A continuous-time value function approach to VSL and VSI valuation

By

Jon Strand

Department of Economics

University of Oslo

Box 1095, Blindern, 0317 Oslo, Norway

jon.strand@econ.uio.no

Abstract

I study a simple analytical model in continuous time where individuals can be either healthy or ill, and have constant transition between the states and constant mortality rates in each state. The value from being in each of the two live states can be represented by a pair of Bellman dynamic programming equations. This leads to simple expressions for lifetime values of remaining in each of the states, values of statistical life in good health (VSL) and of statistical illness (VSI), the value of improved health, and the value of health investments. Changes in background risks have intuitively reasonable effects on VSL and VSI, while a "dead-anyway effect" only appears in the valuation of health investment but not in expressions of VSL or VSI. I argue that my model is most appropriate for VSL and VSI analysis when interventions have immediate effects on health risks, but less so when risks are delayed. More care than hitherto ought thus to be taken, in selecting models for such analyses.

Key words: Value of statistical life; value of statistical illness; mortality risks; background risks.

JEL classification: I12, I18, D11.

1. Introduction

The purpose of this note is to expose and clarify some basic issues related to valuing changes in mortality and morbidity, based on a much simplified but instructive continuous-time model with constant transition (or hazard) rates between three mutually exclusive states: healthy, sick and dead; and exponential remaining times in each of the two former states. As is customary in the literature, we define the value of statistical life in good health (VSL) as the rate of substitution between consumption and mortality risk when initially in a healthy state. We similarly define the value of statistical illness (VSI; as coined by Cameron and DeShazo (2003)) as the rate of substitution between consumption and mortality risk, when initially in the state of illness. Under the continuous-time approach, current valuation of current risk change always departs from the current state. This acknowledgement enormously simplifies the notions and derivations of VSL and VSI, relative to some alternative approaches in the recent literature. In particular, issues such as comorbidities (Bleichrodt, Crainich and Eeckhoudt (2003), background risk (Eeckhoudt and Hammitt (2001)), risk aversion (Eeckhoudt and Hammitt (2004)), and the "dead-anyway effect" (Pratt and Zeckhauser (1996), Breyer and Felder (2002)) are then shown to lose their role in VSL and VSI valuation. Such factors are important under either a discrete formulation of the decision problem facing the individual, or when effects occur with delay, as is typically assumed in these papers. I here however argue that such discretization or delay is often artificial in the context of VSL or VSI valuation, at least when the interventions considered for valuation have immediate effects on the relevant mortality and morbidity rates. Relevant examples are medication or treatment to control blood pressure or cholesterol levels (which affect heart conditions immediately), or stricter road traffic or workplace safety regulations

(which immediately affect accident rates), or efforts to clean the drinking water (with immediate effects on rates of sickness due to water contamination).[1] For such cases the model framework developed here is particularly relevant. The model is also relevant for cases with "competing risks" as analysed e.g. by Dow, Phillipson and Sala-I-Martin (1999) and Chang (2002), given that the hazard rates are constant.[2] I however also consider cases where an intervention has a permanent effect on the mortality or morbidity rate, and show that a "dead-anyway effect" then appears, not in VSL or VSI directly, but in the value of such interventions.

To make the model as simple, intuitive and instructive as possible I concentrate on a stripped-down case where an individual's consumption is constant in a given state (healthy or ill), and risks are time- and age-independent. I first consider instantaneous rate changes in mortality and morbidity rates, and then show that VSL and VSI can be expressed simply as ratios between the expected discounted future utility value of being in the current state, V(H) and V(I) respectively, and the marginal utility of income in the respective states. One implication of this is that the "value of full health" (the value of being in the healthy state instead of in the state of illness) is always greater when the initial state is the ill state, than when one initially is in the healthy state, whenever the marginal utility of income is lower in the ill state (as is considered reasonable). The effects of increases in "background" mortality, morbidity and return-to-full-health rates on VSL and VSI are shown to have intuitively reasonable signs and values (the two first negative and the last positive). In the final section I also note (although do not formally prove) that many of the basic properties of the solutions, such as the basic formulae describing VSL and VSI and the effects of

[1] See also the empirical study by Breyer and Grabka (2001) who find no empirical basis for a dead-anyway effect, on a large German data set.
[2] See e.g. David and Moeschberger (1978) and Elandt-Johnson and Johnson (1980) for the theoretical arguments supporting the current model, under competing risks.

background risks, are not altered when risk levels change throughout an individual's lifetime.

The use of value functions of the Bellman type in continuous time is now widespread in economics, in particular within theoretical and empirical labor market analysis, see e.g. Mortensen and Pissardies (1999), Pissarides (2000) and van den Berg (2003). This tool has turned out to be quite powerful and at the same time intuitively easy both to understand and to handle mathematically, at least in simple contexts similar to those considered here. Basic ideas are that decisions and evaluations are always made sequentially and based on the current state, and that no current commitments are made to actions at future dates. To my knowledge this framework has not so far been applied in health economics settings. I claim that it is extremely well adapted to such settings. A main purpose of this note is then also to stimulate others to undertake research using related approaches.

2. The model

Consider an individual who at can be in only one out of three mutually excluding states: alive and well, with current utility $u(c_1, H)$; alive and ill, with utility $u(c_2, I)$; and dead with utility normalized to zero. c_1 and c_2 denote the consumption levels in the healthy and ill states, which are assumed constant over time for a given state, but may differ between states, typically with $c_1 \geq c_2$.[3] The current utility level u is then constant over time in a given state. I assume that $u(c, H) > u(c, I)$, and $u_c(c, H) > u_c(c,$

[3] We here do not discuss the sources of consumption possibilities. These will typically be labor income, initial financial wealth, and possible insurance schemes. Labor income may be lower in the sick than in the healthy state, while initial insurance contracts to insure the individual against sickness may (partly or fully) counteract the reduction in labor income. A related issue not elaborated further here is that when sickness or life insurance contracts can be continuously renegotiated, changes in death and sickness risk will induce a renegotiation of the initial contract so as to ensure an actuarially fair return to the insurer; see Strand (2003). This is in particular likely to induce a reduction in sick-state consumption when the probability of sickness increases.

I), implying that both the absolute utility level, and the marginal utility of consumption, are higher for given consumption, in the healthy state relative to the ill state. We model time as continuous. An individual who starts from an initial healthy state faces constant and independent Poisson rates λ_D and λ_I at which death and illness occur respectively. When ill the individual faces a constant death rate μ_D, and a rate μ_H at which health is restored to the initial level. This formulation is similar to Cameron and DeShazo (2003), except for two main differences. First, time is here continuous instead of discrete. Secondly, hazard rates are constant here instead of the more general variable rates in Cameron and DeShazo. These changes make for a far simpler formulation, permitting easier derivations and interpretations of results. While our assumption of constant transition rates makes our model less general, it is useful for pedagogical purposes, and for displaying certain principal analytical effects that are far more difficult to derive and interpret under more general formulations. It will also easily be seen that a number of the more important results do not depend on transition rates being constant over time.

Define the Bellman (or value) functions V(H) and V(I), as expected discounted lifetime values of current health and current illness, as follows:[4]

$$rV(H) = u(c_1, H) - \lambda_D V(H) + \lambda_I [V(I) - V(H)] \quad (1)$$

$$rV(I) = u(c_2, I) - \mu_D V(I) + \mu_H [V(H) - V(I)]. \quad (2)$$

r is the (constant) subjective rate of interest. The left-hand sides of (1)-(2) can be interpreted as current returns to two different assets, namely the present discounted

[4] The formulation of Bellman functions in continuous time follows from taking limits of the associated discrete-period functional equations, presented e.g. in Bellman and Kalaba (1965). Derivations are straightforward; for a standard application reference which has had great influence on the labor market literature, and where also a simple derivation is offered, see Shapiro and Stiglitz (1984).

value of currently being healthy, and the present discounted value of currently being sick. These returns have three components. The first is the current utility of consumption (or "current well-being") in the respective state. The second is a component representing the change in asset value when the state changes from the current to the dead state (which is here simply the loss of the whole initial asset value). The third is a component representing the change in asset value when the state changes, from the current to the alternative live state (from healthy to ill in (1), and from ill to healthy in (2)).[5] (1)-(2) can be solved with respect to V(H) and V(I) as follows:

$$V(H) = \frac{1}{D}[(r + \mu_H + \mu_D)u(c_1, H) + \lambda_I u(c_2, I)] \tag{3}$$

$$V(I) = \frac{1}{D}[\mu_H u(c_1, H) + (r + \lambda_D + \lambda_I)u(c_2, I)], \tag{4}$$

where

$$D = (r + \lambda_D + \lambda_I)(r + \mu_H + \mu_D) - \lambda_I \mu_H. \tag{5}$$

V(H) in (3) has a natural interpretation as the "present value of health", and V(I) in (4) as the "present value of illess", at current and constant transition rates between states. V(H) and V(I) are both simple functions of u(H) and u(I), only with different weights to the two utility levels, where V(H) has a relatively greater weight on the

[5] Note the overly simplified structure where mortality and morbidity rates are constant through time. By construction, VSL and VSI will then also be invariant through an individuals' life. This is counterfactual; see e.g. Johansson (2002), Aldy and Viscusi (2003), and Viscusi and Aldy (2003), for recent discussions of models where individuals' VSL values vary over their lifetimes, and the empirical evidence in this regard.

current level u(H), and this relative difference is greater when the discount rate r is higher. We see also e.g. that when μ_H is very large compared to the other parameters, V(H) and V(I) are close. (A spell of illness then lasts for a short time since ill individuals get well fast, and the value of current illness to a large extent reflects the value of current health.) Only the dead state is absorbing in general. In the special case of $\mu_H = 0$, the ill state is also absorbing (as when "illness" represents permanent disability).

Expected remaining lifetimes, EL(H) and EL(I), are here also constant over time for a given state, H or I, and are found as (using techniques similar to those used for deriving (3)-(4))

$$EL(H) = \frac{\mu_H + \mu_D + \lambda_I}{\lambda_D \mu_H + (\lambda_D + \lambda_I)\mu_D} \tag{6}$$

$$EL(I) = \frac{\mu_H + \lambda_D + \lambda_I}{\lambda_D \mu_H + (\lambda_D + \lambda_I)\mu_D}. \tag{7}$$

EL(H) > EL(I) if and only if $\lambda_D < \mu_D$ (as is plausible). The steady-state fractions of healthy and sick individuals in society, F(H) and F(I), given that all have identical hazard rates as given in (6) and (7), can now also be found, assuming that a fraction α start out as healthy and a fraction $1-\alpha$ as sick. We find:

$$F(H) = \alpha \frac{\mu_H + \mu_D}{\mu_H + \mu_D + \lambda_I} + (1-\alpha)\frac{\mu_H}{\mu_H + \lambda_D + \lambda_I} \tag{8}$$

$$F(I) = \alpha \frac{\lambda_I}{\mu_H + \mu_D + \lambda_I} + (1-\alpha)\frac{\lambda_D + \lambda_I}{\mu_H + \lambda_D + \lambda_I}. \tag{9}$$

These expressions simplify further in the (often plausible) case where all start out as healthy (α=1). With constant population, the (constant) ratio of sick to healthy individuals in society is then $\lambda_I/(\mu_H+\mu_D)$. This result in turn e.g. implies that the ratio of healthy to sick is independent of the death rate of healthy individuals. Note also that the average time an individual remains in the healthy and sick states (the average "spells of health and sickness"), are $1/(\lambda_D+\lambda_I)$ and $1/(\mu_D+\mu_H)$, respectively.

3. Willingness to pay for mortality and morbidity changes

We will now derive measures of the individual's willingness to pay (WTP) for changes in value of statistical life in good health (VSL), and in illness (VSI), respectively. This can be identified as the individual's current willingness to pay to reduce the current rate at which death occurs, in each of these two states.[6] Holding V(H) respectively V(I) constant and differentiating with respect to the respective death rates and consumption levels, we find the following simple expressions:

$$VSL = \frac{dc_1}{d\lambda_D} = \frac{V(H)}{u_1(H)} \tag{10}$$

$$VSI = \frac{dc_2}{d\mu_D} = \frac{V(I)}{u_1(I)}. \tag{11}$$

$u_1(i)$ denotes the marginal utility of income in state i = H, I. We find VSL and VSI as simply the present discounted value of currently being in each of the states, evaluated at the respective marginal utilities of consumption in the two states. We have above

[6] Note that this procedure provides us with an appropriate scaling of effects: the scale is such that when λ_D and μ_D respectively are scaled to unity, the expected remaining lifetime of the individual is one period given that no other transition between states occur; and within a very short time span the probability of more than one transition events tends to zero. This implies that the derivatives in (10) and (11) represent VSL (respectively VSI) as defined e.g. by Rosen (1989).

assumed $V(H) > V(I)$, but $u_1(H)$ may be either greater or smaller than $u_1(I)$ (and always greater when $c_1 = c_2$). Whenever $u_1(H) > u_1(I)$, VSL can be either greater or smaller than VSI. This is a well-known result, but is still perhaps not intuitively obvious as the overall value of a longer lifetime is unambiguously higher for a healthy than for a sick individual. It follows because a higher marginal utility of consumption in the healthy state implies that the individual is less willing to give up current consumption in order to gain a longer expected lifetime. Note also that when μ_H becomes large relative to the other parameters, the ratio of V(I) to V(H) approaches unity. If then $u_1(H) > u_1(I)$, VSI > VSL unambiguously. The individual then experiences relatively short spells of illness on the average. The state of illness will then be escaped relatively rapidly, and future lifetime utility is evaluated to a large extent on the basis of healthy-state current utility. The individual is then willing to pay more at the margin to reduce mortality in the ill state, since his or her marginal utility of consumption is lower in that state.

The appropriate interpretation of the VSL and VSI values is in terms of current willingness to pay to enjoy a small change in the parameters λ_D and μ_D for a possibly short period of time, so as to keep V(H) respectively V(I) constant. Note that the expressions (10)-(11) do not depend on the mortality rates λ_D and μ_D being constant over time. In (1)-(2), λ_D and μ_D can be interpreted as <u>current</u> rates of death which do not have to correspond to rates expected farther into the future; such rates only affect the values of V(H) and V(I).

Consider next an individual's "value of full health" (VFH), as the current willingness to pay to reduce the rate of transition from the healthy to the sick state, λ_S (i.e. to postpone sickness), or alternatively, as the willingness to pay to increase the

current rate of transition from the sick to the healthy state, μ_H (to "get well faster"). Differentiating (1), we find this value in the healthy state as:

$$VFH(H) = \frac{dc_1}{d\lambda_s} = \frac{1}{u_1(H)}[V(H) - V(I)]. \tag{12}$$

The corresponding value in the sick state, from (2) is

$$VFH(I) = -\frac{dc_2}{d\mu_H} = \frac{1}{u_1(I)}[V(H) - V(I)]. \tag{13}$$

Since V(H) – V(I) = [(r+μ_D)u(H) - (r+λ_D)u(I)]/D, (12)-(13) can alternatively be written as follows:

$$VFH(H) = \frac{dc_1}{d\lambda_s} = \frac{1}{u_1(H)}\frac{1}{D}[(r+\mu_D)u(H) - (r+\lambda_D)u(I)] \tag{12a}$$

$$VFH(I) = -\frac{dc_2}{d\mu_H} = \frac{1}{u_1(I)}\frac{1}{D}[(r+\mu_D)u(H) - (r+\lambda_D)u(I)] \tag{13a}$$

(12) and (13) (respectively (12a) and (13a)) differ only in terms of the marginal utility of consumption in the two initial states. This implies that the willingness to pay to avoid sickness is unambiguously higher in the sick state than in the healthy state, provided (as is argued here) that the marginal utility of consumption is higher in the healthy than in the ill state. This is a different and stronger result than that regarding relative VSL values in the healthy and ill states, from (10) and (11), where these

differences were ambiguous. Note from (12a)-(13a) that when $\mu_D > \lambda_D$ (life expectancy is shorter when sick than when in health), the last square brackets of (12a) and (13a) are greater, and there is thus a greater value of avoiding sickness.

Another relevant case is where the individual is faced with a given combined risk of death and illness. A relevant case is when some serious condition, such as a stroke, heart attack, traffic accident or cancer case, which is certain to affect the individual's health condition adversely but from which the individual may or may not die, occurs at a given rate. When the individual does not die, he or she then enters the sick state instead. We are here interested in valuing an increase in the survival rate of such a condition. We view $\lambda = \lambda_D + \lambda_I$ as a constant, but each of λ_D and λ_S as variable such that $d\lambda_D = -d\lambda_I$.

(14) $$\frac{dc_1}{d\lambda_D}(d\lambda_I = -d\lambda_D) = \frac{V(I)}{u_1(H)}.$$

The value of statistical life when the alternative to dying is surviving in sickness is here simply the value of sickness evaluated at the marginal consumption value under the healthy state. This is lower than the current value of sickness, VSI, from (11), when the marginal utility of consumption is higher in the H than in the I state.

All values so far derived can be interpreted as "current benefits resulting from current payments". In a number of cases there may however be delays in effects e.g. of treatments for which individuals need to pay currently. I here have in mind factors where there is an average or expected time of delay, but where there is still a possibility that the effect is immediate. Such effects can here be represented by reformulating the model slightly, to allow for a transition to a new state, where the

only difference from the initial state is that the death rate is higher. We now simplify further and assume that the individual in question enjoys constant health and utility while alive, permitting us to define utility as simply u = u(c). We may still define a "state of poor health" (P), simply as a state in which the death risk is higher than initially. Assume that individuals have an initial rate λ_P at which transition to the new state occurs, and that the model is otherwise identical to that used above (e.g., the transition rates λ_D and μ_H are as before). An intervention that increases or reduces λ_P then represents an increase of decrease in life expectancy, from some future date. We here want to derive the individual's willingness to pay to reduce λ_P marginally. This is found, from a calculation essentially similar to (13), as

$$\frac{dc}{d\lambda_P} = \frac{1}{u_1}[V(H) - V(P)], \tag{15}$$

Alternatively we here have

$$\frac{dc}{d\lambda_P} = \frac{1}{u_1}\frac{1}{D_1}(\mu_D - \lambda_D)u(c), \tag{15a}$$

where D_1 equals D from (5) except that λ_I is replaced by λ_P. V(P) now denotes lifetime utility in the high-risk state (corresponding to V(I) above). We see that (15) is quite similar to the VSL expression in (10), as the two expressions differ only in terms of the value changes involved. This implies that the "delay" of mortality effects in the current case has no fundamental implication except in dropping the value increment (from V(H) in the former case, to V(H)-V(P) in the latter). This may appear

surprising, but occurs because there is in fact no discrete delay in this case: the drop in λ_P occurs instantly, which has an instantaneous effect on (potential) mortality.

4. The value of health investments

So far we have only discussed interventions that affect mortality and morbidity transition rates at the point of time of the actual intervention. Other interventions may instead reduce the mortality rates λ_D and μ_D permanently. In this section we want to derive measures of (once-and-for-all) WTP of an individual, for such interventions, which we may call health investments. Note then that we may write, with sufficiently good approximation,

$$V(H) = u(c_1, H)\Delta t + \frac{1}{D}[(r + \mu_H + \mu_D)u(c_1, H) + \lambda_I u(c_2, I)] \tag{16}$$

$$V(I) = u(c_2, I)\Delta t + \frac{1}{D}[\mu_H u(c_1, H) + (r + \lambda_D + \lambda_I)u(c_2, I)], \tag{17}$$

where Δt is a small time interval (and where, formally, V(H) and V(I) on the left-hand sides are defined from a point of time t while the last expressions on the right-hand sides are defined from period $t + \Delta t$). Define now $dR_i = dc_1(i)\,\Delta t$ as a small change in income affecting current consumption only, when in state i = H,I. We may now consider the individual's WTP for a small permanent change in λ_D starting from the healthy state, and a small permanent change in μ_D starting from the ill state, i.e., a change dR_i that will make the individual indifferent. Differentiating the two expressions we then find

$$(18) \qquad WTP(d\lambda_D, H) = \frac{dR(H)}{d\lambda_D} = \frac{r+\mu_H+\mu_D}{D}\frac{V(H)}{u_1(H)} = \frac{r+\mu_H+\mu_D}{D}VSL$$

$$(19) \qquad WTP(d\mu_D, I) = \frac{dR(I)}{d\mu_D} = \frac{r+\lambda_I+\lambda_D}{D}\frac{V(I)}{u_1(I)} = \frac{r+\lambda_I+\lambda_D}{D}VSI\ ,$$

using (10)-(11). These expressions are of a different dimension than the VSL and VSI expressions, since they can be interpreted as one-time investments to reduce future mortality (and not, as in the cases of VSL and VSI, as current expenses for current mortality reductions). This value may be higher in the H than the I state, but not necessarily so. As argued above VSL is typically higher than VSI. Whenever $\mu_D < \lambda_D$, such that if $\mu_H \geq \lambda_I$, we then have WTP(H) > WTP(I). But we have not put very strong restrictions on μ_H and λ_I. Consider e.g. the case where $\mu_H = 0$ (one never gets well after becoming ill, as in the case of a permanent disability), and $\lambda_D + \lambda_I > \mu_D$, which implies that the overall hazard rate from the healthy state (to either illness or death) is greater than the hazard rate from the ill state (to death only). In this case an investment in a lower mortality rate tends to be more valuable in illness than in health, when only the mortality rate in the initial state is affected in both cases.

These last derivations are based on the somewhat artificial assumption that only the mortality rate in health is affected by the investment when the individual is initially in the healthy state, and vice versa.[7] Most interventions are instead likely to affect the mortality rates in both states. Consider then a different special case where

[7] Note that, in the model, a person may start our in the healthy state, then enter the ill state, and later return to the healthy state. The change in λ_D that is valued, the occurs over all those time intervals that the individual is in the healthy state, but there is no change in mortality in the intermittent ill states. This is possible (e.g. when good health involves certain risky activities, such as parachuting and mountain climbing that are ruled out when ill, and where the rates of fatal accidents involved in such activity may be affected by investment in particular equipment), but it is clearly unrealistic in most practical cases.

the absolute change in the rate of mortality is the same in both states. This is valued in the following way, departing from respectively the healthy and the ill state:

(20)

$$WTP(d\lambda_D = d\mu_D, H) = \frac{(r+\mu_H+\mu_D)V(H)+\lambda_I V(I)}{Du_1(H)} = \frac{r+\mu_H+\mu_D}{D}VSL + \frac{\lambda_I}{D}\frac{u_1(I)}{u_1(H)}VSI$$

(21)

$$WTP(d\mu_D = d\lambda_D, I) = \frac{\mu_H V(H)+(r+\lambda_D+\lambda_I)V(I)}{Du_1(I)} = \frac{r+\lambda_I+\lambda_D}{D}VSI + \frac{\mu_H}{D}\frac{u_1(H)}{u_1(I)}VSL\,.$$

We find the two magnitudes to be quite similar, and typically more similar than the two expressions from (18)-(19). These expressions can also be used to derive some other interesting properties of health investments. Consider e.g. the simplified case where $\lambda_I = \mu_H = 0$, i.e., there is not transition between the states (individuals are born, and die, either ill or in health). Then (20)-(21) simplify to (using shorthand notation for the WTP terms)

(22) $$VTP(H) = \frac{1}{r+\lambda_D}VSL$$

(23) $$VTP(I) = \frac{1}{r+\mu_D}VSI\,.$$

For given VSL and VSI respectively, the values of such investments are here affected negatively by the mortality rates (as well as by the interest rate). This is the "dead-anyway effect", where the intuition is that the investments are less valuable since

there will be fewer future states in which their value will be experienced.[8] It should be stressed that this effect does not appear in the VSL or VSI terms directly, but in the derived investment terms.[9]

5. Effects on VSL and VSI of background risks

For the VSL values derived above one can find effects of changes in "background risks", represented by changes in λ_D, λ_I, μ_D and μ_H. Since the $u_1(i)$ are assumed constant in each state, these parameters simply alter VSL and VSI in the same ways as they alter V(H) and V(I) respectively. We find

$$\frac{dVSL}{d\lambda_D} = -\frac{r+\mu_H+\mu_D}{D} VSL < 0 \tag{22}$$

$$\frac{dVSL}{d\lambda_I} = -\frac{1}{u_1(H)}\frac{r+\mu_H+\mu_D}{D}[V(H)-V(I)] < 0 \tag{23}$$

$$\frac{dVSL}{d\mu_D} = -\frac{1}{u_1(H)}\frac{\lambda_I}{D} V(I) < 0 \tag{24}$$

$$\frac{dVSL}{d\mu_H} = \frac{1}{u_1(H)}\frac{\lambda_I}{D}[V(H)-V(I)] > 0 \ . \tag{25}$$

These results are simple, intuitive and easy to interpret. VSL is always reduced when the rate of mortality (in either of the states) or the rate of entering the state of

[8] Note that the calculation done on this very simplified case is quite analogous to that performed in the more complicated model set up by Johannesson, Johansson and Löfgren (1997), where they compare the value of "blips" in the mortality rate, versus permanent changes in mortality. They however fail to emphasize the close relationship between these calculations, and the "dead-anyway effect", as is done here.

[9] "Comorbidities" can in this framework be represented very simply, as an additional factor affecting the death rate, such that, say in (20), $\lambda_D = \lambda_{D1} + \lambda_{D2}$. The formula for WTP(H) would be the same. This implies that the effect of an exogenous comorbidity on WTP(H) (and WTP(I)) is the same as that of a similar "dead-anyway effect".

morbidity increases, and is unambiguously increased when the rate at which the individual returns to the healthy state, after a spell of illness, increases. The results here differ from other similar analyses of background risk such as Bleichrodt, Crainich and Eeckhoudt (2003) and Eeckhoudt and Hammitt (2001), and the analyses related to "dead-anyway" effects, Pratt and Zeckhauser (1996) and Breyer and Felder (2002), which are typically ambiguous and often opposite of those found here.[10] The main factor responsible for the difference in results is the continuous-time formulation used here, together with our assumption that effects are immediate (i.e. when an individual current payment is assumed to affect his or her current mortality or morbidity rates). By contrast, these other studies, often unrealistically, treat time as discrete, which turns out to be a problem masking the main ideas focused on here.

6. Concluding comments

I have in this note presented and analysed a very simple model where time is viewed as continuous and where individuals are in one out of two mutually exclusive states, healthy and ill, while alive. We use the model to derive values of interventions to change the rates of death and transition between the two states, and the value of statistical life (VSL) and statistical illness (VSI), when effects of the interventions are immediate. This permits extremely simple derivations of effects, and easily interpretable and intuitive results. In particular, the continuous-time formulation permits us to avoid most of the problems created by a number of effects, hereunder the so-called "dead-anyway effect" (Pratt and Zeckhauser (1996)) and problems

[10] To be fair, the "dead-anyway" effect derived by Breyer and Felder (2002) relies on quite different model from the current one, with an optimal life insurance (or fair tontine) approach where consumption levels are endogenous, and where an increase in background risk may raise the individual's consumption possibilities in states where he or she survives. See also Strand (2003) for a related model with interpersonal effects. Introducing such effects in the current model would typically render the effect of an increase in λ_D on VSL ambiguous.

related to comorbidities (Bleichrodt, Crainich and Eeckhoudt (2003)), background risk (Eeckhoudt and Hammitt (2001)), and risk aversion (Eeckhoudt and Hammitt (2004).

The point here is not to argue that the latter types of effects are never significant or important. My objective is rather to provide a clearer analysis of the conditions under which they are not important, and what are their implications. In many cases it is clearly appropriate to assume significant time delays in effects (such as with often long minimum latency periods to develop certain cancers, or when current expense has to be made to implement health investment in the future), in which case the current model framework is far less appropriate. The point is however simply to make researchers aware of when a particular type of model is, or is not, the appropriate one.

Note the many very strong other assumptions used in the analysis above. First, mortality and morbidity rates, as well as consumption levels and the marginal utility of consumption, have all been assumed constant in a given state. We have however also noted above that these assumptions do not appear to lead to very fundamental problems for the principal methods of deriving VSL and VSI in section 3. One implication of relevant alternative assumptions (higher mortality rates, and possibly higher marginal utilities of consumption, for older people) would be for VSL and VSI to drop with age, and perhaps more strongly so than in many competing models. A further analysis of such issues is however left to future research. The assumption of exogenous consumption in each state is also strong and at odds with some approaches in the literature that consider the possibility of efficient insurance or savings.

References

Aldy, J. E. and Viscusi, W. K. (2003), Age variations in workers' value of statistical life. NBER working paper, no. 10199.

Bellman, R. and Kalaba, R. (1965), Dynamic programming and modern control theory. New York: Academic Press.

Bleichrodt, H., Crainich, D. and Eeckhoudt, L. (2003), Comorbidities and the willingness to pay for health improvements. Journal of Public Economics, 87, 2399-2406.

Breyer, F. and Felder, S. (2002), The dead-anyway effect revis(it)ed. CESifo working paper 805.

Breyer, F. and Grabka, M. (2001), Is there a dead-anyway effect in willingness to pay for risk reduction? Discussion paper 252, DIW, Berlin.

Cameron, T. A. and DeShazo, J. R. (2003), Toward a general model of demand for health, morbidity and mortality states. Part I: Theoretical analysis. Working paper, UCLA.

Chang, F.-R. (2002), A theory of health investment under competing mortality risks. CESifo Working Paper, 669.

David, H. A. and Moeschberger, M. L. (1978), The theory of competing risks. New York: Macmillan.

Dow, W., Phillison, T. and Sala-I-Martin, X. (1999), Longevity complementaritiesw under competing risks. American Economoic Review, 99, 1358-1371.

Eeckhoudt, L. and Hammitt, J. K. (2001), Background risk and the value of a statistical life. Journal of Risk and Uncertainty, 23, 261-279.

Eeckhoudt, L. and Hammitt, J. K. (2004), Does risk aversion increase the value of mortality risk? Journal of Environmental Economics and Management, 47, 13-29.

Elandt-Johnson, R. and Johnson, N. (1980), Survival models and data analysis. New York: Wiley.

Johannesson, M., Johansson, P. O. and Löfgren, C. G. (1997), On the value of changes in life expectancy: Blips versus parametric changes. Journal of Risk and Uncertainty, 15, 221-239.

Johansson, P. O. (1996), On the value of changes in life expectancy. Journal of Health Economics, 15, 105-113.

Johansson, P. O. (2002), On the definition and age-dependency of the value of a statistical life. Journal of Risk and Uncertainty, 25, 251-263.

Mortensen, D. T. and Pissardies, C. A. (1999), Recent developments in the theory of search in the labor market. In O. Ashenfelter and D. Card (eds.): Handbook of Labor Economics, 2567-2627, Amsterdam: Elsevier Publishers.

Pissarides, C. A. (2000), Equilibrium unemployment theory. Cambridge, Ma.: MIT Press.

Pratt, J. W and Zeckhauser, R. J. (1996), Willingness to pay and the distribution of risk and wealth. Journal of Political Economy, 104, 747-763.

Shapiro, K. and Stiglitz, J. E. (1984), Equilibrium unemployment as a worker discipline device. American Economic Review, 74, 433-444.

Strand, J. (2003), Individual and household value of mortality reductions with intra-household bargaining. Working paper, Department of Economics, University of Oslo.

Viscusi, W. K. and Aldy, J. E. (2003), The value of a statistical life: A critical review of market estimates throughout the world. Journal of Risk and Uncertainty, 27, 5-76.