Below we may use the following notation:
Txbk – "Codinig theory and Cryptography" by Hankerson & others.

Exercises.

Problem set 5.

Task 5.1. Let $f(x) \in \mathbb{F}_2[x]$,
$f(x) = x^5 + x^3 + 1$.

Does $f(x)$ divide $x^{31} + 1$ or not?

Task 5.2. Let $g(x) \in \mathbb{F}_2[x]$,
$g(x) = x^5 + x^4 + 1$.

Does $g(x)$ divide $x^{31} + 1$ or not?

Given $f(x) = x^7 + \ldots + 1 \in \mathbb{F}_2[x]$
and $h(x) = x^7 + \ldots + 1 \in \mathbb{F}_2[x]$.
It is known that f is irreducible over $\mathbb{F}_2$ and
h is not irreducible over $\mathbb{F}_2$.

Task 5.3. What is true:
(a) all such f divide $x^{127} + 1$,
(b) some divide and some do not divide,
(c) all do not divide $x^{127} + 1$?
Why? Explain!

Task 5.4. What is true:
(a) all such h divide $x^{127} + 1$,
(b) some divide and some do not divide,
(c) all do not divide $x^{127} + 1$?
Why? Explain!

Problem set 6.

Let α be a generator of $\mathbb{F}_{64}{}^*$.

Task 6.1. The code in $\mathbb{F}_2^{63}$ is defined by the condition
$f_w(\alpha^{20}) = f_w(\alpha^{21}) = 0$.

What is the dimension of this code? Explain.

Task 6.2. The code in $\mathbb{F}_2^{63}$ is defined by the condition
$f_w(\alpha^{20}) = f_w(\alpha^{32}) = f_w(\alpha^{48}) = 0$.

What is the dimension of this code?
How many errors could be corrected with this code? Why? Explain.