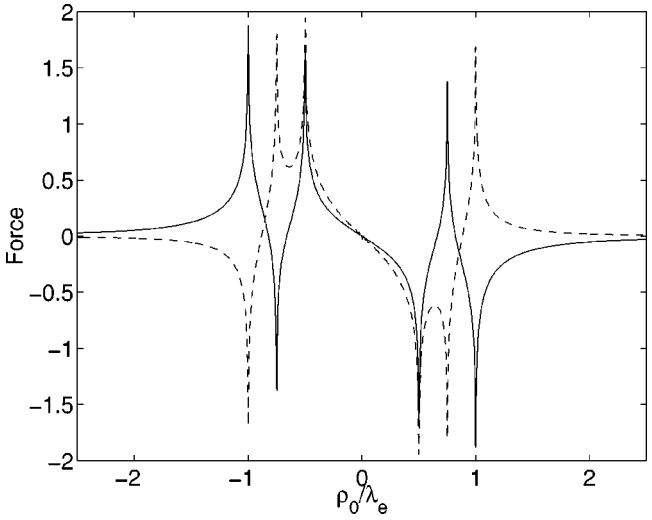


FIG. 4. The force (divided by $\Phi_0 M_1 W_1$) on a vortex near the annular structure of Fig. 3.

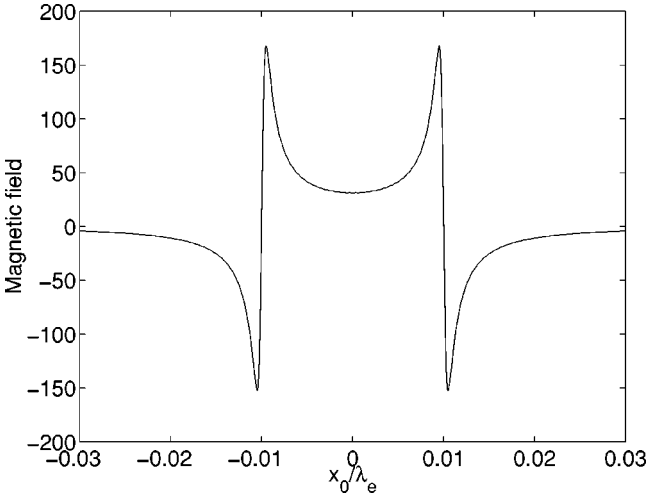


FIG. 5. The magnetic field (divided by $\lambda_e M_0$) generated by a Bloch wall with width $0.02\lambda_e$. Here we have assumed that $z = 0.0005\lambda_e$.

London equation should be solved for this symmetry. Consider a 1D magnetization distribution with the vector pointing in the z direction. Then we may apply the methods presented in this article (using the 1D Fourier transform) and find the following expression for the scalar potential:

$$\phi_m(x,z) = -\frac{\lambda_e}{\pi}\int_0^\infty \frac{k_x M(k_x)\cos(k_x x)}{1+2\lambda_e k_x}\exp(-k_x|z|)dk_x, \tag{41}$$

where we have assumed that the Fourier transform of the magnetization is real. The magnetic field components are given by

$$H_{mz}(x,z) = \frac{\lambda_e}{\pi}\int_0^\infty k_x^2 \frac{M(k_x)\cos(k_x x)}{1+2\lambda_e k_x}\exp(-k_x|z|)dk_x, \tag{42}$$

$$H_{mx}(x,z) = \frac{\lambda_e}{\pi}\int_0^\infty k_x^2 \frac{M(k_x)\sin(k_x x)}{1+2\lambda_e k_x}\exp(-k_x|z|)dk_x. \tag{43}$$

The energy of the system is found to be

$$E = \frac{\Phi_0^2}{4\pi\mu_0\lambda_e} ln\left(\frac{4\pi\lambda_e}{\xi}\right) - \frac{\Phi_0}{\pi}\int_0^\infty \frac{M(k_x)\cos(k_x x_0)}{1+2\lambda_e k_x}dk_x - \frac{\lambda_e\mu_0}{2\pi}\int_0^\infty k_x^2 \frac{M^2(k_x)}{1+2\lambda_e k_x}dk_x. \tag{44}$$

To find the interaction force between the magnetization distribution and the vortex, we simply take the negative derivative of the interaction energy and obtain

$$F(x_0) = -\frac{\Phi_0}{\pi}\int_0^\infty k_x \frac{M(k_x)\sin(k_x x_0)}{1+2\lambda_e k_x}dk_x. \tag{45}$$

As an example, Fig. 5 shows the z component of the magnetic field generated by the following magnetization distribution (which is looks like a Bloch wall separated by in-plane magnetized domains):

$$\mathbf{M} = \begin{cases} M_0\delta(z)\hat{\mathbf{e}}_z & \text{if } -W \leqslant x \leqslant W, \\ 0 & \text{if } |x| \geqslant W. \end{cases}$$

Here we have chosen $W = \lambda_e/50$, $z = 0$, and $t = \lambda_e/2000$. Note in particular the peaks near the edges of the Bloch wall. In general, it is also seen that there is a small field directly above the magnetization distribution which is not screened by the superconductor. It can be shown that the corresponding force on the vortex has a shape similar to that of Fig. 2, with its maximum near the edges of the magnetization distribution.

V. CONCLUSION

The interplay between a superconducting film and a magnetic nanostructure is studied. The generalized London equation for this system is solved, and the magnetic fields, the energy, and the interaction forces are computed. In particular, we focus on how to manipulate vortices using magnetic nanostructures. The formalism presented here could be generalized to arbitrary magnetization vectors, and we therefore hope that it will be useful in future studies.

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