III The MSSM and how it works

1. Basics of Supersymmetry*

SUSY: a symmetry relating scalar and vector bosons and fermions.

SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa:

$$\mathcal{Q}|\text{Fermion}> |\text{Boson}> \quad , \quad \mathcal{Q}|\text{Boson}> |\text{Fermion}>$$

Bosonic fields and fermionic fields have the same mass and same quantum numbers (except for the spin). Combined into superfields.

- **Simplest case is chiral/scalar superfield: S (2dof) and ζ ($\zeta^c = \zeta$).**
- **Another possibility is the vector superfield: A^a_μ and λ_A (2dof).**
- **All fields involved have the canonical kinetic energies**

$$\mathcal{L}_{\text{kin}} = \Sigma_i \Big\{ (D_\mu S_i^*)(D^\mu S_i) + i\overline{\psi}_i D_\mu \gamma^\mu \psi_i \Big\} + \Sigma_a \Big\{ -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{i}{2} \overline{\lambda}_a D \lambda_a \Big\}$$

with D the covariant derivative. [Note that $\psi(\lambda)$ have 4(2) comps.]

- **The interactions are specified by SUSY and gauge invariance:**

$$\mathcal{L}_{\text{int. scal-fer.-gauginos}} = -\sqrt{2} \sum_{i,a} g_a \left[S_i^* T^a \overline{\psi}_{iL} \lambda_a + \text{h.c.} \right]$$

$$\mathcal{L}_{\text{int. quartic scal.}} = -\frac{1}{2} \sum_a \Big(\sum_i g_a S_i^* T^a S_i \Big)^2$$

All interactions are given by the gauge coupling constants.

Everything is completely specified and no adjustable parameter!

*Here, we give only the basic importnat facts which will be needed in the phenomenological discussion that we will have later. For details on the theoretical issues, see the lectures given by I. Aitchinson for instance.

Only freedom: choice of Superpotential W (SUSY and gauge inv!). It gives the scalar potential and Yukawa interactions (fer.–scal.).

– $W \equiv$ function of the superfields z_i only (not z_i^*!).

– Analytic function: no derivative interaction.

– Renormalizability: only terms of dimension 2 and 3.

$$\Rightarrow \mathcal{L}_W = -\sum_i \mid \frac{\partial W}{\partial z_i} \mid^2 -\frac{1}{2}\sum_{ij}[\overline{\psi}_{iL}\frac{\partial^2 W}{\partial z_i \partial z_j}\psi_j + \text{h.c.}]$$

To obtain the interactions explicitly: we take the derivative of W with respect to z_i, and we evaluate in terms of the scalar fields S_i.

Just a few important points to recall, though:

• SUSY cannot be an exact symmetry since no scalars exist with the same mass as known fermions: $\Rightarrow$ must be explicitly broken.•

We need SUSY breaking at low energy to solve the problems:

– Quadratic divergences in the Higgs sector.

– Unification of the coupling constants of $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$.

– Dark Matter problem (existence of a massive stable particle), etc.

• In the breaking, we still need to preserve gauge invariance, renormalizability and the fact that there is no quadratic divergence.

No satisfactory (dynamical) way to break SUSY at the moment: $\Rightarrow$ Breaking by hand (with several possibilities...) .

$\Rightarrow$ "Low energy SUSY" $\equiv$ effective theory at low energy.

2. The Minimal Supersymmetric Standard Model

The MSSM is the most economic (low energy) supersymmetric extension of the SM. It is based on the following assumptions:

- **Minimal gauge group:** $\mathrm{G_{SM}} = \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)}$.

The SM gauge bosons $[B, W_{1-3}$ **and** $g_{1-8}]$ **and spin–**$\frac{1}{2}$ **gauginos** $[\tilde{b}, \tilde{w}_{1-3}, \tilde{g}_{1-8}]$ **are in vector superfields.**

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	**Particle content**
$\hat{G}^a$	8	1	0	G^μ, $\tilde{g}$
$\hat{W}^i$	1	3	0	W_i^μ, $\tilde{\omega}_i$
$\hat{B}$	1	1	0	B^μ, $\tilde{b}$

- **Minimal particle content:**

– Three fermion generations [no $\tilde{\nu}_R$**...] and their spin–0 SUSY, the sfermions** $\tilde{f}_L, \tilde{f}_R$**, combined in chiral supermultiplets.**

– No chiral anomalies and fermion mass generation in a SUSY invariant way (no H^***): two chiral superfields with** $Y = +1$ **and** $Y = -1$**.**

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	**Particle content**
$\hat{Q}$	3	2	$\frac{1}{3}$	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{4}{3}$	$\overline{u}_R$, $\tilde{u}_R^*$
$\hat{D}^c$	$\overline{3}$	1	$\frac{2}{3}$	$\overline{d}_R$, $\tilde{d}_R^*$
$\hat{L}$	1	2	-1	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	2	$\overline{e}_R$, $\tilde{e}_R^*$
$\hat{H}_1$	1	2	-1	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	1	$(H_2, \tilde{h}_2)$

- **R–parity:**

To eliminate terms violating B and L numbers (and proton decay): Discrete and multiplicative symmetry called R-parity or R_p:

$$R_p = (-1)^{2s+3B+L}$$

Then, for ordinary particles: R=+1, and SUSY particles: R=−1. The consequences of R_p conservation are important:

- **SUSY particles always produced in pairs.**
- **SUSY particles decay into an odd number of SUSY particles.**
- **The lightest SUSY particle (LSP) is absolutely stable.**

At this stage, we have a globally supersymmetric Lagrangian. The most general Superpotential compatible with gauge invariance, renormalizability and R–parity conservation is:

$$W = \sum_{\text{i,j=gen}} Y^u_{ij}\, \hat{u}^i_R \hat{H}_2.\hat{Q}^j + Y^d_{ij}\, \hat{d}^i_R \hat{H}_1.\hat{Q}^j + Y^l_{ij}\, \hat{l}^i_R \hat{H}_1.\hat{L}^j + \mu \hat{H}_1.\hat{H}_2$$

- $Y^{u,d,l}_{ij}$ **denote the Yukawa couplings among generations.**
- μ **supersymmetric Higgs–higgsino mass parameter.**

- **Soft SUSY breaking:**

To explicitely break SUSY without reintroducing the quadratic divergences (the so–called soft SUSY–breaking), we add by hand a collection of soft terms (of dimension 2 et 3):

$$\mathcal{L}_{\rm gaugino} = \frac{1}{2}\left[M_1\tilde{b}\tilde{b} + M_2\Sigma_{a=1}^{3}\tilde{w}^a\tilde{w}_a + M_3\Sigma_{a=1}^{8}\tilde{g}^a\tilde{g}_a \;+\; \text{h.c.}\right]$$

$$\mathcal{L}_{\rm sf.} = \Sigma_i m^2_{\tilde{Q},i}\tilde{Q}_i^\dagger\tilde{Q}_i + m^2_{\tilde{L},i}\tilde{L}_i^\dagger\tilde{L}_i + m^2_{\tilde{u},i}|\tilde{u}_{R_i}|^2 + m^2_{\tilde{d},i}|\tilde{d}_{R_i}|^2 + m^2_{\tilde{l},i}|\tilde{l}_{R_i}|^2$$

$$\mathcal{L}_{\rm Higgs} = m_2^2 H_2^\dagger H_2 + m_1^2 H_1^\dagger H_1 + B\mu(H_2.H_1 + \text{h.c.})$$

$$\mathcal{L}_{\rm tr.} = \Sigma_{i,j}\left[A^u_{ij}Y^u_{ij}\tilde{u}_{R_i}H_2.\tilde{Q}_j + A^d_{ij}Y^d_{ij}\tilde{d}_{R_i}H_1.\tilde{Q}_j + A^l_{ij}Y^l_{ij}\tilde{l}_{R_i}H_1.\tilde{L}_j \;+\; \text{h.c.}\right]$$

A rather complicated and problematic potential indeed!

- **Too many parameters and thus not very predictive!**

In the most general case (mixing and phases): 105 free parameters!

- **complex gaugino masses M_1, M_2, M_3 : 6**
- **3×3 hermitian mass matrices $m_{\tilde{F}}$: 45**
- **3×3 complex trilinear coupling matrices A_F : 54**
- **2×2 matrix for the bilinear B coupling : 4**
- **Higgs masses squared, $m^2_{H_1}, m^2_{H_2}$: 2**

111–6 (due to constraints from symmetries and Higgs sector)=115.

- **Too problematic phenomenologically!**

For "generic" sets of these parameters, leads to severe problems:

- **large flavor changing neutral currents [FCNC]**
- **unacceptable amount of additional CP–violation**
- **color and/or charge breaking minima**
- **an incorrect value of the Z boson mass, etc.....**

We need more constrained MSSMs

3. The constrained MSSMs:

a) The phenomenological MSSM:

A phenomenologically viable MSSM is defined by assuming:

(i) all soft SUSY–breaking parameters are real (no new CP viol).

(ii) Mass and trilinear cpls. for sfermions diagonal (no FCNC)

(iii) 1st/2d sfermion generation universality (no pb. with Kaons)

Phenomenological MSSM (pMSSM) with 22 free parameters:

$\tan\beta$**: the ratio of the vevs of the two–Higgs doublet fields.**

$m^2_{H_u}, m^2_{H_d}$**: the Higgs mass parameters squared.**

M_1, M_2, M_3**: the bino, wino and gluino mass parameters.**

$m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{l}}, m_{\tilde{e}_R}$**: 1st/2d generation sfermion mass para.**

$m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}, m_{\tilde{L}}, m_{\tilde{\tau}_R}$**: third generation sfermion mass para.**

A_u, A_d, A_e**: the first/second generation trilinear couplings.**

A_t, A_b, A_τ**: the third generation trilinear couplings.**

– μ, B are determined from EW symmetry breaking, see later.

– A_u, A_d, A_e in general not relevant for phenomenology).

– If you focus on a given sector (Higgs, gauginos, sfermions): only few parameters to deal with and model indep. analyses....

$\Rightarrow$ phenomenologically viable model

b) The minimal Supergravity model:

Almost all problems of uMSSM solved at once if soft SUSY–breaking parameters obey universal boundary conditions at $M_{\rm GUT}$.

Underlying assumption: SUSY–breaking in hidden sector communicating with visible sector through gravitational interactions.

Universal soft terms emerge if interactions are "flavor–blind":

Besides unification of $g_{1,2,3}$ which fix the scale $M_{\rm GUT} \sim 2 \cdot 10^{16}$ GeV:

– **Unification of the gaugino [bino, wino and gluino] masses:**

$$M_1(M_{\rm GUT}) = M_2(M_{\rm GUT}) = M_3(M_{\rm GUT}) \equiv m_{1/2} \tag{1}$$

– **Universal scalar [i.e. sfermion and Higgs boson] masses:**

$$M_{\tilde{Q}_i}(M_{\rm GUT}) \;=\; M_{\tilde{L}_i}(M_{\rm GUT}) = M_{H_i}(M_{\rm GUT}) \equiv m_0 \tag{2}$$

– **Universal trilinear couplings:**

$$A^u_{ij}(M_{\rm GUT}) = A^d_{ij}(M_{\rm GUT}) = A^l_{ij}(M_{\rm GUT}) \equiv A_0\, \delta_{ij} \tag{3}$$

B and μ from requiring of EWSB and minimization of $V_{\rm Higgs}$

$$\begin{aligned} \mu^2 &= \frac{1}{2}\big[\tan 2\beta (m^2_{H_u} \tan\beta - m^2_{H_d} \cot\beta) - M_Z^2\big] \\ B\mu &= \frac{1}{2} \sin 2\beta \Big[m^2_{H_u} + m^2_{H_d} + 2\mu^2\Big] \end{aligned} \tag{4}$$

Only 4.5 param: $\tan\beta$, $m_{1/2}$, m_0 , A_0 , $\mathrm{sign}(\mu)$

All soft breaking parameters at M_S are obtained through RGEs.

c) The Anomaly mediated SUSY model:

In AMSB, SUSY breaking occurs also in hidden sector but is transmitted to visible sector by the super–Weyl anomaly.

Gaugino, scalar masses and trilinear couplings are simply related to scale dependence of gauge and matter kinetic functions.

In terms of (large) gravitino mass $m_{3/2}$, β functions for g_a and Y_i and anomalous dimensions γ_i of chiral superfields, SSB terms are (μ^2 and B are obtained as usual by requiring EWSB):

$$\begin{aligned} M_a &= \frac{\beta_{g_a}}{g_a} m_{3/2} \ , \ A_i = \frac{\beta_{Y_i}}{Y_i} m_{3/2} \\ m_i^2 &= -\frac{1}{4}\left(\Sigma_a \frac{\partial \gamma_i}{\partial g_a}\beta_{g_a} + \Sigma_k \frac{\partial \gamma_i}{\partial Y_k}\beta_{Y_k}\right) m_{3/2}^2 \end{aligned} \tag{5}$$

RG invariant equations valid at any scale (make a predictive model).

In principle, only three inputs parameters $m_{3/2}, \tan\beta$ and sign(μ), but spoiled by tachyonic sleptons $m_{\tilde{L}}^2 < 0$ in general!

$\Rightarrow$ add a non anomalous contribution to soft masses. General:

$$m_{\tilde{S}_i}^2 = c_{S_i} m_0^2 - \frac{1}{4}\left(\Sigma_a \frac{\partial \gamma_i}{\partial g_a}\beta_{g_a} + \Sigma_k \frac{\partial \gamma_i}{\partial Y_k}\beta_{Y_k}\right) m_{3/2}^2 \tag{6}$$

where the input coefficients c_{S_i} depend on the considered model. For instance in minimal AMSB with a universal m_0: $c_F = 1$

Including coefficients c_{S_i}, the input parameters of AMSB are:

$$m_0 \ , \ m_{3/2} \ , \ \tan\beta \ , \ \text{sign}(\mu) \text{ and } c_{S_i} \tag{7}$$

c) The Gauge mediated SUSY model:

In GMSB, SSB transmitted to the MSSM fields via the SM interactions. There are two sectors: one where SUSY is broken and contains messenger fields and one containing the MSSM fields.

The soft mass terms for gauginos and sfermion arise from one–loop and two–loop diagrams involving messenger field exchange, while trilinear couplings are small (generated at two loops).

The source of SSB is a gauge singlet chiral superfield $\hat{S}$ whose scalar and auxiliary components acquire vevs S and F_S. There are $n_{\hat{q}}$ pairs of $\hat{q}, \hat{\bar{q}}$ quark–like and $n_{\hat{l}}$ pairs of $\hat{l}, \hat{\bar{l}}$ lepton–like] messengers and coupled to $\hat{S}$ through a superpotential of the form $\lambda \hat{S} \hat{q} \hat{\bar{q}} + \lambda \hat{S} \hat{l} \hat{\bar{l}}$. SSB parameters are generated at messenger scale $M_{\rm mes} = \lambda S$,

$$M_G(M_{\rm mes}) \;=\; \frac{\alpha_G(M_{\rm mes})}{4\pi} \Lambda \, g\big(\frac{\Lambda}{M_{\rm mes}}\big) \sum_m N_R^G(m) \tag{8}$$

$$m_s^2(M_{\rm mes}) \;=\; 2\Lambda^2 f\big(\frac{\Lambda}{M_{\rm mes}}\big) \sum_{m,G} \big[\frac{\alpha_G(M_{\rm mes})}{4\pi}\big]^2 N_R^G(m) C_R^G(s) \tag{9}$$

$$A_f(M_{\rm mes}) \;\simeq\; 0 \tag{10}$$

where $\Lambda = F_S/S$, $G = \mathrm{U}(1), \mathrm{SU}(2), \mathrm{SU}(3)$, m labels the messengers and s all scalars; f, g are one(two) loop simple functions

Thus, in GMSB model there are six input parameters

$$\tan\beta \;,\; \mathrm{sign}(\mu) \;,\; M_{\rm mes} \;,\; \Lambda \;,\; n_{\hat{q}} \;,\; n_{\hat{l}} \tag{11}$$

Plus the mass of the very light gravitino (which is LSP).

IV. The SUSY particle spectrum

1. The gaugino sector

Winos and higgsinos mix to make the charginos and neutralinos: to get physical states $m_{\chi^\pm_{1,2}}$, $m_{\chi^0_{1-4}}$, diagonalize mass matrices.

The general chargino mass matrix, in terms of M_2, μ and $\tan\beta$, is

$$\mathcal{M}_C = \begin{bmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{bmatrix} \tag{12}$$

$s_\beta \equiv \sin\beta$ *etc*; diagonalized by two real matrices U and V

$$U\mathcal{M}_C V^{-1} \quad \rightarrow \quad U = \mathcal{O}_- \text{ and } \quad V = \begin{cases} \mathcal{O}_+ & \text{if } \det\mathcal{M}_C > 0 \\ \sigma_3\mathcal{O}_+ & \text{if } \det\mathcal{M}_C < 0 \end{cases} \tag{13}$$

σ_3 is to make the masses positive and $\mathcal{O}_\pm$ rotation matrices

This leads to the two chargino masses

$$m^2_{\chi^\pm_{1,2}} = \frac{1}{2}\left\{M_2^2 + \mu^2 + 2M_W^2 \mp \left[(M_2^2-\mu^2)^2 + 4M_W^2(M_W^2 c_{2\beta}^2 + M_2^2 + \mu^2 + 2M_2\mu s_{2\beta})\right]^{\frac{1}{2}}\right\}$$

In the limit $|\mu| \gg M_2, M_W$, the masses are [ϵ_μ =sign(μ)]

$$m_{\chi_1^\pm} \simeq M_2 - M_W^2\mu^{-2}(M_2 + \mu s_{2\beta}) \quad , \quad m_{\chi_2^\pm} \simeq |\mu| + M_W^2\mu^{-2}\epsilon_\mu(M_2 s_{2\beta} + \mu)$$

For $|\mu| \to \infty$, $\chi_1^\pm$ is wino with $m_{\chi_1^\pm} \simeq M_2$ and $\chi_2^\pm$ is higgsino with $m_{\chi_2^\pm} = |\mu|$. In the limit, $M_2 \gg |\mu|, M_Z$, roles of $\chi_1^\pm, \chi_2^\pm$ reversed.

In the case of the neutralinos, the 4-dim mass matrix depends on $\mu, M_2, \tan\beta$ and M_1. In the $(-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, it is

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{bmatrix} \tag{14}$$

Can be diagonalized by a single real matrix Z. For $|\mu| \gg M_{1,2} \gg M_Z$:

$$m_{\chi_1^0} \simeq M_1 - \frac{M_Z^2}{\mu^2}(M_1 + \mu s_{2\beta}) s_W^2 \ , \ m_{\chi_2^0} \simeq M_2 - \frac{M_Z^2}{\mu^2}(M_2 + \mu s_{2\beta}) c_W^2$$

$$m_{\chi_{3/4}^0} \simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1 \mp s_{2\beta})\left(\mu \pm M_2 s_W^2 \mp M_1 c_W^2\right)$$

For $|\mu| \to \infty$, two neutralinos are gauginos with $m_{\chi_1^0} \simeq M_1$ and $m_{\chi_2^0} = M_2$, and two are higgsinos with $m_{\chi_3^0} \simeq m_{\chi_4^0} \simeq |\mu|$. In opposite limit, roles are again reversed: $m_{\chi_1^0} \simeq m_{\chi_2^0} \simeq |\mu|, m_{\chi_3^0} \simeq M_1$ and $m_{\chi_4^0} \simeq M_2$.

Finally, the gluino mass is identified with M_3 at the tree–level

$$m_{\tilde{g}} = M_3 \tag{15}$$

In constrained models with boundary conditions at the high energy scale M_U, the evolution of the gaugino masses given by RGEs

$$\frac{\mathrm{d}M_i}{\mathrm{d}\log(M_{\rm U}/Q^2)} = -\frac{g_i^2 M_i}{16\pi^2} b_i \ , \ b_1 = \frac{33}{5} \ , \ b_2 = 1 \ , b_3 = -3 \tag{16}$$

where in b_i all sparticles contribute to the evolution from Q to M_U. Equations are related to those of the gauge couplings $\alpha_i = g_i^2/(4\pi)$. With inputs at scale M_Z and common value at $M_U \sim 2 \times 10^{16}$ GeV, one obtains at M_S:

$$M_3 : M_2 : M_1 \sim \alpha_3 : \alpha_2 : \alpha_1 \sim 6 : 2 : 1 \tag{17}$$

With norm.factor $\frac{5}{3}$ in α_1, we have $M_1 = \frac{5}{3}\tan^2\theta_W M_2 \simeq \frac{1}{2}M_2$.

2. The sfermion sector

Sfermion system described by $\tan\beta, \mu$ and 3 param.for each species: $m_{\tilde{f}_L}, m_{\tilde{f}_R}$ and A_f. For 3d generation, mixing $\propto m_f$ to be included. Sfermion mass matrix are:

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m_f^2 + m_{LL}^2 & m_f X_f \\ m_f X_f & m_f^2 + m_{RR}^2 \end{pmatrix} \tag{18}$$

with the various entries given by

$$\begin{aligned} m_{LL}^2 &= m_{\tilde{f}_L}^2 + (I_f^{3L} - Q_f s_W^2)\, M_Z^2\, c_{2\beta} \\ m_{RR}^2 &= m_{\tilde{f}_R}^2 + Q_f s_W^2\, M_Z^2\, c_{2\beta} \\ X_f &= A_f - \mu(\tan\beta)^{-2I_f^{3L}} \end{aligned} \tag{19}$$

They are diagonalized by 2×2 rotation matrices of angle θ_f, which turn the current eigenstates $\tilde{f}_L, \tilde{f}_R$ into the mass eigenstates $\tilde{f}_1, \tilde{f}_2$

$$R^{\tilde{f}} = \begin{pmatrix} c_{\theta_f} & s_{\theta_f} \\ -s_{\theta_f} & c_{\theta_f} \end{pmatrix} \quad , \quad c_{\theta_f} \equiv \cos\theta_{\tilde{f}} \quad \text{and} \quad s_{\theta_f} \equiv \sin\theta_{\tilde{f}} \tag{20}$$

The mixing angle and sfermion masses are then given by

$$s_{2\theta_f} = \frac{2 m_f X_f}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \quad , \quad c_{2\theta_f} = \frac{m_{LL}^2 - m_{RR}^2}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \tag{21}$$

$$m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4 m_f^2 X_f^2}\ \right] \tag{22}$$

Mixing very strong in stop sector, $X_t = A_t - \mu\cot\beta$ and generates mass splitting between $\tilde{t}_1, \tilde{t}_2$, leading to light $\tilde{t}_1$; mixing in sbottom/stau sectors also for large $X_{b,\tau} = A_{b,\tau} - \mu\tan\beta$.

In cMSSM with universal m_0 and $m_{1/2}$ at $M_{\rm GUT}$, the RGEs for the scalar masses are simple if Yukawas are small:

$$m^2_{\tilde{f}_{L,R}} = m_0^2 + \sum_{i=1}^{3} F_i(f) m^2_{1/2} \ , \ \ F_i = \frac{c_i(f)}{b_i}\left[1 - \left(1 - \frac{\alpha_U}{4\pi} b_i \log\frac{Q^2}{M_U^2}\right)^{-2}\right] \quad (23)$$

with $c(\tilde{f}) = (c_1, c_2, c_3)(\tilde{f})$ dependin on isospin, Y and color

$$c(\tilde{L}) = \begin{pmatrix} \frac{3}{10} \\ \frac{3}{2} \\ 0 \end{pmatrix}, c(\tilde{l}_R) = \begin{pmatrix} \frac{6}{5} \\ 0 \\ 0 \end{pmatrix}, c(\tilde{Q}) = \begin{pmatrix} \frac{1}{30} \\ \frac{3}{2} \\ \frac{8}{3} \end{pmatrix}, \ c(\tilde{u}_R) = \begin{pmatrix} \frac{8}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}, c(\tilde{d}_R) = \begin{pmatrix} \frac{2}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}$$

With inputs at M_Z, $\alpha_U \simeq 0.041$ and M_U, one obtains

$$m^2_{\tilde{q}_i} \sim m_0^2 + 6m^2_{1/2}\,, \quad m^2_{\tilde{\ell}_L} \sim m_0^2 + 0.52 m^2_{1/2}\,, \quad m^2_{\tilde{e}_R} \sim m_0^2 + 0.15 m^2_{1/2} \quad (24)$$

For 3d gen. squarks, Yukawa couplings to be included. Approximare RGE:

$$m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L} \sim m_0^2 + 6m^2_{1/2} - \frac{1}{3}X_t\,, \quad m^2_{\tilde{t}_R} = m^2_{\tilde{b}_L} \sim m_0^2 + 6m^2_{1/2} - \frac{2}{3}X_t \quad (25)$$

with $X_t \sim 1.3 m_0^2 + 3 m^2_{1/2}$ [?]. This shows that, in contrast to the first two generations, one has generically a sizable splitting between $m^2_{\tilde{t}_L}$ and $m^2_{\tilde{t}_R}$ at the electroweak scale, due to the running of the large top Yukawa coupling. This justifies the choice of different soft SUSY–breaking scalar masses and trilinear couplings for the third generation and the first/second generation sfermions [as well as for slepton and squark masses].

3. The scalar Higgs potential in the MSSM

In MSSM with two Higgs doublet $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$ and $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$, the terms contributing to the scalar potential V come from 3 sources:

- **D terms (quartic interactions):** $V_D = \frac{1}{2}\sum_a \left(\sum_i g_a S_i^* T^a S_i\right)^2$
- **F terms F of the Superpotential:** $V_F = \sum_i \left|\frac{\partial W(z_i)}{\partial z_i}\right|^2 \to \sum_i \left|\frac{\partial W(\phi_j)}{\partial \phi_i}\right|^2$
- **Soft termes:** $V_{\text{soft}} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + B\mu(H_2.H_1 + \text{h.c.})$

- **D terms for the two Higgs fields H_1, H_2 with $Y = +1, -1$:**

$$\begin{aligned} \text{U(1)}_\text{Y} \ &: \ V_D^1 = \left[\frac{g_1}{2}(|H_2|^2 - |H_1|^2)\right]^2 \\ \text{SU(2)}_\text{L} \ &: \ V_D^2 = \left[\frac{g_2}{2}(H_1^{i*}\tau_{ij}^a H_1^j + H_2^{i*}\tau_{ij}^a H_2^j)\right]^2 \end{aligned}$$

with $\tau^a = 2T^a$. Using the SU(2) identity: $\tau_{ij}^a \tau_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$:

$$V_D = \frac{g_2^2}{8}\left[4|H_1^*.H_2|^2 - 2(H_1^*.H_2)(H_2^*.H_2) + (|H_1|^2 + |H_2|^2)\right]^2 + \frac{g_1^2}{8}(|H_2|^2 - |H_1|^2)^2$$

- **F terms [just from $W \sim \mu \hat{H}_1, \hat{H}_2$] and soft terms:**

$$V_F = \mu^2(|H_1|^2 + |H_2|^2) + m_1^2|H_1|^2 + m_2^2|H|_2^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + \text{h.c.})$$

$\Rightarrow$ The scalar potential involving the Higgs bosons:

$$\begin{aligned} V_H \ = \ & (|\mu|^2 + m_1^2)|H_1|^2 + (|\mu|^2 + m_2^2)|H_2|^2 - \mu B\epsilon_{ij}(H_1^i H_2^j + \text{h.c.}) \\ & + \frac{g_2^2 + g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^* H_2|^2 \end{aligned}$$

Development in terms of the components of H_1 et H_2 and setting:

$$\overline{m}_1^2 = |\mu|^2 + m_1^2, \ \overline{m}_2^2 = |\mu|^2 + m_2^2, \ \overline{m}_3^2 = B\mu$$

$$V_H = \overline{m}_1^2(|H_0^1|^2 + |H_1^+|^2) + \overline{m}_2^2(|H_2^0|^2 + |H_2^-|^2) - \overline{m}_3^2(H_1^+ H_2^- - H_1^0 H_2^0 + \text{hc})$$
$$+\frac{g_2^2 + g_1^2}{8}(|H_1^0|^2 + |H_1^+|^2 - |H_2^0|^2 - |H_2^-|^2)^2 + \frac{g_2^2}{2}|H_1^{+*} H_1^0 + H_2^{0*} H_2^-|^2$$

Now we require that the minimum of V_H breaks $G_{\rm SM} \to U(1)_{\rm QED}$.

At $V_H^{\rm min}$ we choose $\langle H_1^+ \rangle$=0 [SU(2)] and at $\partial V/\partial H_1^+$=0 we have $\langle H_2^- \rangle$=0. It is good for QED, and we can ignore the fields H_1^+, H_2^- to simplify:

$$V_H = \overline{m}_1^2|H_0^1|^2 + \overline{m}_2^2|H_2^0|^2 + \overline{m}_3^2(H_1^0 H_2^0 + \text{hc}) + \frac{g_2^2 + g_1^2}{8}(|H_1^0|^2 - |H_2^0|^2)^2$$

Some remarks:

- **Quartic couplings fixed in terms of the gauge couplings, only 3 free parameters: $\overline{m}_1^2, \overline{m}_2^2, \overline{m}_3^2$ (6 and a phase in a general 2HDM).**
- **$m_{1,2}^2 + |\mu|^2$ real, only $B\mu$ can be complex. But any phase in $B\mu$ can be absorbed in phases of $H_1, H_2 \Rightarrow V_H$ (MSSM) conserves CP.**
- **If $B\mu$ is zero, all other terms are positif and thus $V_H = 0$ only if $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. To have SSB (without CCB), we need $\overline{m}_{1,2,3} \neq 0 \Rightarrow$ Connection of gauge symmetry breaking and SUSY breaking!!**

More precisely: in the SM, SSB takes place with ad hoc choice $\mu^2 < 0$. In MSSM, $m_{H_i}^2 > 0$ at $M_{\rm GUT}$ but $t/\tilde{t}$ in RGE make $m_{H_i}^2 < 0$ at M_Z: radiative breaking of the electroweak symmetry.

$\Rightarrow$ Symmetry breaking more natural and elegant than in SM.

The MSSM Higgs boson masses and couplings

To obtain the physical Higgs fields and their masses from V_H, develop $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ into real (corresponding to CP–even and charged Higgses) and imaginary (corresponding to CP–odd Higgs and Goldstones) parts and diagonalize the matrices:

$$\mathcal{M}^2_{ij} = \frac{1}{2}\partial^2 V_H/\partial H_i \partial H_j|_{\langle \mathrm{Re}(H^0_{1,2})\rangle = v_{1,2}, \langle \mathrm{Im}(H^0_{1,2})\rangle = 0, \langle H^\pm_{1,2}\rangle = 0}$$

To be done in the next series of exercices. Get:

$$M_A^2 = -\bar{m}_3^2(\tan\beta + \cot\beta) = -2\bar{m}_3^2/\sin 2\beta$$

$$M^2_{H^\pm} = M_A^2 + M_W^2$$

$$M^2_{h,H} = \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right]$$

The mixing angle α which rotates the fields is ($-\frac{\pi}{2} \leq \alpha \leq 0$)

$$\tan 2\alpha \;=\; \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}} = \frac{-(M_A^2 + M_Z^2)\sin 2\beta}{(M_Z^2 - M_A^2)\cos 2\beta} = \tan 2\beta\, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

We see that we have an important constraint on the lightest h boson mass in the MSSM (however; RC have to be included!)

$$M_h \;\leq\; \min(M_A, M_Z)\cdot|\cos 2\beta| \leq M_Z$$

besides some other (alos important) relations:

$$M_H > \max(M_A, M_Z) \quad \text{and} \quad M_{H^\pm} > M_W$$

4. Summary on how to get the MSSM spectrum

