Construction and Capacity Analysis of High-Rank Line-of-Sight MIMO Channels

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***Abstract*— This paper describes a technique for realizing a high rank channel matrix in a *line-of-sight* (LOS) *multiple-input-multiple-output* (MIMO) transmission scenario. This is beneficial for systems which can not make use of the originally derived MIMO gain given by independent and identically distributed (i.i.d.) flat Rayleigh fading channels. Typical applications are fixed wireless access and radio relay systems. The technique is based on optimization of antenna placement in a uniform linear array. By introducing a new and more general geometrical model than that applied in earlier works, additional insight into the optimal design parameters is gained. A novel analysis of the sensitivity of the optimal design parameters is performed. The LOS transmission matrix is used in a Ricean fading channel model, and performance is evaluated with respect to ergodic capacity, outage capacity, and effective degrees of freedom. The results show that even with some deviation from optimal design, the LOS MIMO case outperforms the i.i.d. Rayleigh case in terms of Shannon capacity.**

***Keywords*—MIMO, Rice channel, line-of-sight, uniform linear array, Shannon capacity.**

I. Introduction

Fixed wireless access (FWA) is a flexible and cost-efficient way to bring network applications such as Internet and telephony to offices and homes. This technology requires no expensive and time consuming digging of cables, and is easily scalable with regard to the applications' demand. Some predictions on the role of FWA in the future communication environment are given in [1] and [2], while a presentation of emerging technologies for upcoming FWA systems can be found in [3].

Multiple-input-multiple-output (MIMO) systems have evolved into one of the most promising enabling technologies for resolving the problem of continued demand for increased bandwidth in wireless communications. The wireless frequency spectrum is a very limited resource that must be used efficiently. MIMO systems facilitate efficient frequency utilization by, among other things, smart signal processing at the transmitter and receiver. An excellent tutorial report on progress in the area of MIMO systems is presented in [4].

Most research efforts in the field of MIMO communications utilize the fading given by the multipath environment. The optimal performance was originally derived for channels that exhibit independent and identically distributed (i.i.d.) flat Rayleigh fading [5]. In this paper we present a transmission scheme that results in high MIMO gain also for channels which are not subject to severe multipath. Some work on this topic has already been done, both for indoor scenarios [6], [7] and outdoor scenarios [8], [9].

In this paper we utilize a new geometrical model for the *line-of-sight* (LOS) component to find the optimal antenna separation with regard to capacity for *uniform linear arrays* (ULAs). In contrast to the models used earlier ([6], [8]), our model does not require the sender and receiver arrays to be parallel. This gives additional insight into the optimal design parameters. A ray tracing technique is used to find the optimal placing of antenna elements in the ULA. In accordance with [9], we will show that our scheme actually performs better for a pure LOS channel than for an i.i.d. Rayleigh channel. The solution will apply nicely to FWA and radio relay systems, as the optimal antenna spacing turns out to be a function of the transmitter and receiver separation (fixed), and since these systems often use very directive high gain antennas (i.e., exhibit limited multipath). A novel analysis of the sensitivity of the optimal design parameters is performed by introducing a deviation factor. The scheme will be analyzed with respect to ergodic capacity, outage capacity, and *effective degrees of freedom* (EDOF) for different Ricean channels.

The rest of the paper is organized as follows. Section II discusses the general Shannon capacity formula for MIMO systems. In section III, MIMO channel models for the LOS and *non-LOS* (NLOS) cases are presented. Performance evaluation measures derived from the capacity formula in section II are presented in section IV. The simulation results of the presented scheme are given in section V, while conclusions are drawn in section VI.

II. Capacity of MIMO systems

A MIMO transmission system employs several transmit and receive antennas when transmitting data over a channel. The number of transmit antennas is denoted N, while the number of receive antennas is denoted M. Assuming slowly varying and frequency flat fading channels, it is common to model the MIMO transmission in complex baseband as [4]

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{r}$ is the $M \times 1$ received complex-valued signal vector, $\mathbf{s}$ is the $N \times 1$ transmitted complex-valued signal vector, $\mathbf{H}$ is the

$M \times N$ complex-valued channel matrix, and $\mathbf{n}$ is the $M \times 1$ complex-valued additive white Gaussian noise (AWGN) vector. The additive noise vector contains i.i.d. circularly symmetric complex Gaussian elements with zero mean and variance σ_n^2, denoted $CN(0, \sigma_n^2)$[1].

We denote the covariance matrix of the transmitted signal by $\mathbf{Q} = E[\mathbf{s}\mathbf{s}^H]$. In practical systems we usually need to fulfill an average transmit power constraint over the array. If the total average transmit power is limited to P_T, then trace$(\mathbf{Q}) \leq P_T$ must hold. In the remainder of this paper we will look at uncorrelated branch sources with equal power, i.e. $\mathbf{Q} = (P_T/N)\mathbf{I}_N$. This is optimal with regard to capacity when $\mathbf{H}$ is unknown at the transmitter [4], and when such sources are used, the channel capacity of a MIMO system described by (1) becomes [5]

$$C = \log_2 \left[\det \left(\mathbf{I}_M + \frac{P_T}{N\sigma_n^2} \mathbf{H}\mathbf{H}^H \right) \right] \quad \text{bit/s/Hz}, \qquad (2)$$

where $\mathbf{H}^H$ is the Hermitian transpose of the $\mathbf{H}$ matrix.

In the following we shall also assume that all receiver antennas experience the same average received power. This average received power, P_R, is a function of P_T, the path loss model, the distance, and the shadowing. The average received *signal-to-noise ratio* (SNR) at one receive antenna then becomes $\bar{\gamma} = P_R/\sigma_n^2$. In the remainder of this paper $\mathbf{H}$ is taken to be the *normalized* channel matrix, which implies that each element in $\mathbf{H}$ has unit average power. By requiring this normalization we make the average SNR independent of $\mathbf{H}$. For convenience we define the variables $U = \min(M, N)$ and $V = \max(M, N)$ for use in the rest of the paper. By doing an eigenvalue decomposition and using the expression above for average received SNR, (2) can be written as [5]

$$C = \sum_{i=1}^{U} \log_2 \left(1 + \frac{\bar{\gamma}}{N} \lambda_i \right) \quad \text{bit/s/Hz}. \qquad (3)$$

Here, λ_i is the ith eigenvalue of $\mathbf{W}$, which is defined as

$$\mathbf{W} = \begin{cases} \mathbf{H}\mathbf{H}^H, & M \leq N \\ \mathbf{H}^H\mathbf{H}, & M > N. \end{cases} \qquad (4)$$

Equation (3) shows that a MIMO system can be viewed as consisting of U parallel *single-input-single-output* (SISO) channels, where each channel has gain λ_i, and an average SNR downscaled with the number of transmitters compared to a SISO system with the same total transmit power.

III. MIMO CHANNEL MODEL

It is common practice to model the channel matrix as a sum of two components, a LOS component and an NLOS component. The ratio between the power of the two components gives the Ricean K factor [10, p. 52]. As discussed in section II we want $\mathbf{H}$ to be normalized, and we can express the normalized channel matrix in terms of K as

$$\mathbf{H} = \sqrt{\frac{K}{1+K}} \cdot \mathbf{H}_{\text{LOS}} + \sqrt{\frac{1}{1+K}} \cdot \mathbf{H}_{\text{NLOS}}. \qquad (5)$$

The entries in the two component matrices are discussed in more detail in the following two sections.

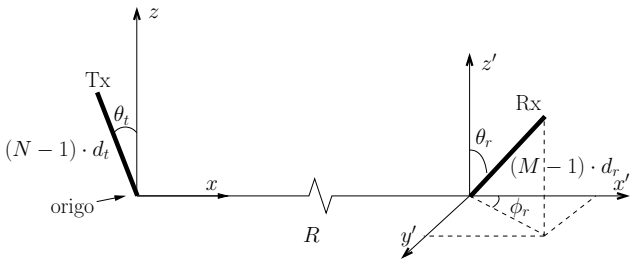


Fig. 1. A general MIMO system with ULAs at both the receiver and the transmitter. The transmitter array is in the xz-plane with origo at the lower end.

A. LOS Channel: Ray Tracing

Here we concentrate on the pure LOS channel. In [6]–[9] it was shown that by placing the antennas in a MIMO system in a smart way, the pure LOS channel matrix actually becomes high rank[2], which corresponds to many nonzero eigenvalues λ_i and thus high MIMO capacity. In this paper we look at a new and more general geometry, using two ULAs with arbitrary orientation (not restricted to be parallel).

Fig. 1 illustrates the system to be analyzed. Due to the ULA assumption, the inter-antenna distances d_t and d_r, at the transmitter and the receiver respectively, are constant. Viewed as system parameters, d_t and d_r can take different values. In the figure, the x-axis is taken to be in the direction from the lower end of the transmit array, which is origo, to the lower end of the receive array. The transmit array is placed in the xz-plane. In Fig. 1 R represents the distance between the sender and receiver, $(N-1)\cdot d_t$ and $(M-1)\cdot d_r$, denote the total array lengths, and the angles θ_t, θ_r, and ϕ_r, are the angles of the local spherical coordinate system at the transmitter and receiver.

Fig. 1 can thus describe any MIMO system employing ULAs with arbitrary orientation. We will now show how we can make $\mathbf{H}_{\text{LOS}}$ a high rank channel matrix by choosing appropriate values for d_t and d_r. The technique used is based on ray tracing.

First we need to define the vector $\mathbf{a}_n^t$ from origo to transmit antenna n, and the vector $\mathbf{a}_m^r$ from origo to receive antenna m (the elements are ordered from element zero at the lower end of the arrays). From Fig. 1 we see that these vectors are given

[1] A circularly symmetric complex Gaussian random variable, $CN(0,\sigma^2)$, is taken to mean a random complex variable, $x + jy$, where the x and y components are independent and normally distributed with $N\left(0, \frac{\sigma^2}{2}\right)$.

[2] The rank of a matrix is equal to the number of linearly independent rows (or columns) of the matrix.

as

$$\mathbf{a}_n^t = (nd_t \sin\theta_t)\mathbf{n}_x + (nd_t \cos\theta_t)\mathbf{n}_z \quad n \in \{0, 1, \ldots, N-1\},$$
$$\mathbf{a}_m^r = (R + md_r \sin\theta_r \cos\phi_r)\mathbf{n}_x + (md_r \sin\theta_r \sin\phi_r)\mathbf{n}_y + (md_r \cos\theta_r)\mathbf{n}_z \quad m \in \{0, 1, \ldots, M-1\},$$

where $\mathbf{n}_x$, $\mathbf{n}_y$, and $\mathbf{n}_z$ denote the unit vectors in their respective directions. By calculating the Euclidean norm of the vector difference between $\mathbf{a}_n^t$ and $\mathbf{a}_m^r$ we can find the *path length,* $r_{m,n}$, between transmit antenna n and receive antenna m:

$$\begin{aligned} r_{m,n} &= \left\|\mathbf{a}_m^r - \mathbf{a}_n^t\right\| \\ &= \big[(R + md_r \sin\theta_r \cos\phi_r - nd_t \sin\theta_t)^2 \\ &\quad + (md_r \sin\theta_r \sin\phi_r)^2 \\ &\quad + (md_r \cos\theta_r - nd_t \cos\theta_t)^2\big]^{1/2} \qquad (6) \\ &\approx R + md_r \sin\theta_r \cos\phi_r - nd_t \sin\theta_t + \\ &\quad \frac{(md_r \sin\theta_r \sin\phi_r)^2 + (md_r \cos\theta_r - nd_t \cos\theta_t)^2}{2R} \qquad (7) \end{aligned}$$

In the last step above some simplifications were made. First we made a Taylor series expansion to the first order of the square root expression, $(1+\Delta)^{1/2} \approx 1 + \frac{\Delta}{2}$. This is a good approximation when $\Delta \ll 1$, which in the present case means that the distance between transmit antenna n and receive antenna m in the x-direction is much larger than the distance between these two antennas in the y- and z-direction. The second simplification was performed in the denominator of the fraction in (7). Here we removed the subtraction of the distance between transmit antenna n and origo in the x-direction, and the addition of the distance between receive antenna m and the lower end of receiver array in the x-direction. This is valid when R is much larger than the array lengths in the x-direction. The two simplifications are thus valid simultaneously if the distance between the lower end of the arrays is much larger than the array dimensions at both the transmitter and receiver. This is a reasonable assumption for our targeted applications.

The symbols transmitted from antenna n will be received at M different receive antennas. Antenna m will experience a path length $r_{m,n}$ as given by (7). This corresponds to an inter-antenna phase difference, and the received vector from transmit antenna n on the M receive antennas can be written as

$$\mathbf{h}_n = \left[\exp\left(\frac{j2\pi}{\lambda} r_{0,n}\right), \ldots, \exp\left(\frac{j2\pi}{\lambda} r_{M-1,n}\right)\right]^T,$$

where λ is the wavelength, each element is normalized as discussed earlier, and $(\cdot)^T$ denotes the vector transpose. The channel matrix is thus given as

$$\mathbf{H}_{\text{LOS}} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{N-1}].$$

We will now derive the optimal antenna separation with regard to Shannon capacity for a pure LOS channel. From linear algebra theory we know that trace $(\mathbf{W}) = \sum_{i=1}^{U} \lambda_i$. The trace of $\mathbf{W}$ for a pure LOS channel matrix can easily be shown to be $N \cdot M$ $(= V \cdot U)$. By maximizing (3) under this constraint, using the method of *Lagrange multipliers*, the optimal solution, is seen to occur when all eigenvalues are equal (i.e., $\lambda_1 = \lambda_2 = \cdots = \lambda_U = V$). One realization of $\mathbf{W}$ that fulfills the requirement of equal eigenvalues is

$$\mathbf{W} = V \cdot \mathbf{I}_{U \times U}, \qquad (8)$$

where $\mathbf{I}_{U\times U}$ is the $U \times U$ identity matrix. Equation (8) is fulfilled when $\mathbf{H}_{\text{LOS}}$ has orthogonal columns for M larger than N, or orthogonal rows for N larger than M.

The situation where M is larger than N will be studied first, and generalizations will be made based on the result. Orthogonality between the different columns in $\mathbf{H}_{\text{LOS}}$ is obtained if the inner product of two received vectors from two different transmit antennas is equal to zero:

$$\begin{aligned} \langle \mathbf{h}_k, \mathbf{h}_l \rangle &= \sum_{m=0}^{M-1} \exp\left(j\frac{2\pi}{\lambda}(r_{m,k} - r_{m,l})\right) \\ &= \sum_{m=0}^{M-1} \exp\left(j2\pi \frac{d_t d_r \cos\theta_r \cos\theta_t}{\lambda R}(l-k)\, m\right) \qquad (9) \\ &= \frac{\sin\left(\pi \frac{d_t d_r}{\lambda R} \cos\theta_r \cos\theta_t (l-k) M\right)}{\sin\left(\pi \frac{d_t d_r}{\lambda R} \cos\theta_r \cos\theta_t (l-k)\right)} = 0 \qquad (10) \\ &\Rightarrow d_t d_r = \frac{\lambda R}{M \cos\theta_r \cos\theta_t} \qquad (11) \end{aligned}$$

When going from (9) to (10) we used the expression of a finite geometric series. From (10) we see that several solutions exist. In (11) we have chosen the solution corresponding to the shortest antenna distance, which is probably the most interesting from a practical design point of view.

From (11), we see that the key design parameter is the product between d_t and d_r, from now on referred to as the *antenna separation product* (ASP). The optimal ASP in (11) depends on the wavelength, transmitter - receiver distance, the number of receive antennas, θ_t and θ_r.

The above result can easily be generalized to any combination of M and N by realizing that a similar result can be obtained when N is larger than M, by switching the transmitter and receiver in Fig. 1. The only difference is that M becomes N in (11). In general, the optimal ASP with respect to Shannon capacity for ULAs therefore becomes

$$d_t d_r = \frac{\lambda R}{V \cos\theta_t \cos\theta_r}. \qquad (12)$$

By inspection of (12) it is apparent that LOS MIMO systems are best suited for scenarios where the transmission distance is constant, such as FWA and radio relay transmission. In practical scenarios, λ, V, and R may be given, whereas the system designer may jointly optimize values of d_t, d_r, θ_t, and θ_r. Comparing (12) to what was found in [6, Eq. (5)] and [8, Eq. (4)], which are valid only for parallel antenna arrays, we see that if we have $\theta_t = 0^\circ$ and $\theta_r = 0^\circ$ (parallel arrays) the same result is obtained. Equation (12) is thus a more general result which supports any orientation of the sender and receiver arrays. Another important new insight to note is that the optimal ASP is independent of the rotation angle, ϕ_r, in Fig. 1.

In the analysis to follow we want to investigate how sensitive the performance is to deviations from the optimal ASP value

found in (12). For this purpose we introduce a *deviation factor*, denoted η, defined as the ratio between the optimal ASP and the actual ASP,

$$\eta = \frac{\text{ASP}_{opt}}{\text{ASP}}. \tag{13}$$

From this definition we see that if the deviation factor is larger than unity, the actual ASP is too small compared to the optimal value in (12), while a deviation factor smaller than one indicates that the actual ASP is too large.

There can be several reasons for η to deviate from zero dB. For example, the optimal ASP may be too large for practical systems so that a compromise is needed, or the parameters in (12) may be difficult to determine with sufficient accuracy. A third reason might be the wavelength dependency. A communication system always occupies a nonzero bandwidth, while the antenna distance can only be optimal for one single frequency. As an example consider the 10.5 GHz license free band (10.000 - 10.680 GHz [11]). If we design a system for the center frequency, the deviation for the lower frequency yields $\lambda_{opt}/\lambda = f/f_{opt} = 10.340/10.000 = 1.034 = 0.145$ dB. Thus the wavelength dependency in this case has small impact on η.

The eigenvalues of the $\mathbf{W}$ matrix can now be found based on the general geometrical model presented in the beginning of this section. We use $\mathbf{H} = \mathbf{H}_{\text{LOS}}$ ($K \to \infty$) in (4), and the relation $|\mathbf{W} - \mathbf{I}\lambda_i| = 0$. The exact eigenvalues can be found numerically by using (6) as the path length. In what follows we will also find approximate eigenvalues $\{\lambda_i\}_{i=1}^{U}$ analytically by using the simplifications described previously, i.e. by using (7) instead of (6). The computation of the eigenvalues gets complicated for large values of U. As an example, the approximate eigenvalues for a 2×2 MIMO system are

$$\lambda_1 = 2 + \sqrt{2 + 2\cos(\pi/\eta)}, \text{ and}$$
$$\lambda_2 = 2 - \sqrt{2 + 2\cos(\pi/\eta)}.$$

For a 3×3 system, when defining $\Phi = \cos\left(\frac{2\pi}{3\eta}\right)$, we get

$$\lambda_1 = 4 - 4\Phi^2, \tag{14}$$

$$\lambda_2 = 2\Phi^2 + \frac{5}{2} + \frac{1}{2}\sqrt{9 + 8\sum_{i=0}^{2}(2+i)\Phi^{2(2-i)}}, \text{ and} \tag{15}$$

$$\lambda_3 = 2\Phi^2 + \frac{5}{2} - \frac{1}{2}\sqrt{9 + 8\sum_{i=0}^{2}(2+i)\Phi^{2(2-i)}}. \tag{16}$$

By inspection we can see, in both examples, that all eigenvalues are equal to N when $\eta = 1$ for these quadratic MIMO systems ($M = N$). This agrees well with what was stated earlier: $\sum\lambda = M \cdot N$, and optimal performance is achieved when $\lambda_1 = \lambda_2 = \cdots = \lambda_U = V$. From (3) we observe that capacity then becomes N times the SISO capacity with the same total transmit power.

As an illustration we have plotted the eigenvalues $\{\lambda_i\}_{i=1}^{3}$ for a 3×3 pure LOS channel in Fig. 2. The plot contains both the numerically calculated eigenvalues found with the procedure described earlier ((6) used as path length), with $R \gg (N-1)\cdot d_t, (M-1)\cdot d_r$, and the approximated eigenvalues calculated in (14)–(16). The analytical results are plotted as points for each integer value of η in dB. As the figure shows, the results from the two procedures are in perfect agreement, and thus the analytical calculations are verified. The figure shows that the optimum (i.e., $\lambda_i = 3$) is reached for a deviation factor equal to 0 dB ($\eta = 1$), and periodically for smaller deviation factors. The explanation for this periodic behavior is that (10) has more than one solution, as discussed previously. As already stated we concentrate on the solution of $\eta = 1$ because this gives the smallest ASP. By examining Fig. 2 together with (3) we can see how the deviation from optimal design values influences the capacity for a pure LOS 3×3 MIMO system.

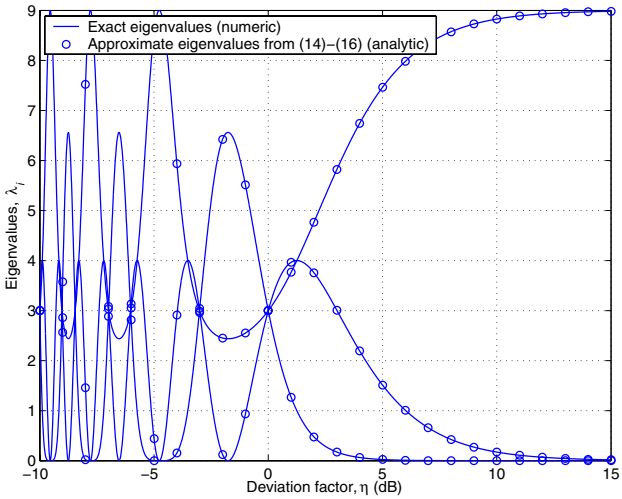


Fig. 2. Eigenvalues (λ_i) for a 3×3 MIMO system as a function of deviation factor (η) in dB for a pure LOS channel.

B. NLOS Channel

The NLOS channel matrix is a result of reflections, diffraction, and scattering from the transmission environment. This component is stochastic, and can only be described by statistical models. A common way of modeling this matrix is by circularly symmetric complex Gaussian elements with zero mean and variance σ^2, denoted $CN(0, \sigma^2)$. The absolute values of these matrix elements will be Rayleigh distributed, and thus it is often referred to as a Rayleigh matrix. If we let the variance of the complex elements equal unity, the matrix will be normalized as required.

IV. System Performance Evaluation

When the channel is stochastic, as in the NLOS case, the capacity given by the expression (3) in section II, becomes a random variable. There exist several different characterizations of the random MIMO capacity, which will be discussed briefly in the next subsections.

1) Ergodic capacity: Ergodic capacity is the mean capacity over all channel realizations for a specific average SNR,

$$\overline{C} = \langle C \rangle = E[C(\bar{\gamma})]$$

where $\langle\ \rangle$ denotes expectation with respect to the channel matrix distribution. Ergodic capacity is used to see how the average capacity changes as a function of average SNR.

2) Outage capacity: Outage capacity is defined as the threshold C_{th} which the system capacity will be below with a given outage probability P_{out},

$$P_{\text{out}}(C_{th}) = \Pr[C \leq C_{\text{th}}].$$

Outage capacity is calculated for a specified average SNR. In contrast to ergodic capacity it characterizes the random behavior of the capacity (not only its mean).

3) Effective degrees of freedom: From (3) we see that the MIMO capacity can be written as a sum of U equivalent SISO channels, each with gain λ_i. When the average SNR and the eigenvalues are so large that $\bar{\gamma}\lambda_i \gg N$, a 3 dB increase in SNR gives approximately a capacity increase of 1 bit/s/Hz for the subchannel. The EDOF [4] quantifies how many of the equivalent SISO channels that effectively exhibit this kind of channel capacity increase:

$$\text{EDOF} = \frac{d}{d\delta} C(2^{\delta}\bar{\gamma})\Big|_{\delta=0} = \sum_{i=1}^{U} \frac{\bar{\gamma}\lambda_i}{N + \bar{\gamma}\lambda_i}$$

From the last expression we see that the EDOF is a simple function of the average SNR, the number of transmit antennas, and the eigenvalues of the $\mathbf{W}$ matrix.

V. Results

Few analytical results exist for the capacity of MIMO systems in Ricean fading, and thus the solutions are often found by simulation over a number of channel realizations. This is also the case here.

The example system that is analyzed in this section is quadratic, with $N = M = 3$ transmit and receive antennas. All results presented in this section are found by numerical simulations over 5000 channel realizations. When we investigate the performance of this system it is convenient to look at the extremal points, as in practice we will lie somewhere in between these points. E.g., from (5) we see that $K = -\infty$ dB gives an NLOS channel matrix, while a large K value gives almost a pure LOS channel. We use $K = 20$ dB as an example of this situation. From Fig. 3 we see that the capacity has some remaining stochastic behavior for $K = 20$ dB ($P_{out}(C_{th})$ is not a perfect step function), but we regard it as sufficient to illustrate the principles. Likewise a deviation factor of $\eta = 0$ dB gives optimal ASP, while $\eta = 30$ dB approximates a total antenna separation mismatch (rank($\mathbf{H_{LOS}}$) = 1, see Fig. 2).

Fig. 3 shows the outage capacity for a 3 ×3 system at $\overline{\gamma} = 20$ dB. Two different scenarios are illustrated, optimal design ($\eta = 0$ dB) and total mismatch ($\eta = 30$ dB). From the figure we see how the outage capacities for the two situations become equal when the Ricean factor decrease and the channel matrix approaches a pure Rayleigh matrix. This is as expected because when $K = -\infty$ dB, the channel is Rayleigh, and independent of the antenna separation. When K increases, the capacity increases for $\eta = 0$ dB towards its maximum, $U\log_2(1+\frac{V}{N}\bar{\gamma})$, while it decreases for η = 30 dB towards $\log_2(1 + M\bar{\gamma})$. This shows that a pure LOS channels with optimal ASP is superior in terms of outage capacity compared to MIMO systems based on i.i.d. Rayleigh channels. This figure is similar to one of

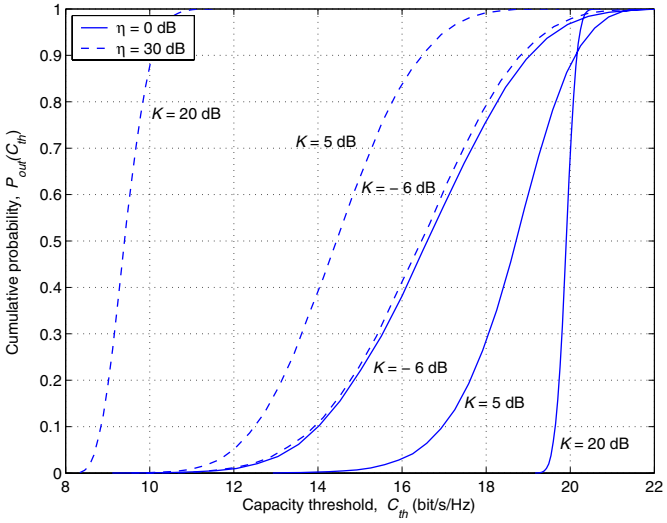


Fig. 3. Outage capacity for a 3×3 MIMO system with average SNR = 20 dB, η = 0 dB (solid lines) and η = 30 dB (dotted lines) for different values of the Ricean K-factor (K = -6 dB, 5 dB, 20 dB).

the figures given in [9], where only the extreme cases were investigated (rank($\mathbf{H_{LOS}}$) = 1 and rank($\mathbf{H_{LOS}}$) = 3). In what follows we will investigate how sensitive the capacity for this system is to deviations from optimal ASP, by allowing η to have values between its extremal values.

In Fig. 4 we have plotted the EDOF for the 3×3 system as a function of the deviation factor, for an average SNR of 5 dB. This is done for different values of K to see how EDOF is influenced by these variables. The figure shows that as K increases the system becomes increasingly dependent on the deviation factor. When $K = -\infty$ dB, the EDOF, and thus the system capacity, is independent of η, as expected. It is

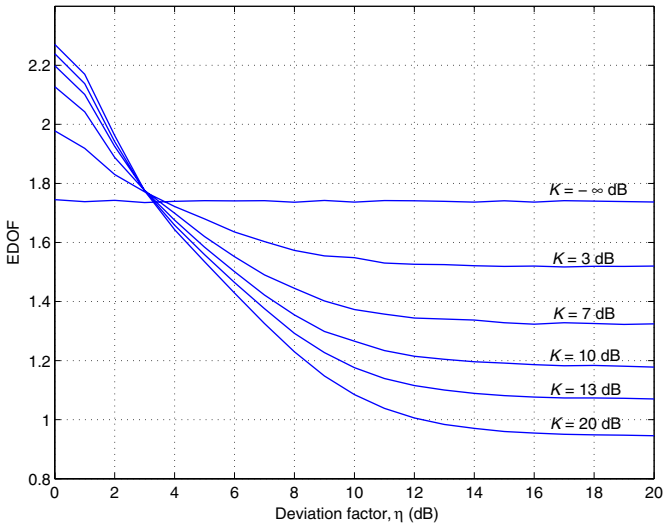


Fig. 4. Effective degrees of freedom for a 3×3 MIMO system as a function of the deviation factor (η), with average SNR = 5 dB and $K = -\infty$ dB, 3 dB, 7 dB, 10 dB, 13 dB and 20 dB.

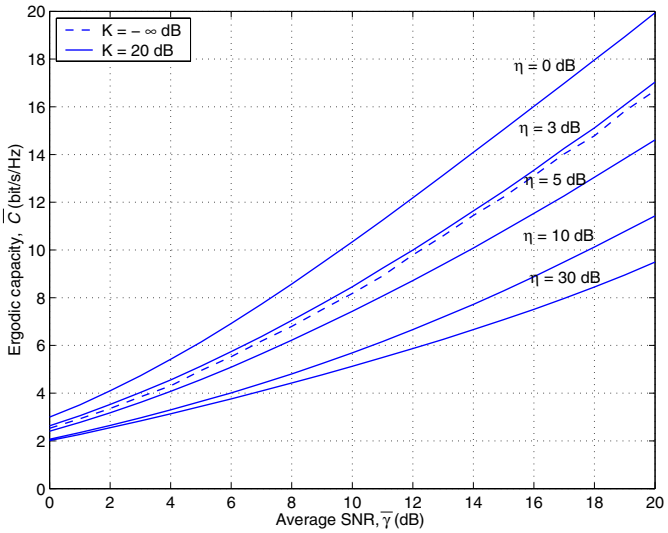


Fig. 5. Ergodic capacity for a 3×3 MIMO system. Solid lines are for $K = 20$ dB ($\sim$ LOS channels) with different values of deviation factor (η) in dB. The dotted line is for $K = -\infty$ (i.i.d. Rayleigh channels).

also interesting to note that at $\eta \approx 3$ dB, EDOF is almost independent of K. Further investigation, not shown here due to lack of space, shows that this point seems to be relatively fixed for different values of the average SNR. For higher channel matrix dimensions the point will not be as well defined (the crossings of $K = -\infty$ dB are more spread out).

Fig. 5 shows how the ergodic capacity decreases as η increases for pure LOS channels. This figure also confirms that these channels perform better than i.i.d. Rayleigh channels with respect to capacity for optimal antenna spacing. We can also verify that when $\eta = 3$ dB for $K = 20$ dB we get almost the same performance as for i.i.d. Rayleigh channels.

VI. Conclusions and Further Work

We have presented results with a new general geometry for the LOS MIMO channels. This geometry allows arbitrary orientation of the transmitter and receiver arrays. The new geometry is utilized to derive the optimal *antenna separation product* (ASP) for *uniform linear arrays* (ULAs), which becomes a function of sender and receiver separation (R), wavelength (λ), dimension of the MIMO system (V), and the spherical angles at the local coordinate systems at the transmitter (θ_t) and receiver (θ_r). Another important result of the optimal design is the independence with respect the rotation angle (ϕ_r).

When we use optimal ASP, the MIMO transmission system performs better in terms of Shannon capacity for pure LOS channels than for traditional i.i.d. Rayleigh channels. For the 3×3 system investigated in section V of this paper, we can tolerate an antenna deviation factor of approximately 3 dB, corresponding to an ASP of half the optimal value, before the ergodic capacity degrades to that of the NLOS case.

The authors are currently working on other ways of making high rank LOS channel matrices, not restricted to ULAs, for different constraints (e.g., maximum array size).

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