Decision Making and Trade without Probabilities

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Abstract

What is a rational decision-maker supposed to do when facing an unfamiliar problem, where there is uncertainty but no basis for making probabilistic assessments? One answer is to use a form of expected utility theory, and assume that agents assign their own subjective probabilities to each element of the (presumably known) state space. In contrast, this paper presents a model in which agents do not form subjective probabilities over the elements of the state space, but nonetheless use new information to update their beliefs about what the elements of the state space are. This model is shown to lead to different predictions about trading behavior in a simple asset market under uncertainty. A controlled laboratory experiment tests the predictions of this model against those of expected utility theory and against the hypothesis that subjects act naïvely and non-strategically. The results suggest that a lack of subjective probabilities does

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not imply irrational or unpredictable behavior, but instead allows individuals to use both what they know and knowledge of what they do not know in their decision making.

Keywords: Decisions under uncertainty; non-expected utility; incomplete preferences; ambiguity

JEL Classifications: M41, D80, D82, D83

When someone faces a new problem, and cannot form a probabilistic assessment, what should count as a rational decision? An approach is to treat all elementary events as equally likely, but how does someone learn what the elementary events are for an unfamiliar problem? Furthermore, even if one does know the events, why is it rational to choose to assign a probability distribution (whether giving equal likelihood to each possibility or not)?

In everyday life, decision settings arise where information is fundamentally new. When a firm begins new research, its managers and even its researchers do not necessarily know what products (if any) they will eventually develop. It is hard to imagine that such a firm can form a reliable estimate of the probability that the research will succeed, or of the expected return on the research, without first knowing what the research produces. At best, a firm might consider somewhat similar research projects, and perhaps concoct a worst-case and a best-case scenario; indeed, it is common to teach MBA students that this is an appropriate way to proceed. One might ask whether we are teaching our MBA students to make decisions rationally, and if so, in what sense?

Standard economic theory has difficulty capturing such settings. Under the axioms of Savage (1954) and of Anscome and Aumann (1963), an economic agent always has enough information to act as if he or she is maximizing expected utility over a unique subjective probability distribution. Moreover, trade between agents, as in the theory of syndicates (Wilson 1968, Amershi and Stoeckenius 1983), or simply communication (Geanakoplos and Polemarchakis 1982) is often enough to imply that everyone converges to the same unique probability distribution. A critical feature of these applications of the expected utility models is that agents always know all possible states of the world, all possible outcomes, and all the mappings from states into outcome space. This assumes away the possibility that a firm can only know in vague terms what fruit new research might bear: in the theories everyone knows what outcomes are possible, and under which circumstances each outcome arises.

Recent theoretical work has described settings that do not lead agents to act as if they fabricate unique probability distributions over all uncertain events. These include models where agents are unaware of the state space (Fagin and Halpern 1988, Dekel et al. 1998), where the collection of maps between states and outcomes is only partially known (Blume et al. 2005), or where states are related to outcomes via correspondences (Stecher 2005a).[1] A general theme of this literature is that individuals do not have a partitional structure describing the states that can emerge (see also Geanokoplos (1989), Shin (1993), or Morris (1996). In these sorts of settings, agents may be able to exclude certain outcomes, even if they cannot be Savage-rational (an exception is Karni (2005)). Refining the set of possible outcomes, without making a probabilistic assessment, is viewed here as a rational (if not neoclassical) form of updating beliefs. We refer to such updating as *rational non-probabilistic belief revision.*[2]

This paper presents an experimental setting where it is possible to distinguish the predictions of expected utility maximization from both those of rational non-probabilistic belief revision and those of agents acting naïvely and non-strategically. In a simple investment setting, agents receive information in the form of bounds on expected returns, without receiving probabilistic information. The setting is designed so that trade can only occur in an expected utility framework if there is a difference in agents' attitudes toward risk or if there are different probabilistic assessments across agents; in the latter case, there are severe restrictions on the possible observable trading behaviors. If instead agents revise their beliefs rationally but non-probabilistically, there is some indeterminacy in equilibrium trading behavior, but certain actions are ruled out.

The paper proceeds as follows: the next section describes the structure of the investment setting, and gives the predicted behavior for agents who (a) maximize expected utility, (b) withhold judgment on probabilities but update beliefs about possible outcomes, or (c) act non-strategically or naïvely.

[1] A related idea occurs in the military operations research literature (Koopman 1940a,b, 1941), where agents have a lower bound and an upper bound on their estimated probability of an uncertain event; this is closely related to the ideas in Gilboa and Schmeidler (1989). To reduce the technical overhead, we focus bounds on possibilities, rather than bounds on expectations. For an alternative axiomatic derivation of imprecise probabilities, see the discussion on Axiom 2 in Saari (2005), based on the framework of Luce (1959).

[2] There is an extensive literature on belief revision that extends beyond the Bayesian or even probabilistic frameworks; see Alchourrón et al. (1985). An economic model of decision-making where beliefs are excluded is provided in Easley and Rustichini (1999).

Section 3 details our experimental procedures. Section 4 reports our results. The final section gives our conclusions.

1 Investment setting

1.1 Overview of the experimental setting

In this experiment, subjects have the opportunity to invest in and trade a financial asset whose return comes from an unknown distribution. Information on the return comes in the form of a range of possible values. A group of subjects consists of four buyers and one seller. The initial range, denoted by $(\underline{\rho}, \overline{\rho})$ is from return of -10 percent to $+50$ percent. This is common knowledge among the buyers and the seller.

The seller is initially endowed with 1000 francs, while the buyers are initially endowed with 1500 francs. At step 1, the seller decides how much of his initial endowment to invest. We denote this investment by S_0. See the first column of Figure 1.

At step 2 (column 2 of Figure 1), the seller receives updated information in the form of new upper and lower bounds on the possible return. This updated range, which we denote by (ρ', ρ''), is strictly between the original bounds of -10 percent and $+50$ percent. So $(\rho', \rho'') \subsetneq (\underline{\rho}, \overline{\rho})$.

At step 3 (column 3), the seller privately decides his reservation price, based on the updated information. At step 4 (column 4), the buyers learn the updated lower bound ρ' and the seller's investment S_0. They do not, however, learn the updated upper bound ρ''. So at this point, the seller knows that the return $\rho \in (\rho', \rho'')$, while the buyers know that $\rho \in (\rho', \overline{\rho})$.

At step 5 (column 5), the buyers bid for the asset in a first-price, sealed bid auction. They can bid up to their endowment of 1500 francs. Thus, trade occurs if the highest bid is above the seller's private reservation price, in which case the highest bidder gets the asset and pays the seller his bid. Otherwise, there is no trade, and the seller keeps the asset. The value of ρ is then announced, and all the subjects are paid accordingly.

Stage 1	*Stage 2*	*Stage 3*	*Stage 4*	*Stage 5*
Seller endowed with 1000 fr to invest	Seller receives new information on ρ	Seller sets private reservation price	Buyers learn ρ' and S_0	Buyers submit their bids in a first-price sealed bid auction
Possible return $(-0.1, 0.5)$	Possible return $(\rho', \rho'') \subsetneq (-0.1, 0.5)$		Possible return $(\rho', 0.5)$	
Seller invests S_0				Buyers can bid up to 1500 fr

Figure 1: Sequence of events in the experiment

1.2 Predicted behavior under expected utility theory

Under expected utility theory, agents act as if they assign a probability distribution to each element of the state space and maximize a utility function. Before the investment is undertaken, the support of the distribution of returns must be some subset of $(\underline{\rho}, \overline{\rho})$. Once the seller receives the revised bounds, the support of the distribution of returns that the seller uses is a subset of (ρ', ρ''). The buyers, after learning ρ', update their subjective distributions on returns, forming a new distribution whose support is a subset of $(\rho', \overline{\rho})$.

In the case where agents are risk-neutral, we have the following results:

Proposition 1.1. *A risk-neutral seller never invests if the expected return is negative and invests his entire endowment if the expected return is positive.*

Proof. Proof A direct consequence of the definition of risk-neutrality. □

An implication of Proposition 1.1 is that any seller who invests strictly between 0 and 1000 (i.e., the full endowment) would, under expected utility theory, reveal a belief that the ex ante expected return is 0. This is true irrespective of the seller's beliefs about the buyers' risk-preferences or subjective probabilities.

Proposition 1.2. *Suppose there is a consensus, atomless initial probability distribution with support* $(\underline{\rho}, \overline{\rho})$. *If agents are risk-neutral, then there will be no trade in the auction, and the buyers will never submit bids above* $(1 + \rho')S_0$.

Proof. Proof After learning the revised upper and lower bounds on the return, the seller forms a reservation price based on ρ' and ρ'', whereas the buyers only know ρ'. If a buyer submits a bid above $(1+\rho')S_0$ and the seller is risk-neutral, then trade occurs only if the bid exceeds the seller's private expectation; the latter is the expectation conditional upon all the information in the economy. That is, a bid above $S_0(1+\rho')$ wins only if it has negative expected value. Consequently, a risk-neutral buyer will only bid $S_0(1+\rho')$ or less, and this must be strictly below the risk-neutral seller's reservation price. There will therefore be no trade. □

Proposition 1.2 provides several tests for risk-neutral expected utility maximization, along with some generalizations.[3] If any buyer in any round makes a bid above ρ', then this buyer cannot simultaneously be risk-neutral and believe that the other subjects are risk-neutral. The technical restriction to atomless distributions with full support rules out situations where buyers can safely bid more than $S_0(1+\rho')$ (e.g., if every return between ρ' and the return implied by the bid has probability 0). Similarly, any seller setting a reservation price at or below the lower bound would be a violation of expected utility maximization for a risk-neutral seller.

1.3 Predicted behavior under rational non-probabilistic belief revision

Our non-probabilistic model of belief revision is based on Stecher (2005b), where agents have incomplete preferences over ambiguous objects. We use here only the assumptions from this model that are necessary to derive predictions in the current setting.

In particular, we assume that individuals who know ranges of possible returns prefer assets with unambiguously higher returns. For example, an asset with a return in $(0.1, 0.2)$ is preferred to one with a return in $(0, 0.1)$. An agent's preference is indeterminate when intervals overlap. Stecher's

[3]The no-trade prediction of Proposition 1.2 holds whenever subjects have homogeneous risk-aversion and start with a common prior. Thus, under the Harsanyi doctrine (Harsanyi 1967, 1968a,b), there should be no trade under homogeneous risk-preferences; see also Aumann (1976). When agents are heterogeneous in their risk-aversion, trade can occur, but only in highly restricted ways; in particular, the least risk-averse agent should win every auction. Similarly, when agents have diffuse priors, the agent with the highest expected return should win every auction. There was no evidence of such trading behavior in our data.

model can be shown to induce an interval ordering on the set of alternatives.[4]

A rational seller in this setting can invest anything between 0 and 1000 francs, and can set his reservation price anywhere between $(1+\rho')S_0$ and $(1+\rho'')S_0$. This is because the seller initially knows that the return can be positive, negative, or zero; if the seller does not form probabilistic beliefs, his preference for this asset is indeterminate. Similarly, his updated belief is that the value of the asset lies somewhere between $(1+\rho')S_0$ and $(1+\rho'')S_0$, but the seller cannot specify a unique expectation in this range.

The fact that the seller cannot form a unique expectation completely alters the optimal strategy for the buyers: the buyers can now rationally bid anything in the interval from $(1+\rho')S_0$ to $(1+\overline{\rho})S_0$. Recall that under the expected utility model, the seller would privately set a reservation price at the expected value of the asset, so that any buyer who submits a potentially winning bid would have a negative expected return. In the non-probabilistic setting, however, there is no unique expected value available to the seller, so buyers can justify making potentially winning bids. Moreover, since the seller can rationally set his reservation price to $(1+\rho')S_0$ (again in contrast with the expected utility setting, where the reservation price must be strictly above this lower bound), any bid below this ex post lower bound is dominated by a bid at $(1+\rho')S_0$. A buyer bidding less than this can never expect to win the asset from a rational seller, while a buyer bidding exactly this amount has some chance of purchasing the asset, and if so will purchase an asset guaranteed to make a positive return (though one that can become arbitrarily small).

The argument stated above shows the following:

Proposition 1.3. *Suppose that the agents do not form subjective beliefs and have incomplete preferences, which are increasing over non-overlapping possible payoffs. Then the seller's reservation price will belong to the interval* $[(1+\rho')S_0, (1+\rho'')S_0]$*, and the buyers' bids will belong to the interval* $[(1+\rho')S_0, (1+\overline{\rho})S_0)$.

[4]For more on interval orders, see Fishburn (1970). Under some circumstances, the semiorders of Luce (1956) also can be mapped to interval orders, so the results here can generalize to settings with complete preferences but with intransitive indifference. The approaches closest to ours are Gilboa and Schmeidler (1989) and Bewley (2002). But Gilboa and Schmeidler assume maximin preferences (that is, their agents always maximize the worst-case scenario), while Bewley assumes an inertia bias (so that agents do not trade except for something unambiguously better).

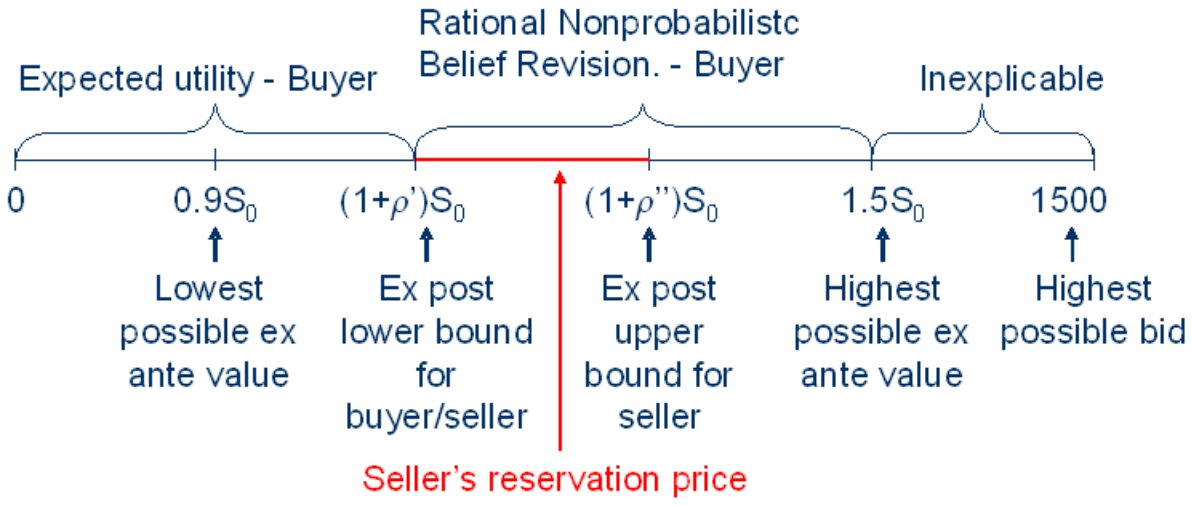


Figure 2: Range of possible bids

Remark. Proposition 1.3 indicates that trade can occur, but trade still need not occur. The indeterminacy of the seller's reservation price makes trade possible, but cannot guarantee trade.

A direct consequence of Proposition 1.3 is the following:

Corollary 1.1. *Under the assumptions of Proposition 1.3, if trade occurs, the market price will belong to the same interval as the buyers' possible bids, namely,* $[(1+\rho')S_0, (1+\overline{\rho})S_0)$.

Proof. Proof A rational seller's reservation price must be at least $(1+\rho')S_0$, so no trade can occur below this price. A rational buyer's bid must be below $(1+\overline{\rho})S_0$, since the asset is guaranteed to return less than this amount. Between the seller's lowest possible bid and the buyers' highest possible price, behavior is indeterminate, so trade is possible. □

Figure 2 presents an overview of the possible bids and possible reservation prices under each model.

1.4 Naïve and inexplicable behavior

The discussion above indicates that trade can occur if buyers and sellers are fully rational but do not specify a unique probability distribution over returns, given what they know. It is also possible that trade can occur if buyers are boundedly rational, in the sense of calculating expected returns naïvely rather than considering the seller's incentives. We consider two forms of naïve buyers. To get clear predictions, we focus attention on the cases where agents treat the return as uniformly distributed over an appropriate interval.[5]

Since the buyer knows only that the asset's value is in $((1+\rho')S_0, (1+\overline{\rho})S_0)$, a simple calculation would be to treat ρ as uniformly distributed between ρ' and $\overline{\rho}$, giving the buyer an expected value at $(1+(\rho'+\overline{\rho})/2)S_0$. On the other hand, if the buyer considers the fact that the seller knows ρ'' but does not consider the seller's strategic choice, the buyer would recognize that $\rho'' \in (\rho', \overline{\rho})$. The principle of insufficient reason would then suggest that the buyer's expectation of ρ'' is $(\rho'+\overline{\rho})/2$, and that the buyer's expectation of ρ would consequently be $(3\rho'+\overline{\rho})/4$.

Note that if the seller's behavior cannot be said to be naïve in the same way that the buyers' behaviors can. This is because, under expected utility theory, the seller optimally chooses a reservation price at the expected value of the asset, and does not benefit from choosing a different reservation price based on conjectures about the buyers' strategy.

Summing up, a naïve buyer may be expected to set a reservation price at either the midpoint of the interval of possible values, or at a corrected midpoint that adjusts for an expected value of ρ''. Bids that are outside these values are inconsistent with boundedly rational risk-neutral expected utility maximization but can be consistent with rational non-probabilistic belief revision. However, any bids above $(1+\overline{\rho})S_0 = 1.5S_0$ are inexplicable under the theories we compare.

[5] This is Laplace's "principle of insufficient reason," and is appealed to in the literature on global games, for example in Morris and Shin (2003). Keynes referred to this idea as the "principle of indifference;" the idea can be made precise by a maximum entropy argument.

2 Experimental Procedure

The experiment was run using the z-Tree software of Fischbacher (1999).

The experiment was run with 20 subjects. Each subject was grouped with four other participants and was in the same group throughout the experiment. There were 20 rounds, and the group composition remained unaltered across rounds.

In each round, one subject in each group was randomly selected to be the seller; other subjects in the group were designated as buyers. The seller received 1000 francs (currency for the experiment) as an initial endowment. He was reminded that the maximum possible return on the investment in every round was 0.5 and that maximum possible loss was -0.1; these values had also been presented in the instructions, which were read aloud to all the subjects before the start of the experiment. After having been reminded of the possible returns and of his endowment, the seller was required to decide how much of the 1000 francs to invest. We denote this investment choice S_0.

After the seller made his investment decision, he privately saw on his computer screen a revised minimum return and a revised maximum return, along with the corresponding range of values in francs. The revised minimum return ρ' was always at least as high as the initial minimum return, and the revised maximum return ρ'' was always at most as high as the initial maximum return. After seeing the revised returns, the seller was told to set the minimum selling price for his asset.

The treatment of the buyers was as follows: in each round, each buyer received 1500 francs as an endowment for that round. The buyers were told the investment made by the seller S_0 and the revised minimum return ρ', and were reminded of the initial maximum return of 0.5. The buyers were also shown the corresponding levels of possible payoffs in francs. The buyers then entered their bids privately on their computers.

After all the buyers submitted their bids, the computer would select the highest bid and compare it to the seller's minimum selling price. If this highest bid was at least as high as the seller's minimum price, trade occurred; otherwise, trade did not occur. When trade occurred, the highest bidder received the asset, and the seller received the winning bid. In case of a tie, one buyer was randomly selected to be the winning bidder.

The true return was determined and payoffs were calculated as follows: If trade did not occur, seller's payoff was set to 1000 francs − the seller's investment + the actual payoff from the asset − the minimum possible ending balance (900 francs):

$$\text{Seller's Payoff if No Trade} = (1000 - S_0) + (1+\rho)S_0 - 900 = 100 + \rho S_0. \quad (1)$$

If trade occurred, seller's payoff was set to 1,000 francs − the seller's investment + the selling price − the minimum possible ending balance (900 francs):

$$\text{Seller's Payoff if Trade} = (1000 - S_0) + \text{ Highest Bid } - 900 = 100 + \text{ Highest Bid } - S_0. \quad (2)$$

Payoffs to the buyers were calculated analogously: if trade did not occur, each buyer's payoff was set to 1500 francs − the minimum possible ending balance (900 francs). If trade did occur, the payoff to the winning buyer was 1,500 francs − the purchase price + the actual payoff from the asset − the minimum possible ending balance (900 francs); the payoffs to the other buyers was 1,500 francs − the minimum possible ending balance (900 francs) = 600 francs:

$$\begin{aligned}\text{Highest Bidder's Payoff if Trade} &= 1500 - \text{ Highest Bid } + (1+\rho)S_0 - 900 \\ &= 600 - \text{ Highest Bid } + (1+\rho)S_0. \quad (3)\end{aligned}$$

$$\text{Other Buyers' Payoffs if Trade} = \text{All Buyers' Payoffs if No Trade} = 1500 - 900 = 600. \quad (4)$$

Note that the minimum possible payoff to the seller assumes that the seller does not sell the asset for a return below the ex ante lower bound of −10%. Analogously, the minimum possible payoff to the buyers assumes the buyers do not bid above the ex ante upper bound of 50%. The earnings from each round were put into a bank account and were unavailable for use in trade in subsequent rounds. At the end of the experiment, each subject's total earnings were converted to Canadian dollars, at the exchange rate of 4 francs to 1¢.

An experimenter read the instructions aloud to the subjects, while the subjects followed their own copies of the instructions. The experimenter then asked the subjects to answer written questions about the experiment. The CIRANO Research Institute in Montreal recruited the subjects and ran the experiment.

We sought in our experiment to insure that in some sense the distributions of revised bounds and actual returns were unknown to the subject and to the experimenter. To achieve this result, we

used quantum bits (qbits). As noted in Calude (2004) (p. 10) and Calude and Svozil (2006) (p. 6), real-world quantum random number generators do not typically produce qbits with the idealized probability of 1/2, even when subjected to unbiasing algorithms. Moreover, Calude and Svozil argue that no sequence of qbits is Turing-computable.

We generated the revised bounds and actual returns using qbits. The procedure works as follows: each qbit was determined by firing a photon at a semi-transparent mirror; a device then attempted to determine whether the photon had passed through the barrier. If the photon was detected, a qbit was set equal to 1; otherwise, it was set equal to zero. A string of 8 qbits constructed in this way was converted to a real number. [6]

3 Results

The experiment produced 71 rounds across the four groups of subjects (referred to henceforth as group-rounds).[7] The sellers' average investment was typically not all or nothing: only one group-round (1.4%) had nothing invested, while in 14 group-rounds (19.7%), the sellers invested everything.[8] Thus, 79.9% of investment decisions were inconsistent with risk-neutral expected utility maximization but consistent with rational non-probabilistic decision-making.

Among the buyers, 32.9% of the bids were at or below the ex post lower bound $(1+\rho')S_0$, and were thus in a range consistent with expected utility theory. In the last eight rounds of the experiments,

[6] Although the distributions we used are uncomputable (by the arguments in Calude and in Calude and Svozil), the sampling distribution we actually employed is difficult to distinguish from a uniform distribution (plots and tests available upon request). In our analysis, we assess the degree to which the data are consistent with a subject's assuming a uniform distribution.

[7] A data entry error in Round 12 for Group 1 produced a revised lower bound on the return that exceeded the revised upper bound. Consequently, the last nine rounds for Group 1 were excluded from the analysis. Inclusion of these data do not substantively change the results.

[8] The average investment was 606.55 francs. It was over 500 francs in all but the first round (where average investment was 444.75 francs), and under 900 francs in all but the last round (where it was 923.75), and had a general upward trend through the experiment. A simple OLS regression of average contribution on the round number had an intercept of 553.25 francs and a slope of 13.12 francs, indicating that average investment typically rose 13.12 francs per round. The R^2 was 0.34.

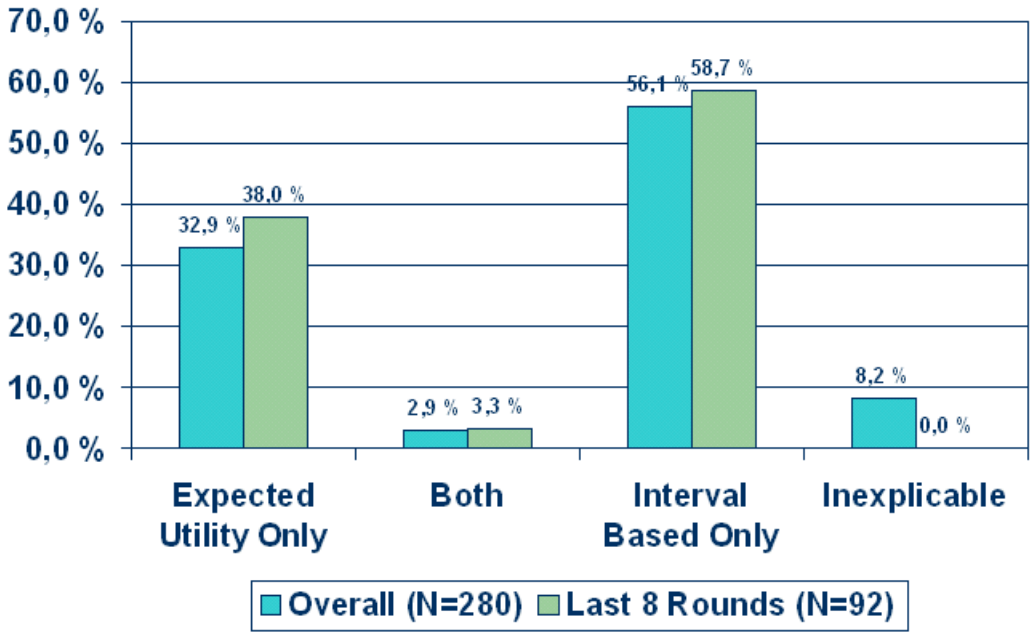


Figure 3: Frequency of bids consistent with each theory

this increased to 38.0%. Only 2.9% of the bids were exactly at the ex post lower bound (the only point consistent with both theories); this did not change significantly in the last eight rounds of the experiment. We observed 56.1% of the bids between the ex post lower bound $(1+\rho')S_0$ and the ex ante upper bound $1.5S_0$, consistent with rational non-probabilistic belief revision. The remaining bids were inexplicable (above the ex ante upper bound). All of the inexplicable bids occurred during the first 12 rounds, and 43.5% of these bids were due to a single subject. See Figure 3.

We did not find strong support for the models of boundedly rational expected utility maximization described in Subsection 1.4. Recall that a naïve buyer who considers only his interval of possible returns and behaves according to Laplace's principle of insufficient reason would have an expected value of $(1+(\rho'+\overline{\rho})/2)S_0$, while a naïve buyer who considers the seller's upper bound but not the seller's strategy would have an expected value of $(1+(3\rho'+\overline{\rho})/4)S_0$. Only four bids (1.4%) were within a rounding error of either of these naïve expectations. Even if we consider all bids weakly between either of these naïve expectations as consistent with boundedly rational expected utility maximization, we have only 18.6% of the bids consistent with the theory; this represents 33.1% of those explained by non-probabilistic rational belief revision. However, there were two

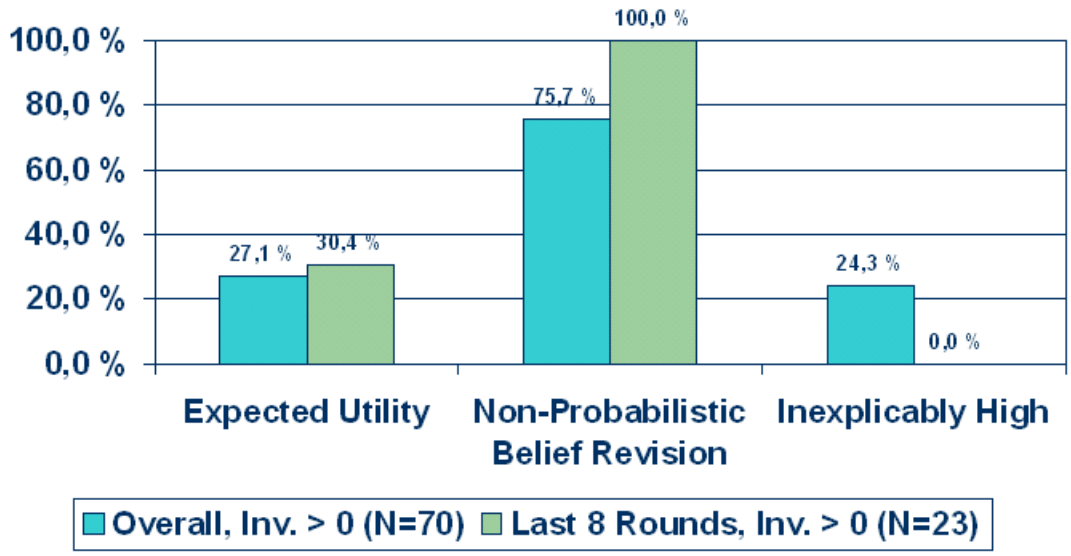


Figure 4: Frequency of prices consistent with each theory

subjects—both in Group 2—who bid weakly between the naïve expectations more than half the time (52.9% of the bids for Subject 7 and 58.8% of the bids for Subject 11).

The equilibrium market behavior was as follows: in 27.1% of the group-rounds, there was no trade, which is consistent with expected utility maximization. This increased to just over 30% in the last eight rounds. In 75.7% of the group-rounds, market prices were consistent with equilibria under rational non-probabilistic belief revision; by the last eight rounds, this was true of every market price. There were some inexplicably high market prices during the first 12 rounds, but none in the last eight rounds of the experiment. See Figure 4.[9]

4 Conclusion

This experiment provides insight into how people behave when they have enough information about a problem to know what is possible but insufficient data to form meaningful probabilities. We contrast the predictions of expected utility theory, which posits that people will form beliefs

[9]These translated into earnings as follows: an average subject earned C$26.90, with earnings ranging from C$20.72 to C$29.75.

about probabilities, with those of belief-free models.

In our experiment, trade should not have occurred in any round if subjects were expected utility maximizers,[10] while under rational non-probabilistic belief revision, trade is possible though not a given. The range of bids consistent with expected utility theory does not overlap with those consistent with rational non-probabilistic belief revision. Additionally, there were some possible bids that would be inconsistent with either model, and some focal bids suggestive of boundedly rational decision-making. Thus, our set-up gave us the ability to make clear distinctions between the models we were testing and to reject both models.

Approximately 30% of our buyers made bids consistent only with expected utility theory, while almost 60% were consistent only with non-probabilistic belief revision. There was not strong support for a naïve model of boundedly rational expected utility maximization, but in the early rounds there were some bids that were inexplicable under either theory.

The results point to an explanation for the findings of Chen and Plott (1998), who study behavior in first-price sealed bid auction experiments. Chen and Plott find that their subjects behave consistently with the predictions of game theory, and thus seem to be rational, with the sole exception being their formation of beliefs. They attribute subjects' behavior to a form of "limited rationality,"[11] and suggest that the answer may lie in a belief-free model. Our results are largely consistent with those of Chen and Plott, and provide evidence that rational non-probabilistic models can explain behavior that remains paradoxical under traditional expected utility theory.

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[10] This does not mean subjects would not participate under expected utility theory. In each round, the amount of subject participation is the same: subjects enter bids or asking prices; the only difference is in what values subjects are predicted to enter.

[11] Their notion of limited rationality is different from the one in the literature, e.g. Lipman (1991).

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Instructions

This is a computerized experiment in the economics of decision-making. This experiment will last approximately one hour.

To make a profit, you will trade a financial asset, which may lose or gain money. At the end of the experiment, we will pay you a show-up fee of $10 plus any profits you will have made. We will pay you in cash, privately, immediately after the experiment ends. You must sign and date a compensation receipt form in order to receive your payment.

There are some rules you must follow:

(1) Do not talk to others at any time during the experiment.
(2) If you have any questions during the experiment, please raise your hand. An experimenter will come to your location and answer your questions.

You are free to withdraw from the experiment at any time, for any reason. If you choose to do so, please raise your hand. In this case, you will be paid your $10 show-up fee as you leave.

Grouping of the Participants

The computer will assign you to a group with four other participants. There are 20 rounds of this experiment. You will be in the same group through all 20 rounds.

At the beginning of each round, the computer will select one person from your group as a seller for that round; the other four members of your group will be buyers for that round. In the next round, the computer may select any member of your group as the new seller. *If you are the seller, you will be given 1,000 francs of currency at the beginning of the round. Otherwise, if you are a buyer, you will be given 1,500 francs of currency at the beginning of the round.* The computer will tell you whether you are a buyer or a seller.

Seller's Investment Decision

If you are a seller, you will see the following table on your screen:

Minimum Return	-10%
Maximum Return	+50%
Your Investment	FR_____

The minimum possible return will *always* be -10% of your investment. That is, the worst that could happen in any round is that you could lose 10% of your investment. The maximum possible return will always be 50%. That is, the best that could happen in any round is that you could gain 50% of what you invest.

You may invest as much of the 1,000 francs as you like. The third line in the above table shows where you will enter the amount you wish to invest.

Example 1

Suppose you decide to invest 500 francs. Then your minimum possible payoff is 450 francs. This is calculated as follows:

	Your investment	500 francs
Less:	10% of your investment	50 francs
	Minimum payoff	450 francs

Your maximum possible payoff is 750 francs. This is calculated as follows:

	Your investment	500 francs
Add:	50% of your investment	250 francs
	Maximum payoff	750 francs

Remember that the true return will be somewhere between the minimum and maximum returns, so your true payoff will be between the minimum and maximum payoffs.

Seller's Minimum Price

If you are a seller, then after you enter your investment, you will see the following table on your screen. Only you will see this table.

Revised Minimum Return (in percent)	
Revised Minimum Payoff (in francs)	
Revised Maximum Return (in percent)	
Revised Maximum Payoff (in francs)	
Minimum price for which you are willing to sell your asset	FR_____

This table shows your updated minimum and maximum payoffs. Initially, you were told that the return would lie somewhere between -10% and +50%. When you see the above screen, you will know that the true return will lie somewhere between the revised minimum return and the revised maximum return. In the fifth row of the above table, you will enter the minimum price for which you are willing to sell the asset.

Example 2

Suppose you decide to invest 100 francs, and that you later learn that the revised minimum return is -5%. Then revised minimum payoff is 95 francs. This is calculated as follows:

	Your investment	100 francs
Less:	5% of your investment	5 francs
	Revised minimum payoff	95 francs

If the revised maximum return is 30%, then the revised maximum payoff is 130 francs. This is calculated as follows:

	Your investment	100 francs
Add:	30% of your investment	30 francs
	Revised maximum payoff	130 francs

The screen you will see in this case is:

Revised Minimum Return (in percent)	-5%
Revised Minimum Payoff (in francs)	95 francs
Revised Maximum Return (in percent)	30%
Revised Maximum Payoff (in francs)	130 francs
Minimum price for which you are willing to sell your asset	FR_____

Buyer's Bidding Decision

If you are a buyer, you will see the following table on your screen after the seller privately sees the revised information and privately sets the minimum selling price:

Seller's Investment (in francs)	
Minimum Return (in percent)	
Minimum Payoff (in francs)	
Maximum Return (in percent)	+50%
Maximum Payoff (in francs)	
Your Bid	FR _____

The first line in the above table states how many francs the seller has invested. The second line is the *seller's revised minimum return.* That is, you will learn the same minimum possible return that the seller learns. The third line converts this minimum return into a payoff.

Line four tells you the *initial* maximum return—that is, what the seller was told, before making the investment, that the highest return could be. You will not learn the seller's revised maximum return. *The initial maximum return is always 50%.* Line five converts this 50% into a payoff.

On the last line of the above table, you will enter what you are willing to pay for the asset. You may enter any amount of your 1,500 francs for that round.

Example 3:
Suppose you see the following numbers on your screen:

Seller's Investment (in francs)	1000 francs
Minimum Return (in percent)	5%
Minimum Payoff (in francs)	1050 francs

Maximum Return (in percent)	50%
Maximum Payoff (in francs)	1500 francs
Your Bid (in francs)	FR _____

The revised minimum payoff of 1050 francs is calculated as:

	Seller's investment	1000	francs
Add:	5% of the investment	50	francs
	Revised minimum payoff	1050	francs

The maximum payoff is calculated as

	Seller's investment	1000	francs
Add:	50% of the investment	500	francs
	Maximum payoff	1500	francs

How the market price will be determined

After all four buyers in your group submit their bids, the computer decides whether trade will occur. If the highest bid is at least the minimum price the seller will accept, then trade will occur; otherwise, no trade will occur.

It trade occurs, the asset will be sold to the highest bidder. If two or more buyers have this highest bid, then the computer will randomly select one of these buyers and sell the asset to the selected buyer.

The computer will then determine the return and the payoff. This will be between the revised minimum and the revised maximum.

Determination of Profits

After determining the true payoff, the computer will calculate the profit for all the members in your group. If you are a buyer, you will see the following screen:

Trade occurred / did not occur	
Actual return (in percent)	
Actual payoff (in francs)	
Your bid (in francs)	
Highest bid (in francs)	

Initial balance (in francs)	1500 francs
Less: Your bid, if you bought the asset (in francs)	

Add: Actual payoff, if you bought the asset (in francs)	
Less: Smallest possible ending balance (in francs)	900 francs
Net profit for this round (in francs)	
Total profit to date	

If you are a seller, you will see the following screen:

Trade occurred / did not occur	
Actual return (in percent)	
Actual payoff (in francs)	
Your minimum acceptable selling price (in francs)	
Highest bid (in francs)	

Initial balance (in francs)	1000 francs
Less: Your investment (in francs)	
Add: Actual payoff, if trade did not occur (in francs)	
Add: Highest bid, if trade occurred (in francs)	
Less: Smallest possible ending balance (in francs)	900 francs
Net profit for this round (in francs)	
Total profit to date	

Profits are determined in the following way:

If you are the seller and you do not sell the asset, your profit will be 1000 francs – francs invested + the actual payoff from the asset – the minimum possible ending balance (900 francs).

If you are the seller and you do sell the asset, your profit will be 1,000 francs – francs invested + selling price – the minimum possible ending balance (900 francs).

If you are a buyer and you do not purchase the asset, your profit will be 1,500 francs – the minimum possible ending balance (900 francs) = 600 francs.

If you are a buyer and you do purchase the asset, your profit will be 1,500 francs – your purchase price + the actual payoff from the asset – the minimum possible ending balance (900 francs).

Example 4

Suppose the seller's investment is 500 francs and the screen for a buyer shows:

Trade occurred	
Actual return (in percent)	30 %
Actual payoff (in francs)	650 francs
Your bid (in francs)	600 francs
Highest bid (in francs)	600 francs

Initial balance (in francs)	1500 francs
Less: Your bid, if you bought the asset (in francs)	600 francs
Add: Actual payoff, if you bought the asset (in francs)	650 francs
Less: Smallest possible ending balance (in francs)	900 francs
Net profit for this round (in francs)	650 francs
Total profits to date	

Since this buyer made the highest bid, he or she bought the asset. The bottom row shows the total profits from this round and all previous rounds. We have left this blank in the example, but you will see this figure during the experiment.

The seller's screen in this case will appear as:

Trade occurred	
Actual return (in percent)	30 %
Actual payoff (in francs)	650 francs
Your minimum acceptable selling price (in francs)	580 francs
Highest bid (in francs)	600 francs

Initial balance (in francs)	1000 francs
Less: Your investment (in francs)	500 francs
Add: Actual payoff, if trade did not occur (in francs)	
Add: Highest bid, if trade occurred (in francs)	600 francs
Less: Smallest possible ending balance (in francs)	900 francs

Net profit for this round (in francs)	500 francs
Total profits to date	

The bottom row shows the total profits from this round and all previous rounds. We have left this blank in the example, but you will see this figure during the experiment.

Example 5

Suppose the seller's investment is 100 francs and the screen for a buyer shows:

Trade did not occur	
Actual return (in percent)	10%
Actual payoff (in francs)	110 francs
Your bid (in francs)	102 francs
Highest bid (in francs)	115 francs

Initial balance (in francs)	1500 francs
Less: Your bid, if you bought the asset (in francs)	
Add: Actual payoff, if you bought the asset (in francs)	
Less: Smallest possible ending balance (in francs)	900 francs
Net profit for this round (in francs)	600 francs
Total profits to date	

Since this buyer did not make the highest bid, the buyer only had the smallest possible ending balance of 900 francs deducted, leaving him or her with 600 francs for the round. The bottom row shows the total profits from this round and all previous rounds. We have left this blank in the example, but you will see this figure during the experiment.

The seller's screen in this case will appear as:

Trade did not occur	
Actual return (in percent)	10%
Actual payoff (in francs)	110 francs
Your minimum acceptable selling price (in francs)	120 francs
Highest bid (in francs)	115 francs

Initial balance (in francs)	1000 francs

Less: Your investment (in francs)	100 francs
Add: Actual payoff, if trade did not occur (in francs)	110 francs
Add: Highest bid, if trade occurred (in francs)	
Less: Smallest possible ending balance (in francs)	900 francs
Net profit for this round (in francs)	110 francs
Total Profits to date	

Since the highest bid was less than the minimum acceptable selling price, trade did not occur. The bottom row shows the total profits from this round and all previous rounds. We have left this blank in the example, but you will see this figure during the experiment.

What You Will Be Paid

The profit you accumulate in each round (after deducting the minimum possible payoff of 900 francs) will not be available to you for trading in subsequent rounds. However, your profits from each round will be totaled, and paid to you *in dollars* at the end of the experiment, at the rate of 4 francs / 1 cent.

Questions

Below, please write down your answers to the following questions. In a few minutes, an experimenter will review the correct answers with you.

1. You are the seller. If you invest 850 francs, what is the maximum amount you can lose, and what is the maximum amount you can gain?
2. You are the seller, and your investment is 200 francs. The revised minimum return is -5%, and the revised maximum return is 30%. What are the lowest and highest possible payoffs?
3. You are the buyer, and you learn that the seller invested 600 francs and the (revised) minimum return is -5%. What is the maximum possible payoff?
4. The minimum price for which seller is willing to sell the asset is 700 francs, and highest bid is 689 francs. Will trade occur?
5. Suppose trade does not occur, the seller invested 400 francs, and the actual payoff is 380 francs. What is the seller's net profit, and what is each buyer's net profit?
6. Suppose trade occurs, and the seller has invested 600 francs. The highest bid is 650 francs, and the actual payoff is 670 francs. What are the payoffs to the seller, the buyer making the highest bid, and the other buyers?