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Impact of Second Order Wave Spectra on Heave, Pitch, and Roll Wave Measurements

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DIRECTIONAL SPECTRUM: $S(\omega)D(\theta, \omega)$

$$D(\theta, \omega) = \frac{1}{2\pi} \left(1 + \sum_{n=1}^{\infty} \{a_n(\omega) \cos(n\theta) + b_n(\omega) \sin(n\theta)\} \right)$$

THE STANDARD ANALYSIS (heave(h), pitch (x), and roll(y):

$$S(\omega) = C_{hh}(\omega)$$

$$a_1(\omega) = \frac{Q_{hx}(\omega)}{[C_{hh}(\omega)(C_{xx}(\omega) + C_{yy}(\omega))]^{1/2}},$$

$$b_1(\omega) = \frac{Q_{hy}(\omega)}{[C_{hh}(\omega)(C_{xx}(\omega) + C_{yy}(\omega))]^{1/2}},$$

$$a_2(\omega) = \frac{C_{xx}(\omega) - C_{yy}(\omega)}{C_{xx}(\omega) + C_{yy}(\omega)},$$

$$b_2(\omega) = \frac{2C_{xy}(\omega)}{C_{xx}(\omega) + C_{yy}(\omega)}$$

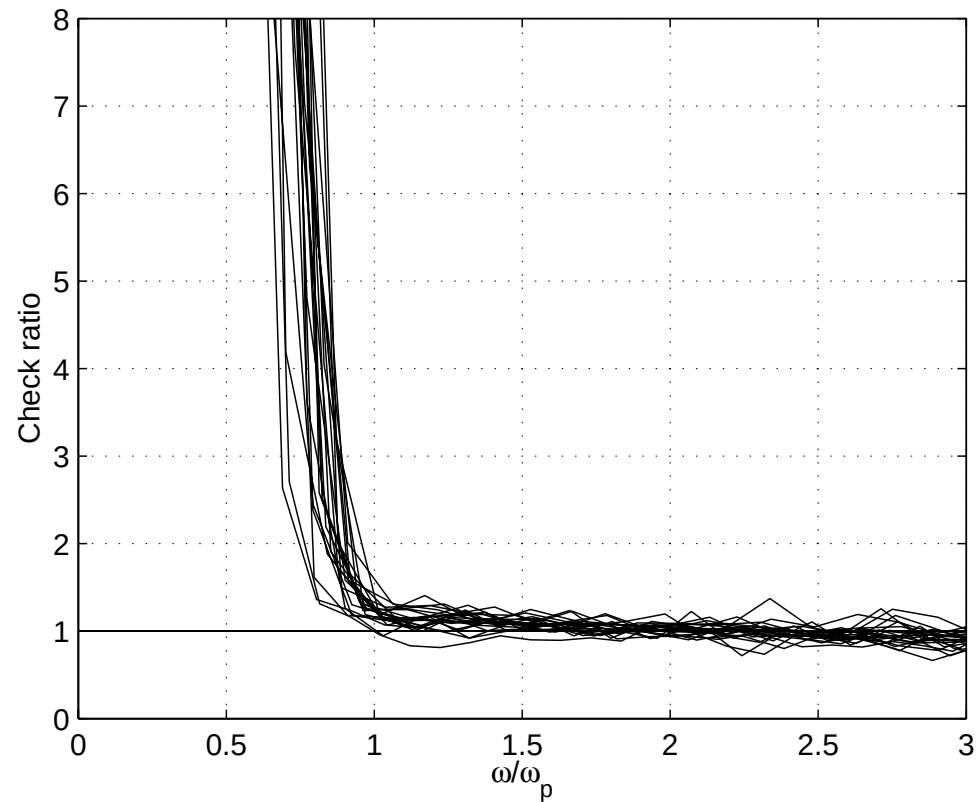
$$R(\omega) = ((C_{xx}(\omega) + C_{yy}(\omega)) / C_{hh}(\omega))^{1/2} / k_{lin}(\omega)$$

Directional spread:

$$\sigma_1(\omega) = \sqrt{2(1 - r(\omega))},$$

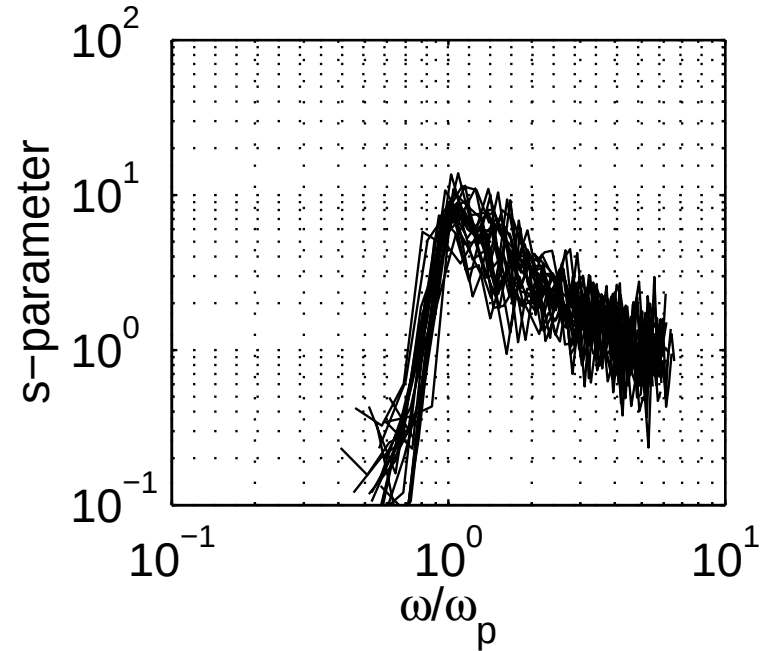
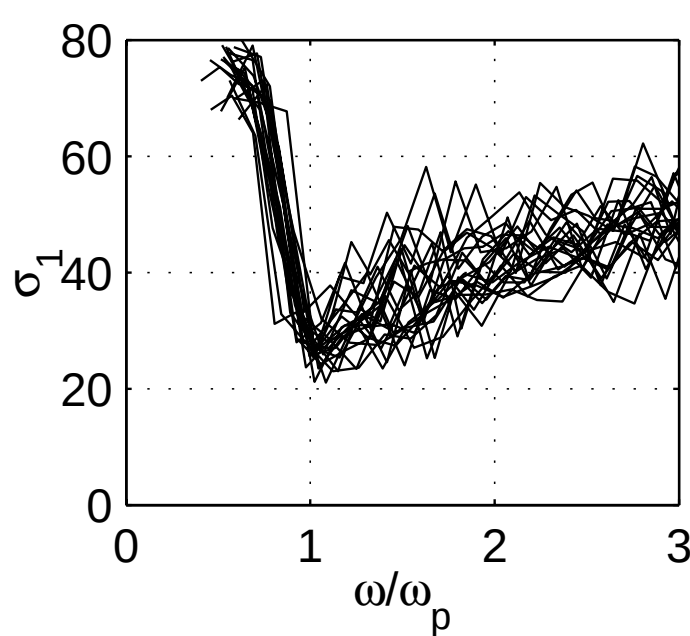
$$r(\omega) = \sqrt{a_1^2(\omega) + b_1^2(\omega)}$$

Field observations from the WADIC experiment (Wavescan buoy)



$Hm0 > 6m$, 22 records

Field observations from the WADIC experiment (Wavescan buoy)



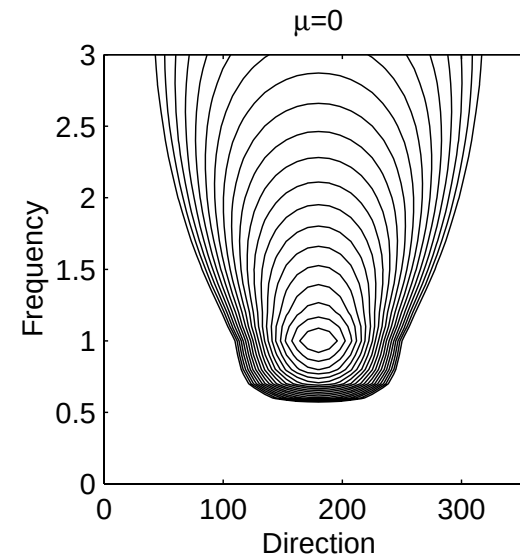
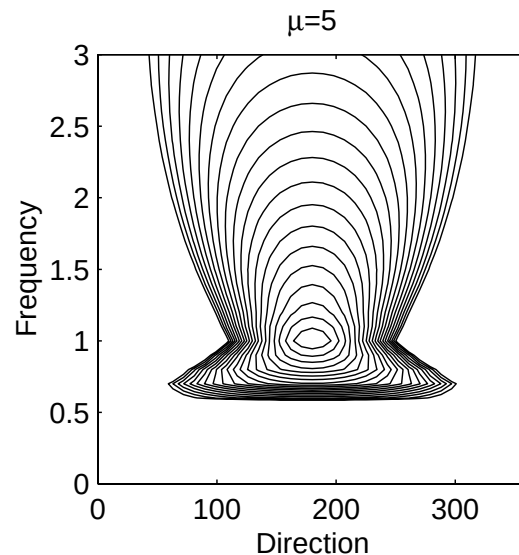
$$s = \frac{2}{\sigma_1^2} - 1$$

$Hm0 > 6m$, 22 records

Synthetic spectrum based on a standard JONSWAP spectrum and the cos-2s directional distribution (Misuyasu parametrization)

$$\sim S_J \left(\frac{f}{f_p} \right) \cos^{2s(f/f_p)} \left(\frac{\theta - \theta_0}{2} \right),$$

$$s(f/f_p) = s_p \left(\frac{f}{f_p} \right)^\mu, \mu = \begin{cases} 5, & f < f_p \\ -2.5 & f > f_p \end{cases}$$



THE GENERAL CASE

The ocean surface:

$$\eta(\mathbf{x}, t) = \int_{\mathbf{k}, \omega} e^{i(\mathbf{k}\mathbf{x} - \omega t)} dZ(\mathbf{k}, \omega)$$

The full 3D spectrum:

$$d\chi(\mathbf{k}, \omega) = E\left(dZ(\mathbf{k}, \omega) \overline{dZ(\mathbf{k}, \omega)}\right)$$

Wavenumber spectrum

$$\Psi(\mathbf{k}) d^2 k = \int_{d^2 k} \int_{\omega=0}^{\infty} d\chi(k, \omega)$$

Linear wave theory (no currents):

$$d\chi(\mathbf{k}, \omega) = [\delta(\omega - \sigma(k))\Psi(\mathbf{k}) + \delta(\omega + \sigma(k))\Psi(-\mathbf{k})] d^2 k,$$
$$\sigma(k) = \sqrt{gk \tanh(kh)}$$

GENERAL INTERPRETATION OF H/P/R-DATA (suppressing the dependence of ω):

$$a_1 = \langle k_x \rangle / \langle k^2 \rangle^{1/2}$$

$$b_1 = \langle k_y \rangle / \langle k^2 \rangle^{1/2}$$

$$a_2 = \left(\langle k_x^2 \rangle - \langle k_y^2 \rangle \right) / \langle k^2 \rangle$$

$$b_2 = \left(2 \langle k_x k_y \rangle \right) / \langle k^2 \rangle$$

$$R = \langle k^2(\omega) \rangle^{1/2} / \left(\omega^2 / g \tanh(kh) \right)$$

$$\langle g \rangle(\omega) = \int_{\mathbf{k}, [\omega, \omega + \Delta\omega]} g(k, \omega) d\chi(\mathbf{K}) / \int_{\mathbf{k}, [\omega, \omega + \Delta\omega]} d\chi(\mathbf{K})$$

H/P/R data (assuming linear waves) collected in currents may be analyzed using the general theory (Barstow and Krogstad, 1984):

- *The current influence is strongest for high frequencies*
- *Mean direction and directional spread slightly influenced*
- *Check ratio rather strongly affected (more by an opposing current to the waves)*

Currents can not explain the strange low-frequency behaviour

OTHER CAUSES?

Spectral leakage:

To some extent (R. Ezraty, A. Cavanié, Ocean. Acta, 1981)

Horizontal motion:

Simple analysis indicates that the horizontal motion of a H/P/R buoy is less damaging than expected since the leading order perturbation due to the horizontal motion vanishes.

Noise:

Increases the spread at very low spectral levels (Tucker)

THE SECOND ORDER SPECTRUM

First order spectrum $d\chi_1(\mathbf{K}), \mathbf{K} = (\mathbf{k}, \omega)$

- supported on the dispersion surface $|\omega| = \sqrt{gk \tanh(kh)}$

Second order spectrum (4th order in wave slope), Laing (1986):

$$d\chi_2(\mathbf{K}) = \int_{\mathbf{K}'} G(\mathbf{K}, \mathbf{K}_1) d\chi_1(\mathbf{K}_1) d\chi_1(\mathbf{K} - \mathbf{K}_1)$$

$$G(\mathbf{K}, \mathbf{K}_1) = \frac{1}{2} \left[f_2(\mathbf{K}, \mathbf{K}_1) / \left(g - \omega^2 / k \tanh(kh) \right) \right]^2$$

$$f_2(\mathbf{K}, \mathbf{K}_1) = \frac{1}{2} \left[\omega_1^2 - \omega_1 \omega + \omega^2 \right. \\
\left. - \omega_1 (\omega - \omega_1) \langle \mathbf{k}_1, \mathbf{k} - \mathbf{k}_1 \rangle \coth(k_1 h) \coth(|\mathbf{k} - \mathbf{k}_1| h) \right. \\
\left. - \omega \omega_1 \langle \mathbf{k}, \mathbf{k}_1 \rangle \coth(kh) \coth(k_1 h) \right. \\
\left. - \omega (\omega - \omega_1) \langle \mathbf{k}, \mathbf{k} - \mathbf{k}_1 \rangle \coth(kh) \coth(|\mathbf{k} - \mathbf{k}_1| h) \right]$$

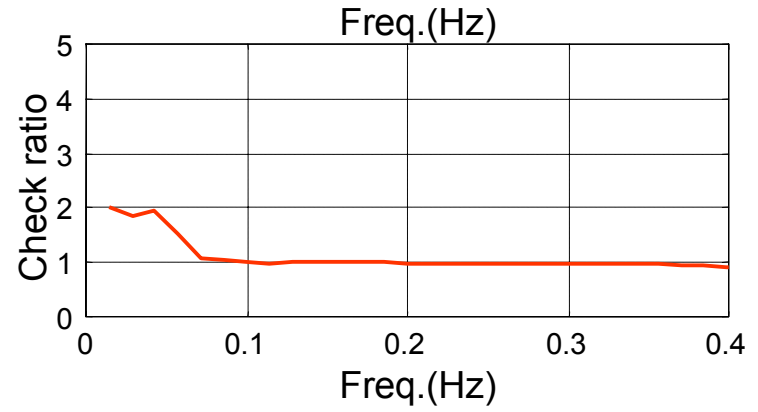
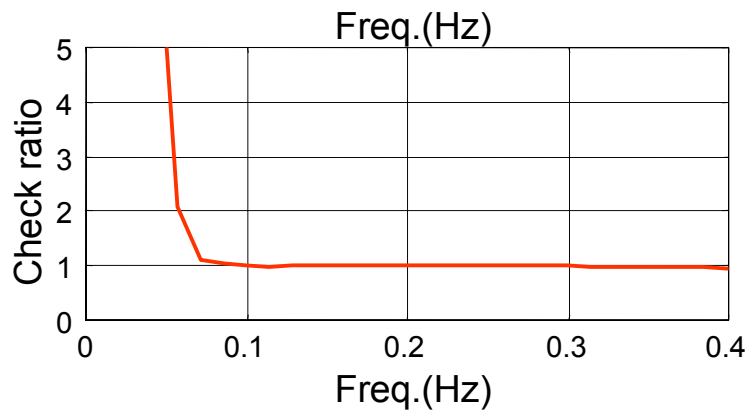
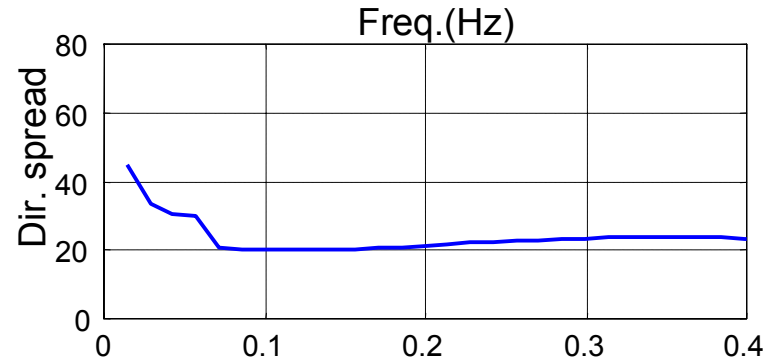
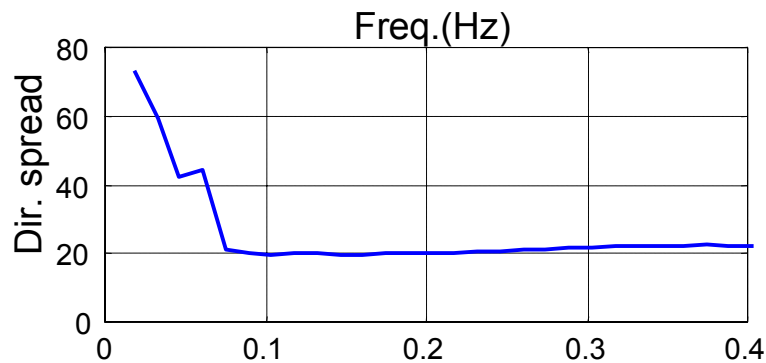
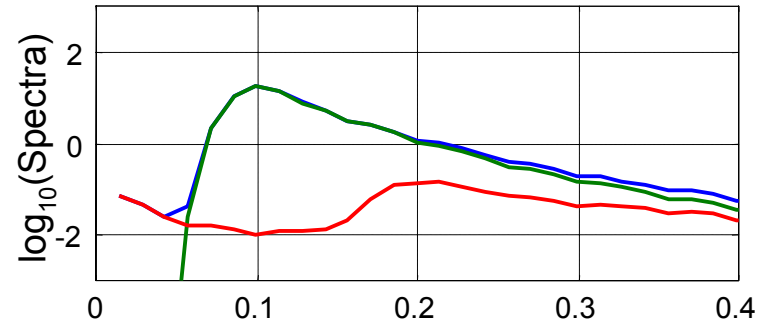
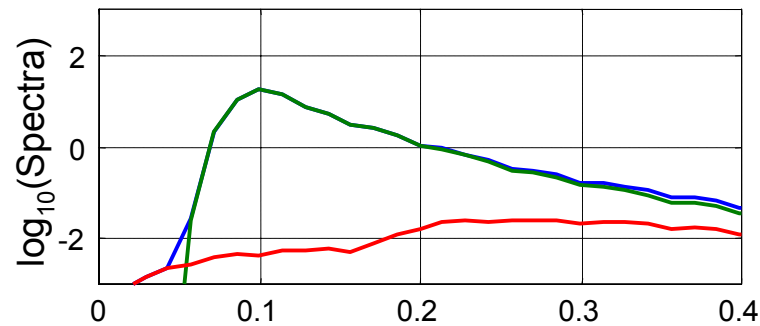
$$\langle \mathbf{k}_1, \mathbf{k}_2 \rangle = \mathbf{k}_1 \cdot \mathbf{k}_2 / |\mathbf{k}_1| |\mathbf{k}_2|$$

A simple numerical implementation assuming
a discrete first order spectrum:

$$\chi_1(\mathbf{K}) = \sum_{j=1}^N s_j \left\{ \delta(\mathbf{K} - \mathbf{K}_j) + \delta(\mathbf{K} + \mathbf{K}_j) \right\} \mathbf{K}_j \text{ on the dispersion surface.}$$

$$\begin{aligned} \chi_2(\mathbf{K}) = \sum_{j,n=1}^N s_j s_n \big\{ & G(\mathbf{K}, \mathbf{K}_j) \delta(\mathbf{K} - (\mathbf{K}_j + \mathbf{K}_n)) + G(\mathbf{K}, \mathbf{K}_j) \delta(\mathbf{K} - (\mathbf{K}_j - \mathbf{K}_n)) \\ & + G(\mathbf{K}, -\mathbf{K}_j) \delta(\mathbf{K} - (-\mathbf{K}_j + \mathbf{K}_n)) + G(\mathbf{K}, -\mathbf{K}_j) \delta(\mathbf{K} - (-\mathbf{K}_j - \mathbf{K}_n)) \big\} \end{aligned}$$

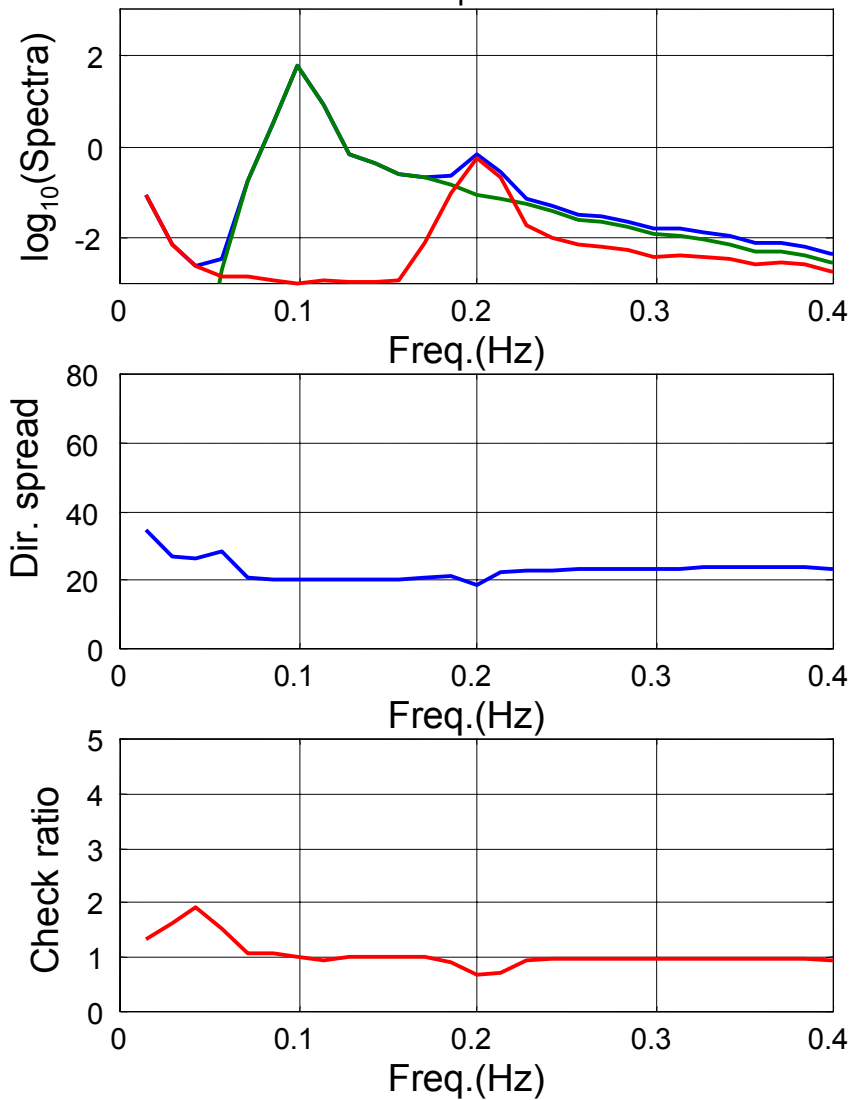
$Hm0=4m, T_p=10s, \sigma_p=20^\circ, \gamma=2$

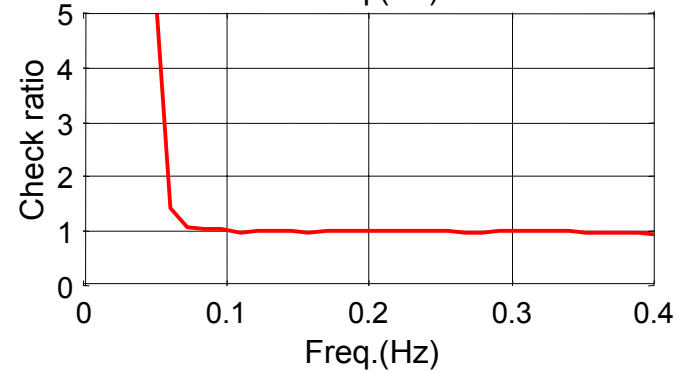
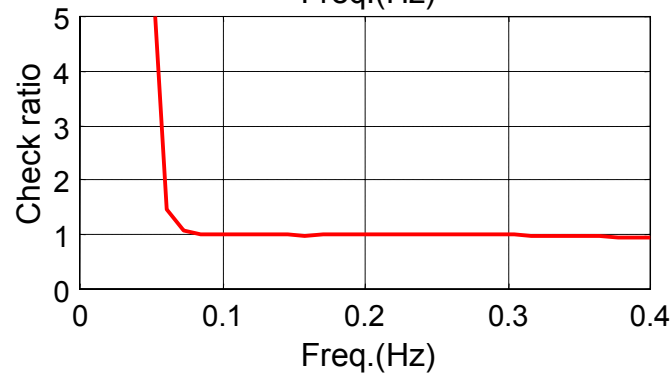
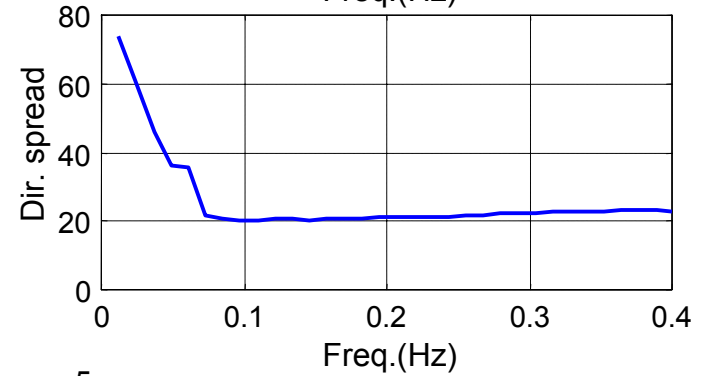
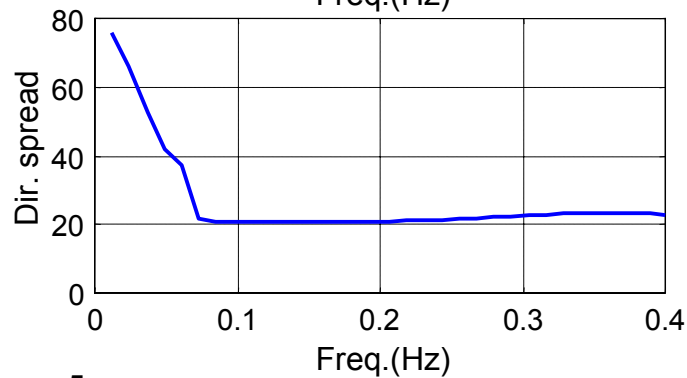
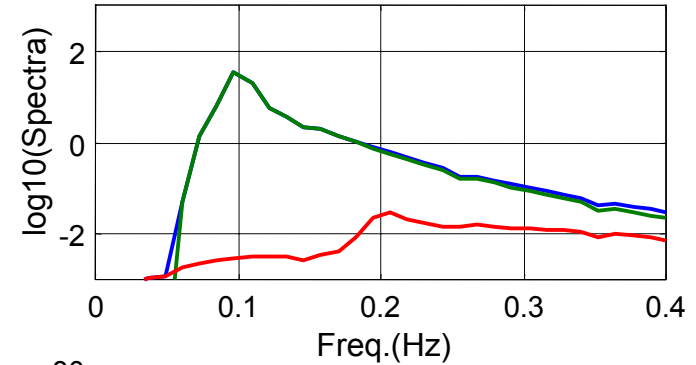
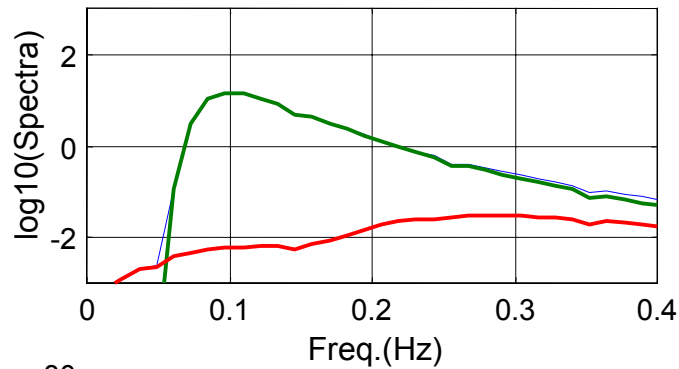


Deep water

Depth = 20m

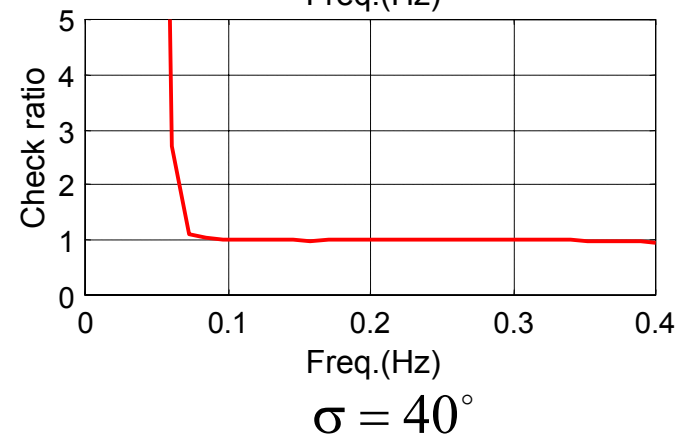
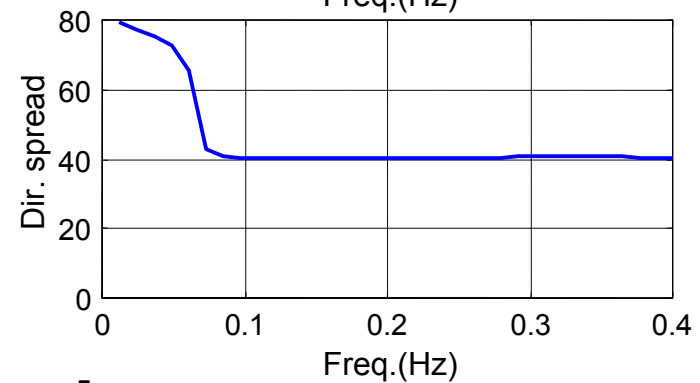
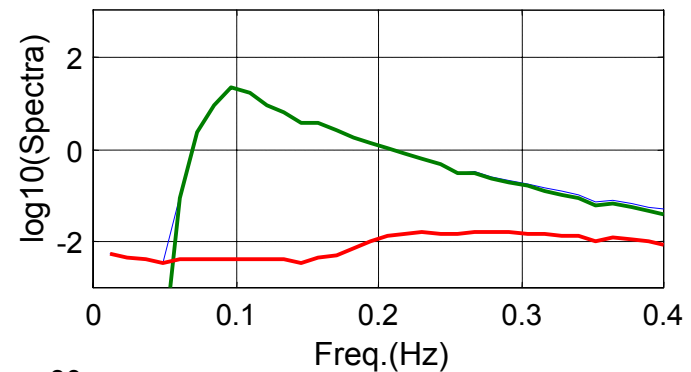
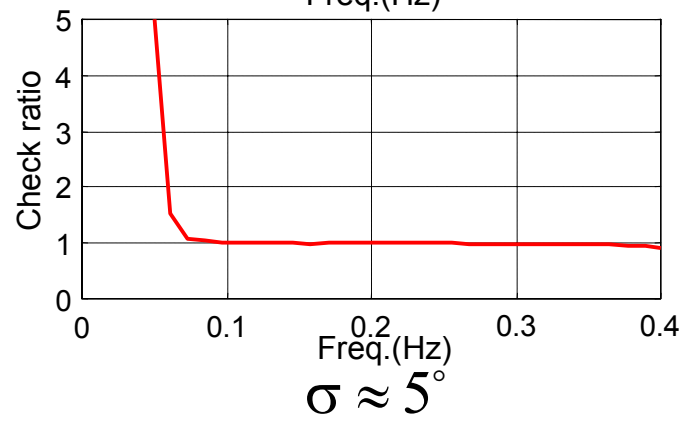
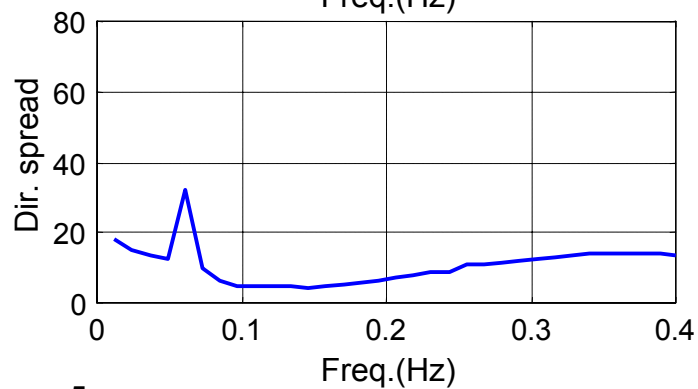
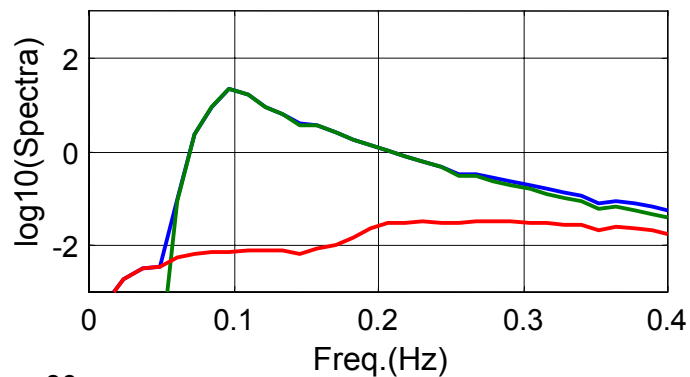
$Hm0=4m, T_p=10s, \sigma_p=20^\circ, \gamma=100, h=20m$





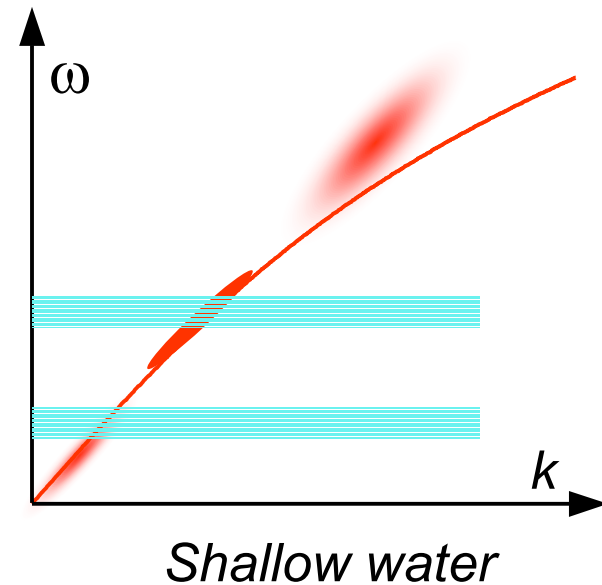
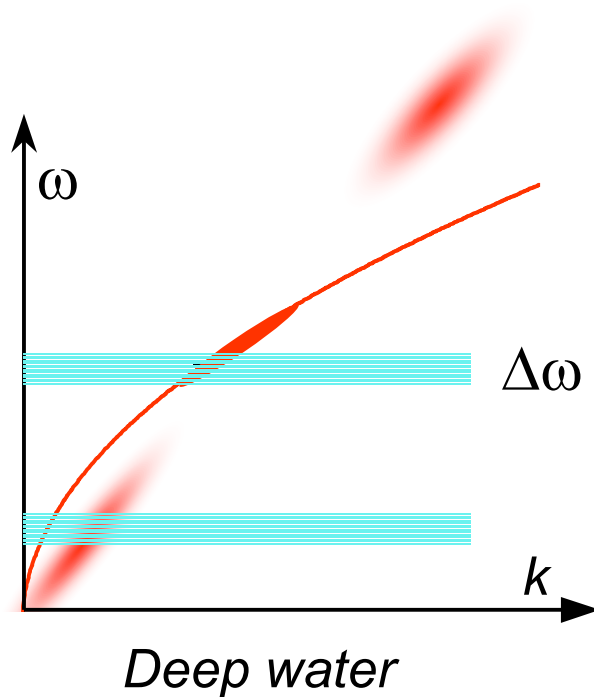
$\gamma=1$ (PM-spectrum)

$\gamma=7$ (JONSWAP-North Sea)



Qualitative explanation why deep water shows a stronger influence than shallow water

$$\langle k^2 \rangle(\omega) = \int_{\mathbf{k}, [\omega, \omega + \Delta\omega]} k^2 d\chi(\mathbf{K}) / \int_{\mathbf{k}, [\omega, \omega + \Delta\omega]} d\chi(\mathbf{K})$$



CONCLUSIONS:

- The strong increase in the directional spread below the spectral peak that is always observed in real spectra, may partly be due to the impact of the second order spectrum
- The strong deviation from 1 in the dispersion ratio below the spectral peak may also be due to the second order spectrum
- The increase in spread above the peak is not explained by the second order spectrum