Final Exam – Answers and Comments.

Problem 1.

Answer:$d = 6$.
One should write arguments why $d = 6$ "works" and why $d = 5$ would not.

Problem 2.

Answer: $k = 3$, $d = 4$ and the matrix could be

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Finding G is simple, then one gets k, and it is left to check the weight for sums of basic vectors without calculational mistakes.

Problem 3.

Answer: $g(X) = X^5 + X^4 + X^2 + 1$.
It could be done with EA, or using the prime decomposition of the polynomial involved (it happens that the book has the decomposition for $X^{15} + 1$).

Problem 4.

Answer: $n = 23$ and let me skip writing the statement of the theorem.

Problem 5.

Answer: 010.100.000.000.010.110.100.110
There are many solutions possible. Basically one needs to check if a vector belongs to the code space for various combinations of erasures, but some tricks or pure luck can shortened the checking.

Problem 6.

Answer: $j_1 = 30$, $b_1 = \alpha^3$ and $j_2 = 13$, $b_2 = \alpha^3 + \alpha^2 + \alpha + 1$.
Here we use the standard procedure, and $N = 2$, $u_2 = X^2 + X + \beta + 1 = X^2 + X + \alpha^2 + \alpha$. Then $\tilde{a_1} = \alpha$, $\tilde{a_2} = \alpha + 1$. The rest follows.

Problem 7.

Here we notice that rows 1-4 and 6-7 do contain errors. Thus the row 5 contains no error and 6 rows have at least one error each. We have 2 more errors spread in these six rows, so we conclude that each row has at most three errors. This permit us to correst errors row by row using the correction procedure for G_{24}.

Problem 8.

Answer: $k = 25$.
Here we can use $m = -1$ that gives the generating polynomial of degree 6 over $\mathbb{F}_2$ with roots

$$\{1, \alpha, \alpha^2, \alpha^4, \alpha^8, \alpha^{16}, \}$$

One sees that $\mathbb{F}_2{}^*$ has 31 element and splits into the Galois group orbit $\{1\}$ and 6 orbits by 5 elements. We need at least two orbits, so $6 = 1 + 5$ is the minimal number of roots. Many over choices give $10 = 5 + 5$ roots and thus $k = 21$ which is not minimal.