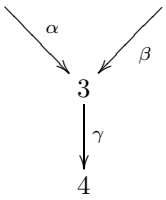
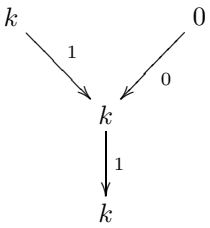
MA3203 - Problem sheet 3 - 2006

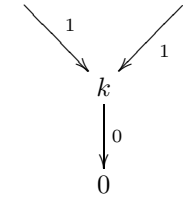
Problem 1. Given $\Lambda = k\Gamma/\langle\rho\rangle$, where Γ is the quiver 

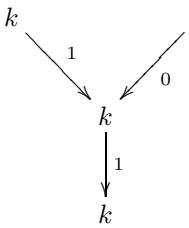
with relations $\rho = \{\gamma\beta\}$ and k is a field.

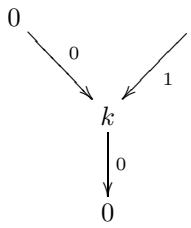
(a) Find the representations corresponding to Λe_1, Λe_2, Λe_3 and Λe_4.

(b) Find a composition series of the Λ-module Λe_1. What is the length of Λe_1?

(c) Find all the homomorphisms from the representation 

to the representation 

, and from the representation 

and to the representation 

.

Problem 2. Let Λ be a finite dimensional algebra over a field k. Let M be a left Λ-module. Show that the following are equivalent.

(a) M is a finitely generated Λ-module.

(b) M has finite length.

(c) M as a vector space over k is finite dimensional.

Problem 3. Let $\mathfrak{r}$ be the radical of a ring Λ, and let

$$A \xrightarrow{f} B \xrightarrow{g} C \to 0$$

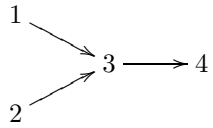
be an exact sequence of Λ-modules and homomorphisms (i.e. $\operatorname{Im} f = \operatorname{Ker} g$ and g is onto; f does not need to be mono).

Show that the sequence

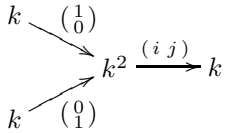
$$A/\mathfrak{r}A \xrightarrow{\bar{f}} B/\mathfrak{r}B \xrightarrow{\bar{g}} C/\mathfrak{r}C \to 0$$

also is exact, where the maps $\bar{f}$ and $\bar{g}$ are induced by $\bar{f}(a+\mathfrak{r}A) = f(a)+\mathfrak{r}B$ and $\bar{g}(b) = g(b)+\mathfrak{r}C$.

Problem 4. Let k be a field and let Γ be the quiver

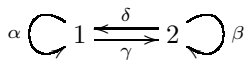


For an ordered pair (i,j) of elements in k, let M_{ij} be the representation given by



(a) Determine for which (i,j) the representation M_{ij} is indecomposable and for which (i,j) it decomposes.
(b) Prove that if M_{ij} and M_{rs} are indecomposable then they are isomorphic. Is the same true if M_{ij} and M_{rs} decomposes?

Problem 5. Let k be a field and Γ the quiver



with relations $\rho = \{\delta\gamma - \alpha^2, \alpha^3 - \alpha^2, \gamma\delta - \beta^2, \beta^3 - \beta^2, \alpha\delta - \delta\beta, \gamma\alpha - \beta\gamma\}$.

(a) Show that the dimension of $k\Gamma/\langle\rho\rangle$ over k is 12.
(b) Show that the subspace of $k\Gamma/\langle\rho\rangle$ spanned by $\alpha^2, \gamma\alpha^2, \alpha^2\delta, \beta^2$ is a ring which is isomorphic to $M_2(k)$-ring of 2×2-matrices over k.

Problem 6. We say that a ring Λ is local if the nonunits of Λ (elements in Λ without multiplicative invers) form an ideal in Λ.

Problem 7. Let Λ_c be the algebra over $\mathbb{C}$ with basis $\{e_0, e_1, e_2, e_3\}$ over $\mathbb{C}$, where c is a given complex number. The multiplication is given by the following multiplication table:

	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_1	e_3	e_3	0
e_2	e_2	$-e_3$	ce_3	0
e_3	e_3	0	0	0

For which c and c' are the algebras Λ_c and $\Lambda_{c'}$ isomorphic?

Challenge. Find a quiver with relations ρ_c over $\mathbb{C}$ such that $\Lambda_c \cong \mathbb{C}\Gamma/\langle\rho_c\rangle$.