Obligatory problem set 2

April 6, 2003

Each student registered for the spring 2003 exam in MA 001 must hand in acceptable solutions to two obligatory problem sets. This is the second problem set.

Each student prepares and hands in an individual solution to his or her group-teacher or to the mathematical institute, 7th floor in the N. H. Abels building. The group-teacher then corrects and grades the solutions. Those qualifying for a passing grade (4.0/E or better) are considered accepted.

Give a mathematical justification for all answers. Give exact answers when possible.

Remember to write your full name and the number of your group on the solution you hand in.

The absolute deadline for handing in solutions is **Friday 25. April 2003 kl. 14.30**

Only applications with documentation from a doctor may give postponement on delivering in the solutions. Applications are sent to Matematisk Institutt v/ Heidi Raude (heidimr@math.uio.no)

Problem 1

Some radioactive material will decay quickly, but others continue to emit radiation for a long time. We will look at two of these materials, that we call material A and B. The radioactivity is measured in the unity curie. The material's half-life is the time spent before the radioactivity has dropped to half of its original value. At time t = 0 we start with given quantities of materials A and B.

The table shows radioactivity after N years:

Radioactivity after	10 years	25 years	50 years	100 years	10 000 years
A	400	172	43	2.6	
B	0.4				0.12

Let f(t) and g(t) be the radioactivity (in curie) in material A and B resp. after t years.

a) Plot the data for material A in a logarithmic coordinate system. (Logarithmic scale for radioactivity, linear scale for time.) Pick a straight line best suited for the data. Find the radioactivity after 75 years by reading from the plot.

b) Find a mathematical expression for f(t).

c) Find the half-life for material A.

We will now use the integrals

$$F(N) = \int_0^N f(t)dt$$

and

$$G(N) = \int_0^N g(t)dt$$

as a measure of how much the environment is damaged due to the radiation from material A and B during the first N years.

d) Given $g(t) = 0.4e^{(-\lambda t)}$ (curie) where $\lambda = 1.2 \cdot 10^{-4}$ (years^{-1}).Find the integral $G(N)$.

e) Find the integral $F(N)$ and determine which of the materials that does most damage to the environment during the first 10 000 years.

Problem 2

a) Determine when the function is 0, and find extremes for the function

$$f(x) = 2x + \frac{3}{x}, x \neq 0$$

b) Find the functions inflexion points (if they exist) and sketch the graph of the function.

Problem 3

a) Solve the integrals

i) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$

ii) $\int_1^3 x^2 e^x dx$

b) Find the solution to the differential equation with the given initial condition.

$$\frac{dN}{dt} = 8N + 2, \quad N(2) = 4$$

c) Find the limit

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

Problem 4

The size of a population of rodents change periodically. The population is at its largest every 4.th year, and is then 550 animals. The smallest population size is 150 animals. Use trigonometric functions to make a function describing the population size as a function of time.