Below we use the following notation:
Txbk – "Codinig theory and Cryptography" by Hankerson & others.

Exerceses.

Problem set 1.

Let a code $\mathcal{C} \subset \mathbb{F}_2^{10}$ be generated by the following row vectors

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \end{matrix}$$

Task 1.1. Write an encoding map (choose one that you consider convenient) and describe the corresponding "decoding for codewords" procedure.

Task 1.2. What are n, k, d numbers for this code? Why?

Problem set 2.

Let the generating matrix for a binary code be

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

Task 2.1. Is it possible to use this code in a way that any 1 or 2 errors are corrected, but 3 errors are detected? If YES, describe how it could be done. If NO, describe why?

Task 2.2. We would like to use this code and we are interested in detecting as many errors as possible. How many errors could be detected? Describe how it could be done.

Task 2.3. What is the minimal distance for this code?

Problem set 3.

Task 3.1. What is the largest n such that you can claim : there exists a binary $[n, k, d]$-code with $n - k = 5$, $d = 6$? Explain.

Task 3.2. What is the largest k such that you can claim : there exists a binary $[n, k, d]$-code with $n = 15$, $d = 5$? Explain.

Task 3.3. What is the smallest n such that you can claim : there exists a $[n, k, d]$-code over $\mathbb{F}_3$ (ternary code) with $k = 5$, $d = 3$? Explain.

Problem set 4.

Let us consider the smallest linear cyclic code in $\mathbb{F}_2^{15}$ containing the word

$$001110001011100.$$

Task 4.1. Find the generating polynomial for this code.

Task 4.2 Write a generating matrix of this code that has a standard form (has the identity matrix I_k inside).