LDPC-coded adaptive modulation on slowly varying Nakagami-fading channels

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ABSTRACT

Adaptive coded modulation is a promising idea for bandwidth-efficient transmission on time-varying, narrowband wireless channels. On power limited Additive White Gaussian Noise (AWGN) channels, low-density parity-check (LDPC) codes are a class of error-control codes which have demonstrated impressive error-correcting qualities, under some conditions performing even better than turbo codes. We propose a combination of these two principles: A rate-adaptive LDPC-coded modulation scheme designed for slowly varying flat fading channels. The scheme's theoretical performance is evaluated by calculating its bandwidth efficiency for a given target bit error rate (BER) on channels with Nakagami Multipath Fading. Finally we extend the scheme to a potentially zero-error hybrid FEC-ARQ system, without degrading bandwidth efficiency.

1 INCREASING WIRELESS SPECTRAL EFFICIENCY BY LINK ADAPTATION

Future generations of wireless communication systems will have to include some form of *link adaptation*, i.e., *rate-adaptive transmission schemes*, in order to increase the throughput of information to the levels demanded by high-quality multimedia services. This is because wireless and mobile channels exhibit time-varying channel signal-to-noise ratio (CSNR), usually caused by multipath reflections, shadowing, and the relative movement between transmitter and receiver in a mobile system. Upcoming Wireless Local Area Network (WLAN) standards such as HiperLAN/2 are already beginning to explore link adaptation as a means towards increased spectral efficiency, by means of punctured convolutional codes and QAM modulation [1].

One variant of the link adaptation idea is to use constant power transmission, but to allow for on-line adaptation of channel coding and modulation in such a way that highly bandwidth efficient multilevel coding is used when the CSNR is high, and less bandwidth efficient coding (more redundancy) is used when the CSNR is low. This assumes that the transmitter has access to the *channel state information* (CSI) at any time. For slowly varying frequency-flat channels (those with limited mobility and thus limited Doppler shifts, and with limited time dispersion) the CSNR is the same at all frequencies and can also be regarded as nearly constant for shorter periods of time (less than the *coherence time*). This makes it possible to feed back relevant CSI to the transmitter using a feedback or return channel.

During a time interval of length less than the coherence time the channel can now be well approximated by an AWGN channel. An efficient transmission scheme designed for an AWGN channel with the same stationary signal-to-noise ratio will therefore be a good scheme for a wireless channel, as long as the channel is frequency-nonselective and the scheme does not require processing of blocks longer than the channel coherence time.

When choosing the individual channel codes employed in an adaptive scheme, one should therefore ensure that each code has satisfactory performance on an AWGN channel within some certain CSNR range. One could say that the principle of adaptive coded modulation effectively reduces a time-varying channel to a "bank" of temporally multiplexed AWGN channels, each of which is to be used to represent some finite (small) interval on the CSNR axis. This is beneficial since the search for good codes on AWGN channels is a problem which has been subject to much research and large breakthroughs in the last decade.

2 THE FLAT FADING CHANNEL MODEL

The *flat-fading* channel is a much-used model for narrowband transmission over wireless and mobile communication channels, and the one to be used here. On a

flat-fading channel, all signal frequencies are attenuated by the same factor. In a broadband wireless system the channel is frequency-selective, but the flat-fading model may still be used to model *one subchannel* if multicarrier (OFDM) modulation is used, such as is the case e.g., for HiperLAN/2.

The received signal y_k on a flat-fading channel can be written at a given time instant k in the complex baseband domain as

$$y_k = a_k x_k + z_k \tag{1}$$

where a_k is the *fading amplitude*, x_k is the transmitted channel symbol (in general complex), and z_k is complex AWGN. The fading amplitude is also in general complex, but may be assumed to be real-valued if *perfect coherent detection* is assumed at the receiver end. We shall adopt this commonly used assumption throughout this paper.

The fading amplitude a (for notational simplicity we suppress the time dependence from now on) can be described as a stochastic variable, whose distribution we shall assume to be known. One particularly interesting distribution is the *Nakagami* distribution [2]:

$$p_A(a) = \frac{2}{\Gamma(m)} \left(\frac{m}{2\sigma^2}\right)^m a^{2m-1} \mathrm{e}^{-ma^2/2\sigma^2} \tag{2}$$

Here, $m > \frac{1}{2}$ is the Nakagami parameter, σ^2 is the fading amplitude variance, and $\Gamma(x)$ is the gamma function defined by

$$\Gamma(x) = \int_0^\infty y^{x-1} \mathrm{e}^{-y}\, \mathrm{d}y, \qquad x > 0. \tag{3}$$

The Nakagami distribution is of such interest because it allows for modelling of a wide range of fading environments by varying m, while yielding closed-form expressions for many interesting features of the channel, such as channel capacity. For $m = 1$ the model corresponds to the *Rayleigh fading* model which is common in mobile communications when there is no line-of-sight (LOS) path between transmitter and receiver. Increasing m will model less severe fading environments, such as Rician fading, which is useful in LOS environments. The Rician K factor will then be related to the Nakagami parameter by [3]

$$m = \frac{(1+K)^2}{1+2K}. \tag{4}$$

A Rayleigh fading channel with ideal Maximum Ratio Combining (MRC) of m receive antennas will also behave like a Nakagami-m channel. As m is increased towards infinity, the Nakagami fading channel converges towards an AWGN channel.

If a is Nakagami distributed, the CSNR will be Gamma distributed as shown in Equation (5), γ and $\overline{\gamma}$ being the instantaneous and expected CSNR respectively [4].

$$p_\gamma(\gamma) = \left(\frac{m}{\overline{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} \exp\left(-m\frac{\gamma}{\overline{\gamma}}\right), \qquad \gamma \geq 0 \tag{5}$$

Under the assumption of perfect knowledge of the channel state information (CSI) both on the receiver and transmitter side, the *maximum average spectral efficiency* (MASE), or channel capacity, of a Nakagami multipath fading (NMF) channel has been shown to be given by [5, Eq. 23]

$$\mathrm{MASE} = \log_2(\mathrm{e})\mathrm{e}^{m/\overline{\gamma}} \sum_{k=0}^{m-1} \left(\frac{m}{\overline{\gamma}}\right)^k \Gamma\left(-k, \frac{m}{\overline{\gamma}}\right) \tag{6}$$

where $\Gamma(x, y)$ is the socalled *complementary incomplete Gamma function* [4], which is commonly available in numerical software such as MATLAB or Maple. The MASE provides an ultimate performance benchmark for the spectral efficiency of any practical communication system communicating reliably over this channel. It has also been shown that in order to efficiently exploit the channel's capacity over time without excessive delay, a transmitter should switch between codes of different spectral efficiencies—in other words adapt itself to the changes in channel quality as done by adaptive coded modulation [6].

3 ACHIEVING CAPACITY ON AWGN CHANNELS: LDPC CODES

We have stated that good performance on an AWGN channel is a suitable performance criterion when considering channel codes for inclusion in an adaptive coded modulation scheme. In [7, 8] it is shown that a subclass of seemingly simple binary block codes, socalled *Low-Density Parity Check* (LDPC) codes, almost reaches the Shannon channel capacity—i.e. the ultimate performance limit—for AWGN channels with BPSK modulation. The reason for this record-breaking performance of LDPC codes can at least partly be explained by the construction of the codes, to be explained below. According to the proof of Shannon's Channel Coding Theorem [9] the channel capacity can be reached by using a code consisting of very (in principle infinitely) long codewords picked at random from some suitable distribution, in combination with a decoder performing an exhaustive nearest neighbour search.

A nearest neighbour search without any constraints imposed is an NP-complete problem. However, the LDPC codes, invented by Gallager [10], circumvent some of the complexity difficulties. As implied by their name, limiting the space of possible codes to LDPC codes provides an additional property in the controlled sparseness (i.e., very low density of non-zero elements in each column) of their parity check matrices $\mathbf{H}$. This reduces the number of equations to be solved by the decoder and makes practical decoding possible by means of iterative methods similar to those used in turbo decoders.

3.1 Basic Principles of LDPC Codes

To be more precise, an LDPC code is by definition a linear error-correcting block code, which is specified by a very sparse parity check matrix $\mathbf{H}$. A *regular* LDPC code has an $(N-K) \times N$ $\mathbf{H}$ matrix with a constant low number t of 1s in each column, placed at random. An *irregular* LDPC code has a non-uniform distribution of 1s in the rows and columns.

In any case, the parity check matrix may be generated simply by running a binary random generator (in the reg-

ular case, with probability t/N for emitting 1, and probability $1 - t/N$ for emitting 0). This is in effect very similar to the code construction used by Shannon in his proof of the Channel Coding Theorem, but with the constraint of matrix sparseness added in order to be able to find viable decoding algorithms. If the matrix found does not provide a good code, the random generator is simply run once more. Theoretically, however, the probability of finding a good code by such a random construction is very close to 1 if the code length N is large [11].

If the parity check matrix has linearly independent rows, the rate R of the code is K/N information bits per coded bit. In order to reduce the probability of low weight codewords (i.e., provide better distance properties in the code), it is common—for regular LDPC codes—to impose the constraint $t \geq 3$, and to not allow any two columns in $\mathbf{H}$ to have an overlap of non-zero bits by more than 1.

By using Gauss-Jordan elimination on the parity check matrix we may obtain the $N \times K$ *generator matrix* $\mathbf{G}$, satisfying $\mathbf{HG} = 0$. A codeword $\mathbf{t}$ is, as usual for a block code, generated as $\mathbf{t} = \mathbf{Gs}$ where $\mathbf{s}$ is a K-dimensional information bit vector and all algebraic operations are performed modulo 2 ($\mathbf{t}$ and $\mathbf{s}$ are defined as column vectors).

4 RATE-ADAPTIVE LDPC-CODED MODULATION SYSTEM

The impressive error-correcting performance of LDPC codes on power-limited AWGN channels, in combination with the potential of increased bandwidth efficiency in wireless applications demonstrated by adaptive coded modulation, makes us consider the following question: Are LDPC codes also good codes to use in combination with higher-order modulations, and subsequently in a rate-adaptive coded modulation system designed to exhibit good performance on bandwidth-limited time-varying channels? This is the topic of the remainder of this paper.

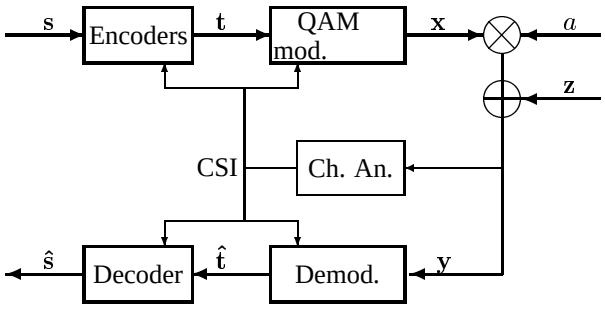


Figure 1: Adaptive LDPC-coded transmission system. The channel analyzer (Ch. An.) finds the CSI and submits this information to the transmitter and receiver through a noiseless zero-delay return channel.

Figure 1 shows the communication system under analysis. Taking the design principles for adaptive moded modulation presented in [4] as a starting point, we have replaced the multidimensional trellis codes of that paper with I LDPC codes ("Encoders" in the figure), followed by Gray code mappings to I M-QAM constellations of varying sizes ("QAM mod." in the figure) [12]. For each M-QAM constellation we construct an LDPC code such that the lengths of all codes equal a constant number l of M-QAM symbols (the *channel vector length*) in the corresponding constellation. The encoder and decoder hence have a given set of I LDPC generator matrices $\mathbf{G}$ and parity check matrices $\mathbf{H}$, respectively. When code n is used, a source vector $\mathbf{s}$ consisting of $K = nl$ information bits is encoded to a binary codeword $\mathbf{t}$ of size $N = (n + k)l$. Here $N - K = kl$ is the number of parity check bits in $\mathbf{t}$. The number of parity check bits to be transmitted in *each channel symbol* $\mathbf{x}[j]$ $j = 1, \ldots, l$ is then equal to k, whereas the number of *information* bits in each channel symbol (which equals the code's *rate*, or spectral efficiency) is n. This is accomplished by letting the QAM modulator map $\mathbf{t}$ to a complex channel vector $\mathbf{x}$ of length l, by taking l distinct sets of $n+k$ bits from $\mathbf{t}$ and mapping each such set into a complex channel symbol.

The subsequent communication of the M-QAM symbol vectors is assumed done by means of a slowly varying Nakagami-m fading channel, which we assume to be well approximated by an AWGN channel for a period of time at least as long as the transmission time of one channel vector $\mathbf{x}$ of length l. Such a vector will then be subjected to an (almost) constant attenuation factor, and the added channel noise $\mathbf{z}$, assumed to be a complex AWGN vector with variance N_0.

The demodulator in the system uses maximum likelihood soft decision detection on $\mathbf{y}$ [12], while the subsequent decoder uses an iterative belief propagation decoding algorithm, explained in [13, 11], to make an estimate $\hat{\mathbf{s}}$ of $\mathbf{s}$.

5 SYSTEM OPTIMIZATION AND PERFORMANCE ANALYSIS

5.1 Forward Error Correction

Having briefly described the component codes, the method of mapping from code bits to channel symbols, and the method of demodulation and decoding, we now present the parameter optimization and performance analysis of the overall rate-adaptive system. By finding each code's bit-error rate as a function of CSNR it is possible to tell for which CSNR range each code is acceptable, i.e., for which CSNRs it will perform better than a specified target bit error rate BER_0. This is done by simulating each relevant combination of LDPC code and M-QAM modulation on an AWGN channel [12]. The lower threshold CSNR for code n providing acceptable BER performance on an AWGN channel is denoted γ_n. This value is found by solving the equation $\text{BER}(\gamma_n) = \text{BER}_0$ for each code, $\text{BER}(\gamma)$ being the bit error rate for code n as a function of the CSNR γ. By always selecting the acceptable code producing the highest spectral efficiency for a given CSNR value, the spectral efficiency of the overall system will be maximized, while still maintaining the BER requirements.

For the particular code construction we use, each modulation symbol will carry n information bits and k parity

bits, where n is the code index. The *average spectral efficiency* (ASE) of the scheme is the expected value of the code rate, averaged over all component codes, and therefore a function of the expected CSNR $\overline{\gamma}$:

$$\text{ASE} = \sum_{n=1}^{I} n \cdot P_n \quad [\text{bits/s/Hz}] \tag{7}$$

where P_n is the probability for γ being in the interval $[\gamma_n, \gamma_{n+1}]$. For a Nakagami-m channel P_n is given by

$$P_n = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma)\,\mathrm{d}\gamma = \frac{\Gamma(m, \frac{m\gamma_n}{\overline{\gamma}}) - \Gamma(m, \frac{m\gamma_{n+1}}{\overline{\gamma}})}{(m-1)!} \tag{8}$$

with $p_\gamma(\gamma)$ being defined by Equation (5).

5.2 Adaptive Coded Modulation with Hybrid FEC/ARQ Protocol

Several papers report that LDPC codes under certain conditions have never made undetected errors [11, 14, 15]. This property is probably related to the *minimum distance* properties of LDPC codes. However, finding this minimum distance is a problem which is known to be NP complete [16]. Here, we simply assume—based on reported empirical results—that our LDPC codes have very good minimum distance properties, and that the decoder therefore always is able to detect whether a block is correctly decoded or not.

Under this assumption we have calculated the ASE using a suggested hybrid *Automatic Repeat reQuest* (ARQ) protocol, potentially corresponding to zero-error output. Codeword retransmission in this system is performed each time the LDPC decoder reports an undetected error, otherwise the system relies on FEC as before. A hybrid FEC-ARQ system will thus be very similar to the pure FEC system described earlier, with the exception of new threshold CSNRs $\{\gamma_n\}$ and the inherent use of retransmission.

Note that all necessary retransmissions of a block of information bits are assumed done by means of the code used in the first transmission of the block. An underlying assumption in this section is therefore that the CSNR does not move from one CSNR interval to another during the time used for all these retransmissions. I.e., the assumption of slowly varying fading is more restrictive here than in the pure FEC case. This restriction can in principle be relaxed by allowing the code to vary with each retransmission, but the analysis would then become significantly more involved.

A tight lower bound on the spectral efficiency (SE) for code n at a CSNR γ in the hybrid FEC-ARQ system can be shown to be [17]

$$\text{SE}(\gamma, n) = [1 - \text{BLER}_n(\gamma)] \cdot n \tag{9}$$

where $\text{BLER}_n(\gamma)$ is the probability of a block of l channel symbols being incorrectly decoded (and detected as such) when using code n at CSNR γ. Now, for all code pairs $(n-1, n)$ there must exist a CSNR value giving the same spectral efficiency SE for both codes. By solving the equation

$$\text{SE}(\gamma_n, n-1) = \text{SE}(\gamma_n, n)$$

we can thus find the ARQ-optimized CSNR thresholds, assuming that $\text{BLER}_{n-1}(\gamma) = 0$ for CSNR values $\geq \gamma_n$:

$$\begin{aligned}
\text{SE}(\gamma_n, n-1) &= \text{SE}(\gamma_n, n) \\
&\Updownarrow \\
(1 - \text{BLER}_{n-1}(\gamma_n))(n-1) &= (1 - \text{BLER}_n(\gamma_n)) \cdot n \\
&\Updownarrow \\
n - 1 &\approx (1 - \text{BLER}_n(\gamma_n)) \cdot n \\
&\Updownarrow \\
\text{BLER}_n(\gamma_n) &\approx \frac{1}{n}
\end{aligned} \tag{10}$$

from which γ_n may be derived. The ASE when using the hybrid FEC-ARQ scheme is now given by

$$\text{ASE}_h = \sum_{n=1}^{I} n \int_{\gamma_n}^{\gamma_{n+1}} (1 - \text{BLER}_n(\gamma)) \cdot p_\gamma(\gamma)\ \mathrm{d}\gamma. \tag{11}$$

6 NUMERICAL RESULTS

6.1 System Specification

We have evaluated an example system designed according to the guidelines we have described, on a range of Nakagami fading channels ($m = 1, 2, 4$). A code set has been designed, with code lengths $N = (n+k)l$ ranging from 400 to 1800 bits, and with constant l=200 [17]. We have used $I = 8$, i.e., 8 M-QAM constellations with 8 corresponding LDPC codes. The constellations range from 4-PSK to 512-QAM, offering an uncoded throughput from 2 to 9 bits/symbol. However, the corresponding binary code rates used are $n/(n+1)$, $n \in \{1 \ldots, 8\}$, producing effective information rates between 1 and 8 bits/symbol. For the LDPC codes with rates 1/2, 2/3, 3/4, and 4/5 we have constructed parity check matrices with column weight $t = 3$, while the rest of the codes have column weight $t = 4$. This is done to ensure that all codes have sufficient minimum distance [14].

We have used the target bit-error rate $\text{BER}_0 = 10^{-3}$ in the FEC system. The same code set was used in both the FEC and the hybrid FEC-ARQ system.

6.2 Performance Results

The bit error rates (BER) and block error rates (BLER) for the 200-symbol codes are presented in Figures 2 and 3. Based on these graphs, we have found $\gamma_1, \ldots, \gamma_8$, and calculated the spectral efficiencies for the two systems (FEC and FEC-ARQ hybrid) when used on AWGN channels, that is, with constant CSNR. The AWGN spectral efficiencies are shown in Figure 4. When combining the AWGN spectral efficiency results with the probability density function for a Nakagami-m fading channel (using Equations (7) and (11) with $m = 1, 2, 4$) we get the results as shown in Figures 5 and 6. The differences between the channels' MASE and the spectral efficiencies of our two proposed systems are shown in Figures 7

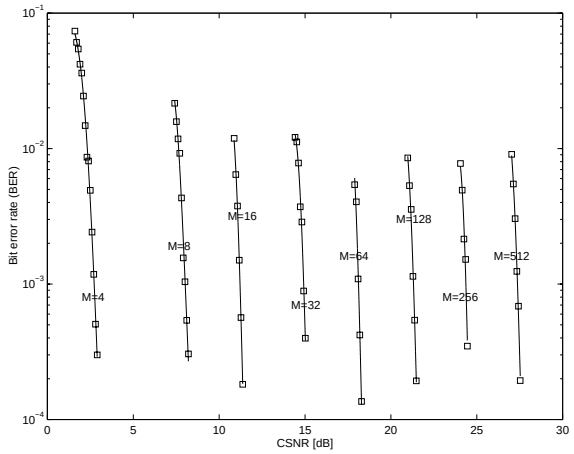


Figure 2: Bit-error-rate as a function of CSNR on AWGN channel.

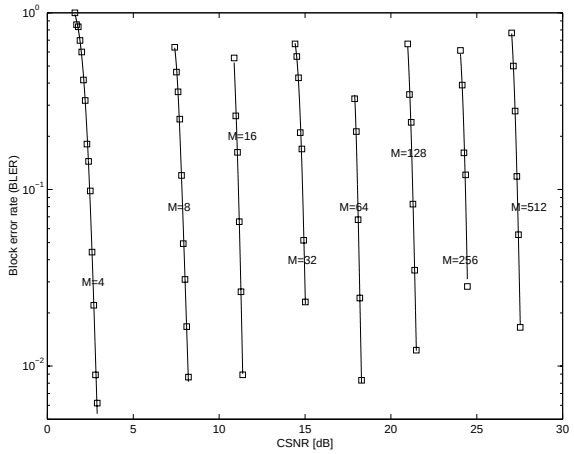


Figure 3: Block error rate as a function of CSNR on AWGN channel.

and 8. It can be seen that the ASE of both systems lie less than 2 bits/s/Hz below the channels' theoretical MASE over a wide range of CSNR values. The ASE increases with increasing Nakagami parameter, reflecting the fact that increased m can interpreted as less severe fading.

It is worth noting that for small CSNR values the hybrid FEC-ARQ system has slightly *higher* spectral efficiency than the FEC system; otherwise their performances are very similar regarding spectral efficiency. This rather counterintuitive behaviour can be attributed to the fact that the ARQ system has higher spectral efficiency in the transition regions when the CSNR values are small, as witnessed by Figure 4. Stated simply, the system can transmit at a higher average rate when we allow for retransmission, and the rate reduction which comes as a result of retransmissions does not destroy all of this spectral efficiency gain. The mechanisms behind this property are not yet fully explored.

7 CONCLUSIONS

Our results confirm that adaptive block-coded modulation has the potential of offering significantly better bandwidth efficiency than what is the case in today's wireless systems, as long as the channel variations are not too rapid. This conclusion holds over a wide range of CSNRs

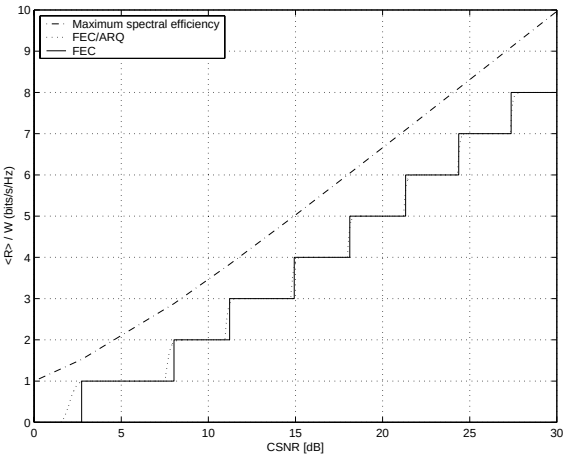


Figure 4: Spectral efficiencies of the constructed component codes when FEC and hybrid FEC/ARQ protocols are used on an AWGN channel.

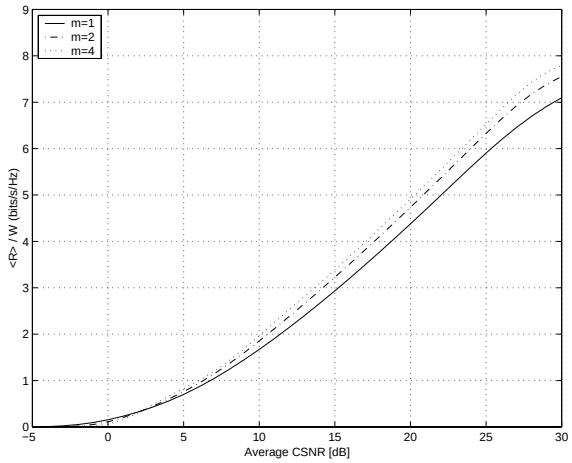


Figure 5: Average spectral efficiency of proposed FEC system as a function of average CSNR on Nakagami-m fading channels; $m = 1, 2, 4$.

and fading environments. Based on our results we also believe that the particular choice of LDPC codes in combination with multilevel QAM modulation constellations is a very promising code choice for such a system.

In addition, the observation that a zero-error system (hybrid FEC-ARQ) slightly outperforms a non-zero-error (FEC) system—regarding spectral efficiency—suggests that using LDPC codes in conjunction with ARQ may reduce the need for code concatenation (e.g., outer Reed-Solomon codes) to reduce the target BER further. More work is needed on the performance degradations experienced when the idealized assumptions of a perfect, zero-delay return channel, and perfect channel estimation are relaxed.

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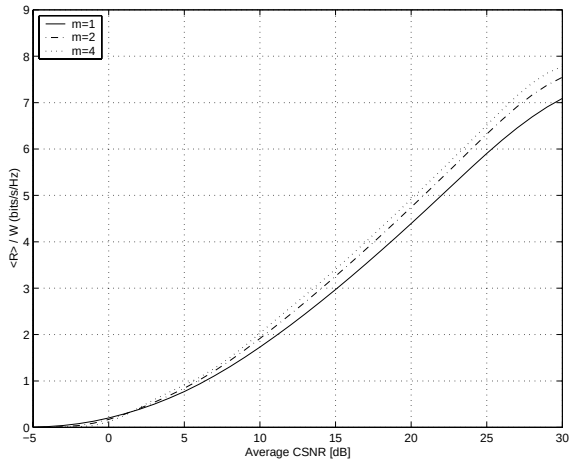


Figure 6: Average spectral efficiency of proposed hybrid FEC/ARQ system as a function of average CSNR on Nakagami-m fading channels; $m = 1, 2, 4$.

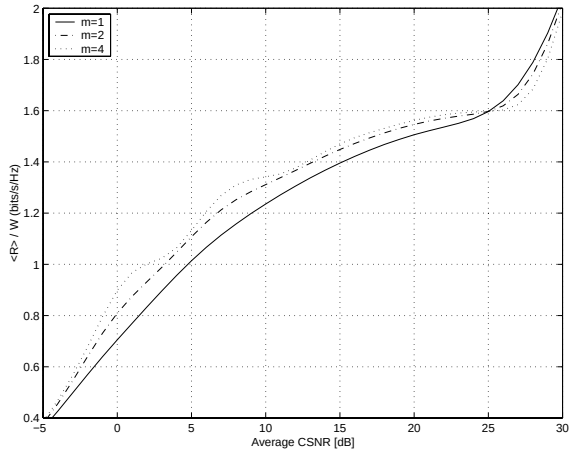


Figure 7: Difference between MASE and average spectral efficiency of proposed FEC system as a function of average CSNR on Nakagami-m fading channels; $m = 1, 2, 4$.

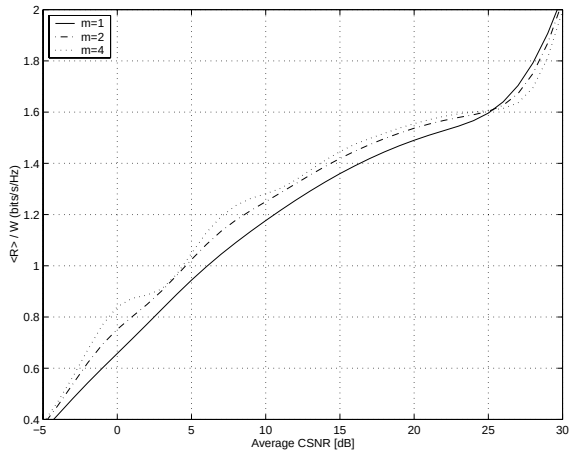


Figure 8: Difference between MASE and average spectral efficiency of proposed FEC/ARQ hybrid system as a function of average CSNR on Nakagami-m fading channels; $m = 1, 2, 4$.

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