LØSNINGSSKISSE EKSAMEN 9 AUG 2006

Industriell statistikk; TMA 4260

1 a)

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = \frac{1}{8}(5.9 + \ldots + 6.2) = \underline{6,475}$$

$$\bar{y} = \frac{1}{7}\sum_{j=1}^{7} y_j = \frac{1}{7}(5.8 + \ldots + 6.3) = \underline{5,814}$$

$$s_x^2 = \frac{1}{8-1}\sum_{i=1}^{8}(x_i - \bar{x})^2 = \underline{0,26}$$

$$s_y^2 = \frac{1}{7-1}\sum_{j=1}^{7}(y_j - \bar{y})^2 = \underline{0,21}$$

$$H_0: \sigma_x^2 = \sigma_y^2, \quad H_1: \sigma_x^2 \neq \sigma_y^2$$

$$F = \frac{s_x^2}{s_y^2} = 1,24 \quad \blacktriangleleft \quad f_{0.95,7,6} = 4,21$$

$$1,24 > \frac{1}{f_{0.95,6,7}} = 0.26$$

Dvs at H_0 aksepteres.

$H_0: \mu_x = \mu_y$, $H_1: \mu_x > \mu_y$

Antar at $\sigma_x^2 = \sigma_y^2$ fra første del av oppgave.

$$s_p^2 = \frac{\sum_{i=1}^{8}(x_i - \bar{x})^2 + \sum_{j=1}^{7}(y_j - \bar{y})^2}{8+7-2} = \underline{0,239}$$

$$T = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{8} + \frac{1}{7}}} = \underline{2,61}$$

Forkaster H_0 dersom $T > t_{8+7-2,\alpha}$, nivå α.

p-verdi $= P(t_{13} > 2,61) = \underline{0,011}$

1b) Wilcoxon ranks

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
y	5.1	5.5	5.6		5.8		6.0		6.3	6.4					
x				5.7		5.9		6.2			6.5	6.6	6.8	6.9	7.2

$w_x = (4+6+\ldots 15) = \underline{83}$

$w_y = (1+2+\ldots 10) = \underline{37}$

$u_y = 37 - \frac{7 \cdot 8}{2} = \underline{9}.$

p-verdi $= P(u_y \leq 9) = \underline{0,014}$ { fra tabell med $n_y = 7$, $n_x = 8$.

Nokså lik som 1a).

(2) a) i) $\hat{\beta}_1 = 0,0379$

$$\beta_L = \hat{\beta}_1 + t_{18-4,0,975} \cdot \text{Std}(\hat{\beta}) = \hat{\beta}_1 - 2.145 \cdot 0,0396$$

$$\beta_H = \hat{\beta}_1 + t_{18-4,0,025} \cdot \text{Std}(\hat{\beta}_1) = \hat{\beta}_1 + 2,145 \cdot 0,0396$$

$$95\%\text{ INT} = (\beta_L, \beta_H) = (-0,047,\ 0,123)$$

ii) $H_0: \beta_2 = 0, \quad H_1: \beta_2 \neq 0.$

$$T = \frac{\hat{\beta}_2}{\text{Std}(\hat{\beta}_2)} = \frac{0,01496}{0,00415} = 3,6.$$

$$p\text{-verdi} = 2 \cdot P(t_{14} > 3.6) \approx 0,004$$

iii) $\hat{y} = 38,3 + 0,015 \cdot 8000 - 0,00000135 \cdot 8000^2$

$$= 71,6$$

b) Generelt for full modell

$$SST = SSR + SSE$$

dvs. $249,2 = 137,1 + 112,1$

Ved ulike tilpasninger av 'subsets' ser vi at R^2 er størst for $[1, T, V, V^2]$, men den er nesten like høy for $[1, V, V^2]$. Differansen i $R^2 = 55 - 52.1 = 3.1$.

$$R^2[1, T, V, V^2] = 0.55 = \frac{137.1}{249.2} = \frac{SSR}{SST}$$

$$R^2[1, V, V^2] = 0.521 = \frac{SSR(1, V, V^2)}{249,2},$$

dvs at $SSR(1, V, V^2) = 0.521 \cdot 249,2$

$= 129,8$

Videre er:

$$F = \frac{SSR - SSR(1, V, V^2)}{s^2} \sim f_{1,14}.$$

$$F = \frac{137,1 - 129,8}{2.83^2} = 0,91.$$

$$f_{1,14,0,95} = 4.6.$$

Da $F < f_{1,14,0.95}$ aksepteres en hypotese om at $[1, V, V^2]$ er en like god modell som $[1, T, V, V^2]$.

(2c) Regresjonsmodell er:

$$\hat{y}(V) = 38,3 + 0,015\,V - 0,00000135\,V^2$$

$$\frac{\partial \hat{y}}{\partial V} = 0,015 - 2 \cdot 0,00000135\,V = 0$$

$$V_{opt} = \frac{0,015}{2 \cdot 0,00000135} = \underline{5556\ \frac{m^3}{h}}$$

$$V_{opt} = -\frac{\hat{\beta}_2}{2\hat{\beta}_3}, \qquad \frac{\partial V_{opt}}{\partial \hat{\beta}_2} = -\frac{1}{2\hat{\beta}_3}$$

$$\frac{\partial V_{opt}}{\partial \hat{\beta}_3} = \frac{\hat{\beta}_2}{2\hat{\beta}_3^2}$$

Linearisering gir at

$$V_{opt} \approx -\frac{\hat{\beta}_2}{2\hat{\beta}_3} - \frac{1}{2\hat{\beta}_3} \cdot (\hat{\beta}_2 - \beta_2) + \frac{\hat{\beta}_2}{2\hat{\beta}_3^2}(\hat{\beta}_3 - \beta_3)$$

$$Var[V_{opt}] = \frac{1}{4\hat{\beta}_3^2} \cdot Var(\hat{\beta}_2) + \frac{\hat{\beta}_2^2}{4\hat{\beta}_3^4} \cdot Var(\hat{\beta}_3)$$

$$= \frac{1}{4 \cdot 0,00000135^2} \cdot 0,00415^2 + \frac{0,015^2}{4 \cdot 0,00000135^4} \cdot 0,00000039^2$$

$$\approx \underline{1536^2}$$

2d)

Residualplottet mot order i Figuren (nederst til høyre) viser at målingene trolig er korrelerte i tid. Blant annet er residualene mellom 5 og 11 alle positive.

$$y_i = \beta_0 + \beta_1 \cdot T_i + \beta_2 V_i + \beta_3 \cdot V_i^2 + \varepsilon_i$$

$$i = 1, \ldots, 18.$$

$$z_1 = (1-0.9^2)\beta_0 + \beta_1 \cdot (1-0.9^2)T_1 + \beta_2 (1-0.9^2)V_1 + \beta_3 (1-0.9^2)V_1^2 + \eta_1$$

$$= \left[1-0.9^2,\ (1-0.9^2)T_1,\ (1-0.9^2)V_1,\ (1-0.9^2)V_1^2\right] \cdot \underline{\beta} + \eta_1$$

der $\mathrm{Var}(\eta_1) = \frac{\sigma^2}{(1-0.9^2)^2} \cdot (1-0.9^2)^2 = \underline{\sigma^2}$

$$z_i = y_i - 0.9 y_{i-1}$$

$$= \beta_0 (1-0.9) + \beta_1 (T_i - 0.9 T_{i-1}) + \beta_2 (V_i - 0.9 V_{i-1}) + \beta_3 (V_i^2 - 0.9 V_{i-1}^2) + \underbrace{\varepsilon_i - 0.9\varepsilon_{i-1}}_{\eta_i}$$

$$= \left[1-0.9,\ T_i - 0.9 T_{i-1},\ V_i - 0.9 V_{i-1},\ V_i^2 - 0.9 V_{i-1}^2\right] \cdot \underline{\beta} + \eta_i$$

der $\mathrm{Var}(\eta_i) = \sigma^2$ og uavhengig av ε_1.

Endelig:

$$\underline{Z} = \begin{bmatrix} 1-0.9^2 & (1-0.9^2)T_1 & \cdots \\ & T_2 - 0.9 T_1 & V_2^2 - 0.9 V_1^2 \\ & \vdots & \\ 1-0.9^2 & T_{18} - 0.9 T_{17} & V_{18}^2 - 0.9 V_{17}^2 \end{bmatrix} \cdot \underline{\beta} + \left(\underline{\eta} \sim \text{uavhengige}\right)$$

3 a)

Fra fortegnsmatrisa har vi at $D = ABC$. Konfunderingsmønsteret blir da

$$D = ABC \quad , \quad C = ABD$$
$$B = ACD \quad , \quad AC = BD$$
$$A = BCD \quad , \quad AB = CD$$

Vi antar at høyere ordens samspill er lik 0. Samt at $BD = 0$ for å estimere AC.

$$\hat{A} = -\left[\frac{21,3 + \ldots + 21,8}{8}\right] + \left[\frac{27,2 + \ldots + 24,4}{8}\right]$$

$$= \underline{1,09}.$$

$$\underline{\hat{C} = -4,6} \quad , \quad \underline{\hat{AC} = 1,66}$$

Estimat for variansen:

$$s^2 = \frac{(y_{11} - \bar{y}_1)^2 + \ldots (y_{82} - \bar{y}_8)^2}{8} = \frac{1,665}{8}$$

$$= \underline{0,208}$$

$$\sigma_{effekt} = \sqrt{\frac{4 \cdot 1,665}{8}} = \underline{0,32}.$$

$\underline{t_{8,0.05} = 1,86.}$ $\underline{A, AC \text{ og } C \text{ er alle signifikante.}}$

3b)

Prediktert respons:

$$\hat{y} = \bar{y} + \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}\begin{pmatrix} \hat{A} \\ \hat{C} \\ \hat{AC} \end{pmatrix}$$

$$= 25{,}13 + \frac{1.09}{2} + \frac{4.6}{2} + \frac{1.66}{2}$$

$$= \underline{27{,}15}$$

$$Var(\hat{y}) = Var(\bar{y}) + \frac{1}{4}Var(\hat{A}) + \frac{1}{4}Var(\hat{C}) + \frac{1}{4}Var(\hat{AC})$$

$$= \frac{\sigma^2}{16} + \frac{3}{4}\cdot\sigma_{eff}^2$$

$$= \frac{0.208}{16} + \frac{3}{4}\cdot 0.32^2 = \underline{0.09}$$

$$\underline{Std(\hat{y}) = 0.3}$$