

NEUTRINO OSCILLATIONS
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Description: Offer an overview of the neutrino oscillations theory as of today, and provide an insight into the experimental aspects of this physical theory.

This project thesis was carried out under guidance of Professor Kjell Mork.

Abstract

The purpose of this project thesis is to offer an overview of the theory behind neutrino oscillations, and review empirical data from numerous neutrino detection experiments conducted over the world. The experimental setups and technical data of the experiments are also briefly reviewed. Best-fits for the important mixing angle and mass-parameters are presented and discussed. It is clear that compelling evidence exist of neutrino oscillations as a real phenomena.

Preface

This project thesis is organized as follows. In chapter 1, background and relevant history concerning the neutrino is provided to offer some insight in the neutrino oscillation idea. Chapter 2 deals with neutrinos found in space, and characteristics of solar, atmospheric and supernova neutrinos are presented. The physics of reactor and accelerator neutrinos is accounted for in this chapter. Also, the possibility that neutrinos contribute significantly to dark matter is briefly discussed. Next, standard neutrino oscillation theory is accounted for in chapter 3. The analysis includes two-flavor analysis, three-flavor analysis, matter-induced effects and the question of CPT -invariance. An alternative explanation to neutrino oscillations is also mentioned. In chapter 4, an overview of the most significant neutrino experiments performed is given. A description of the detectors, key idea behind how to find best-fit values for parameters, and results are listed for each experiment. Finally, a conclusion of this project thesis is given in chapter 5. References are listed at the end.

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Chapter 1

The Neutrino

The neutrino is a tiny particle with virtually no mass. It is the lightest member of the lepton family, and therefore related to the electron, muon and tau particles, but it possesses no electric charge. Hence "neutrino" is a very fitting name, since it means "little neutral one". This elementary particle is emitted in weak processes, such as radioactive β decay, and can only be absorbed by the same type of weak interaction. The discovery of the neutrino is actually linked to the β decay. A series of experiments in 1930 revealed that the electron in the β decay (as it was thought to be at the time)

$$n \rightarrow p + e^-$$

displayed a variation in kinetic energy, though the final energy should be set by the masses of the parent and daughter nuclei. This was such a dramatic result that even abandoning the conservation of energy was considered an option [1]. Fortunately, a different solution to the problem was presented by Wolfgang Pauli. For if an additional particle was emitted with the electron during the β decay, it could carry off the missing energy. It had to be extremely light and electrically neutral, due to conservation of charge, and Pauli began to work on a theory for β decay where this "little neutral one" was taken into account. This proved successful. The correct β decay process is then

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Later, in the mid-1950s, direct experimental verification of the neutrinos existence was finally at hand. A big part of the problem with detecting neutrinos had been that it did not interact enough to leave any collision-trace, and it was unaffected by electrical forces. But in 1956 the two physicists Cowan and Reines set up a large water tank, with steel shielding to prevent anything but neutrinos to enter the water tank, and looked for the reaction [2]

$$\bar{\nu}_e + p \rightarrow n + e^+$$

The results were decisive. Furthermore, later experiments in 1962 [1] showed that there was a second kind of neutrino involved in decay processes, making it necessary to distinguish between the electron-neutrino ν_e and the muon-neutrino ν_μ . The existence of a third type of neutrino, the tau-neutrino ν_τ , was established at Fermilab as late as 2000, although indirect evidence had been found earlier [2].

The neutrino then comes in three different flavors, each with an upper mass limit as illustrated in table 1.1 with data from [3], and each with its own antiparticle. [1, 2]

Table 1.1: Neutrino mass overview found in [3].

Neutrino type	Upper mass limit
ν_e	3.0 eV
ν_μ	0.19 MeV
ν_τ	18 MeV

Neutrinos are today famous for their peculiar oscillating behaviour. The whole theory emerged after years of confusion over results from the neutrino detector Homestake in the late 50's. This first of solar neutrino experiments was designed to capture neutrinos originating from the sun, where they are formed as biproducts of fusion reactions. A problem soon became evident when roughly two thirds of the neutrinos expected from the Standard Solar Model (SSM) [4] turned out to be missing. Later experiments confirmed this deviation from the prediction of SSM. To discard the SSM would be a dramatic move, since it is very successful in determining other physical parameters of the sun [5].

In order to protect the SSM, the theory of neutrino oscillation was proposed. This states, in short, that the neutrinos produced in the sun's core change identity to another flavor while in transit to Earth. Throughout the 90's a series of advanced neutrino detector experiments were performed, which data pointed to that oscillations were actually taking place. Neutrinos were simply missing and the oscillation idea seemed plausible: if, say, an electron-neutrino traveled from sun to Earth and suddenly changed identity to a tau-neutrino, it would not be registred by a detector expecting to find an electron-neutrino. This would explain the absence of neutrino flux compared to the SSM. The following chapters will investigate this neutrino oscillation theory.

Chapter 2

Neutrinos in different scenarios

To provide a better understanding of the role neutrinos are playing in cosmos, this section describes how neutrinos behave and in what shape and context they might appear throughout space and here on Earth.

2.1 Atmospheric neutrinos

Atmospheric neutrinos are produced when cosmic rays enter the Earth's upper atmosphere, collide with nuclei, and create decaying pions. The pion disintegrates by the process

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (2.1)$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad (2.2)$$

while the muon decays further by the reactions

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \quad (2.3)$$

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e. \quad (2.4)$$

It is easily seen from equations 2.1-2.4 that the expected ratio between the number of produced muon-neutrinos and electron-neutrinos is 2. This ratio is not exact due to correcting effects such as other disintegrating hadrons, for instance kaons, and long-lived muons that do not decay before they hit Earth, as a result of time dilation. The uncertainty in the flux ratio is estimated to roughly 5% [6, 7]. However, the experimental data of this scenario is such that the neutrino oscillations theory is strengthened. Though the predicted ratio of produced ν_μ and ν_e is roughly 2, the ratio between data and prediction is [8]

$$\frac{(\nu_\mu/\nu_e)_{\text{data}}}{(\nu_\mu/\nu_e)_{\text{theory}}} = 0.63 \pm 0.03 \pm 0.05,$$

when it should be 1. The ratio uncertainty is given as $\pm_{\text{stat}} \pm_{\text{syst}}$, as in the rest of this document. We see that the 5% uncertainty in flux ratio isn't enough to justify this deviation. At first sight, it seems that some of the produced ν_μ simply vanish, while there is an abundance of ν_e compared to prediction. This could be another argument for neutrino oscillations.

Another strong implication of neutrino oscillations is the measured up/down asymmetry for atmospheric neutrinos. Cosmic rays that are responsible for the pion decay are deflected by the Earth's magnetic field, randomizing in a fashion that would suggest that the neutrino flux at the Earth's surface would be the same for equal zenith angles, as illustrated in figure 2.1, taken from [6].

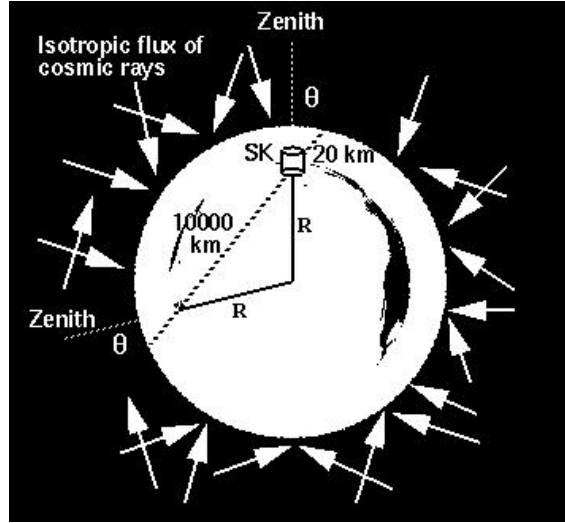


Figure 2.1: The zenith angle θ . [6]

One would then expect to detect the same amount of neutrinos whether they are coming "up" or "down". However, experimental data shows a deviation from what theory predicts also in this case. Let Φ_{up} be the measured neutrino flux from the defined "up" direction (the 10000 km distance in figure 2.1) and Φ_{down} the corresponding neutrino flux from the "down" direction (the 20 km distance in figure 2.1). Then a measure for the up/down asymmetry would be

$$\lambda = \frac{\Phi_{up} - \Phi_{down}}{\Phi_{up} + \Phi_{down}}$$

If the flux was independent of the distance traveled by the neutrinos, then λ should be zero. However, it is not. There is a significant asymmetry displayed in the flux ratio, quantitatively given by $\lambda = 0.296 \pm 0.048 \pm 0.01$ [6]. This asymmetry is displayed in figure 2.2 found in [8], and is particularly relevant for the ν_μ . The high-energy ν_e actually show a consistency of zero asymmetry, as first predicted. The asymmetry of the ν_μ could be explained by neutrino oscillations, since the greater distance a neutrino travels, the higher the probability for an oscillation to occur. In addition, the fact that neutrinos passing through Earth must interact with matter before they reach any detector, could be of significance for the oscillation probability, the so-called MSW-effect (see section 3.2). The exact probability amplitude dependence upon parameters such as distance traveled will be treated in chapter 3. It is seen from figure 2.2 that the flux at $\cos \theta = 1$ is larger than at $\cos \theta = -1$, which would be consistent since the neutrinos at $\theta = 0$ have traveled a shorter distance than those at $\theta = \pi$.

These results imply that the two-flavor oscillations $\nu_\mu \rightarrow \nu_\tau$ are dominant for atmospheric

neutrinos, since there was no significant asymmetry for the ν_e -events. The results for mixing angle and mass square difference parameters for atmospheric neutrinos are treated in chapter 4 and summarized in chapter 5.

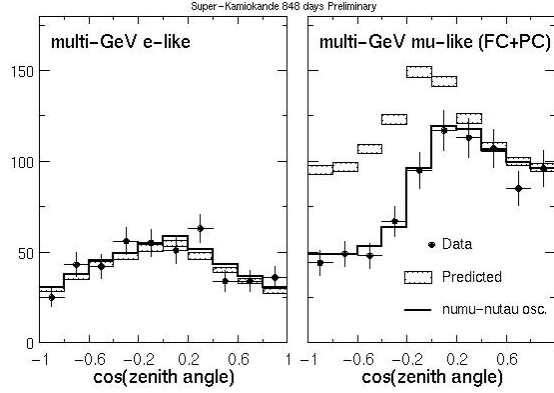


Figure 2.2: Asymmetry in flux distribution as a function of zenith angle. Number of events is plotted against $\cos \theta$. Data is acquired from the Super-Kamiokande detector during a period of 848 days [8].

2.2 Solar neutrinos

Neutrinos are produced in the sun's interior as a result of the β -decay reactions taking place. The vast majority of neutrinos produced in this fashion escapes from the sun's surface without interacting with any matter in the layers of the sun, due to its small interaction cross section. The flux of solar neutrinos is of the order $10^{11} \text{ cm}^{-2} \text{ s}^{-1}$ at the Earth's surface [9], and the dominating processes that produce neutrinos in the sun are shown in table 2.1 [3].

Table 2.1: Solar neutrino energy overview [3].

Reaction	Upper energy limit E_ν in MeV
$p + p \rightarrow d + e^+ + \nu_e$	0.42
$p + p + e^- \rightarrow d + \nu_e$	1.44
${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$	0.86
${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$	14.6

Figure 2.3 taken from [10] shows the energy spectrum of solar neutrinos according to the currently preferred standard solar model known as BP04. This model operates with key parameters such as radius and mass of the convective zone equal to $0.715 R_\odot$ and $0.024 M_\odot$, a central temperature of $15.72 \times 10^6 \text{ K}$, helium abundance of 0.640 and heavy element to hydrogen ratio by mass equal to $Z/X = 0.0229$ [4].

There is a number of other modified solar models which offer a flux prediction of solar neutrinos, as illustrated in table 2.2, found in [4]. The data is presented to give an impression of

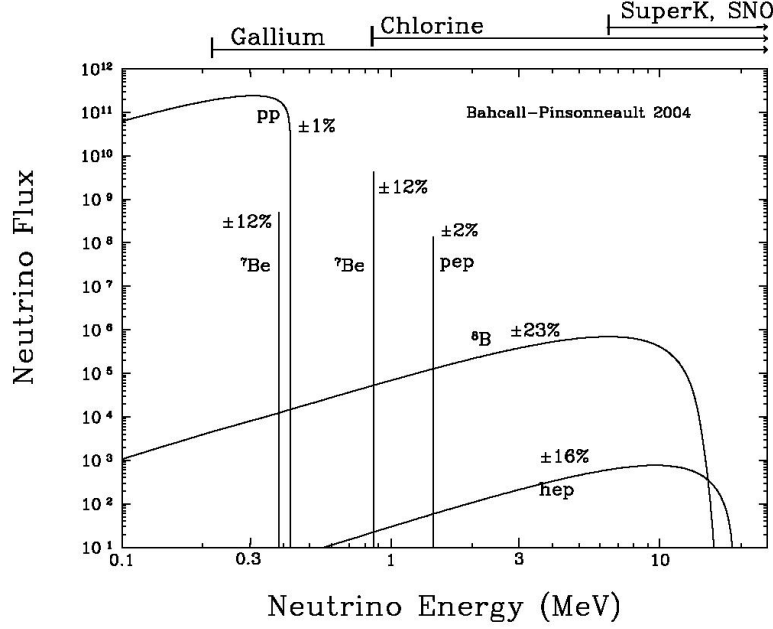


Figure 2.3: Solar neutrino energy spectrum. The flux of the neutrino sources with continuous energies is given as number of neutrinos $\text{cm}^{-2} \text{s}^{-1} \text{MeV}^{-1}$, while the line sources are given as number of neutrinos $\text{cm}^{-2} \text{s}^{-1}$. Both fluxes are at the Earth's surface. The theoretical uncertainties are shown as percentages [10].

the variations in these models.

Table 2.2: Neutrino flux predictions from various solar models found in [4]. The fluxes are given in units of 10^{10} (pp), 10^9 (${}^7\text{Be}$), 10^8 (pep , ${}^{13}\text{N}$, ${}^{15}\text{O}$), 10^6 (${}^8\text{B}$, ${}^{17}\text{F}$) and 10^3 (hep) $\text{cm}^{-2} \text{s}^{-1}$. Capture rates in SNU (see introduction to chapter 4) for chlorine and gallium solar neutrino experiments are given in the last two rows, disregarding oscillations.

Source	BP04	BP04+	BP00	Nucl.	EOS	Comp.	${}^{14}\text{N}$
pp	5.94 (1 ± 0.01)	5.99	5.95	5.94	5.95	6.00	5.98
pep	1.40 (1 ± 0.02)	1.42	1.40	1.40	1.40	1.42	1.42
hep	7.88 (1 ± 0.16)	8.04	9.24	7.88	9.23	9.44	7.93
${}^7\text{Be}$	4.86 (1 ± 0.12)	4.65	4.77	4.84	4.79	4.56	4.86
${}^8\text{B}$	5.82 (1 ± 0.23)	5.28	5.05	5.79	5.08	4.62	5.77
${}^{13}\text{N}$	5.71 ($1^{+0.37}_{-0.35}$)	4.06	5.48	5.69	5.51	3.88	3.23
${}^{15}\text{O}$	5.03 ($1^{+0.43}_{-0.39}$)	3.54	4.80	5.01	4.82	3.36	2.54
${}^{17}\text{F}$	5.91 (1 ± 0.44)	3.97	5.63	5.88	5.66	3.77	5.85
Cl	8.50 (± 1.80)	7.70	7.60	8.50	7.60	6.90	8.20
Ga	131 (1^{+12}_{-10})	126	128	130	129	123	127

Earth's matter density will influence the oscillation probability of the incoming neutrinos by

the so-called MSW effect (see section 3.2). During day and night there will then be a difference in survival probability for a given flavor, dependent on how far the neutrino has to travel through Earth. Also, there is a seasonal variation of the neutrino flux from the sun, as a result of the eccentricity of the Earth's orbit [7].

A full treatment of the experimental results of solar neutrino parameters is given in chapter 4 and summarized in chapter 5.

2.3 Supernova neutrinos

A supernova is practically speaking the death of a giant star. It is truly a titanic event, releasing incredible amounts of energy. Most of this energy is emitted through neutrinos, and on rare occasions a few of these energetic neutrinos find their way to Earth. It is interesting to study how these neutrinos impact on Earth, and find ways to distinguish them from atmospheric and solar neutrinos in detectors. If the neutrino oscillation theory is non-unique for the the latter, one should expect to find evidence of this behaviour from cosmic events such as supernovae.

The supernova source of neutrinos is rather different from its atmospheric and solar relatives with respect to neutrino energies and path length traveled from the source to Earth. The baseline length L (distance from source to detector) is of magnitude light years, while experiments conducted here on Earth are $\sim 10^2$ km at most. Also, supernova neutrinos are in general much more energetic than solar neutrinos, often with energies as high as 20 MeV. [11]. Bright supernovae that can be seen with the naked eye occur very rarely, the last two took place in 1987 and 1604 [11]. However, supernova events in general are not so rare, several are observed each year, although very distant. The observation of a neutrino flux from a distant supernova does not only depend on advanced equipment and correct experimental setups, but also a great amount of luck, since observable neutrino fluxes are estimated to occur during a period of $\sim$ one minute per century [12].

But in 1987, a small number of neutrinos originating from a distant supernova known as SN 1987A were detected at the Kamiokande observatory. The sighting of the supernova was followed by a search at the Kamiokande detector for a three-day period. One looked for the electron scattering reaction $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$, and made an attempt to distinguish supernova neutrinos from other sources by selecting events that produced recoiling electrons with $E_e \leq 50$ MeV, and were separated by at least 20 μ sec, which would eliminate the possibility of a muon-decay. This analysis resulted in the identification of 12 events that were most likely from the supernova neutrino flux, as illustrated in table 2.3 found in [13].

Although this number was statistically too small to offer any substantial information about neutrino parameters, the discovery of these neutrinos was evidence of a neutrino upper mass limit. The neutrinos arrived at different times during an interval of 13 seconds, before the light from the supernova could be observed. Given that the neutrinos started off at the same time, this would indicate that they move at different velocities and hence have different masses, limited by the short interval of arrival. The fact that they arrived within such a short period of time would mean that the velocities are almost equal, but this crucial piece of information

Table 2.3: Event information for the supernova neutrino burst from SN 1987A detected at Kamiokande. [13]

Event number	Event time (sec)	Electron energy (MeV)
1	0	20.0 ± 2.9
2	0.107	13.5 ± 3.2
3	0.303	7.5 ± 2.0
4	0.324	9.2 ± 2.7
5	0.507	12.8 ± 2.9
6	0.686	6.3 ± 1.7
7	1.541	35.4 ± 8.0
8	1.728	21.0 ± 4.2
9	1.915	19.8 ± 3.2
10	9.219	8.6 ± 2.7
11	10.433	13.0 ± 2.6
12	12.439	8.9 ± 1.9

supported the idea of neutrinos having mass eigenstates that differed from their natural flavor states. [11, 13, 14]

It was found that the SN 1987A emitted 10^{58} neutrinos during a 10 second period, corresponding to a total energy of the magnitude 10^{46} J, with an average of 15 MeV per neutrino [11, 13]. Today, there are only a few neutrino detectors that are capable of identifying supernova neutrinos, including Super-Kamiokande, KamLAND and SNO (see section 4.2) [12]. In order to separate the supernova neutrinos from other neutrino sources, one must take advantage of their high energies. One way of doing so would be to look for γ -rays resulting from excitation of nuclei in a scintillation detector. When neutrinos with sufficient energy, *i.e.* from a supernova, impact on such nuclei, the result is excited nuclei which by γ -decay returns to ground state shortly after. These energetic photons are the signature of such a reaction, and can easily be distinguished from low-energy solar and atmospheric neutrinos. Examples of such identification reactions are the decay of the excited state $I^\pi = 1^+$ in ^{12}C , resulting in a 15 MeV photon, or decay of ^{15}O and ^{15}N , resulting in a photon with energy ≥ 5 MeV [12].

It is difficult to produce firm statements about the oscillations of neutrinos emanating from supernovae. Part of the problem is that the neutrinos most likely will travel through Earth before they reach the detector, and thus be affected by the MSW-effect (see section 3.2), making it hard to say something specific about how the supernova core and envelope is relevant for oscillation probabilities. The neutrino flux here on Earth can be expressed by the initial flux of the supernova by

$$\Phi_{\nu_e, \text{Earth}} = p_{\nu_e \rightarrow \nu_e} \Phi_{\nu_e, \text{initial}} + (1 - p_{\nu_e \rightarrow \nu_e}) \Phi_{\nu_x, \text{initial}},$$

where $\nu_x \in \{\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\}$. However, the initial flux Φ_{initial} is at best poorly known, and the Kamiokande detector only produced a well defined spectrum for the $\bar{\nu}_e$. Should we be so lucky to experience a supernova within 10 kpc of Earth, it is expected that the Super-Kamiokande detector would find between 7000-12000 events from the $\bar{\nu}_e + p \rightarrow n + e^+$ reaction alone. Such a sizeable amount of data would doubtlessly be a goldmine when it comes to studying

oscillations of supernova neutrinos. [15].

2.4 Dark matter

The dark matter issue originates from the fact that the observed rotational speed of galaxies is not consistent with the total visible mass of stars in the galaxy. There appears to be some form of invisible mass present in the galaxies, and it seems that visible mass makes up a much smaller fraction of the universe than non-luminous matter does [11]. The neutrinos could provide an explanation to this so called dark matter problem.

This dark matter is crucial when it comes to predicting the fate of the universe. It is related to the mass density Ω_m of the universe, a parameter that predicts whether the universe will continue to expand forever, eventually halt its expansion, or in the worst case scenario collapse in a "big crunch" [11]. The alternatives correspond to $\Omega_m < 1$, $\Omega_m = 1$ (so called critical density) and $\Omega_m > 1$. Many observations indicate that the matter density of the universe is less than critical. These include high-redshift Type 1a supernovae, evolution of galaxy clusters, high baryon content of galaxies and galaxies at very high redshift [16]. At the same time, there is evidence for a critical mass density $\Omega_m = 1$, such as the local anisotropic nature of cosmic background radiation and large-scale distribution of galaxies [17].

The best models that fit experimental data favor the critical density universe and divide dark matter into a cold fraction (CDM) and hot fraction (HDM). Cold dark matter is believed to consist of MACHOs (Massive Compact Halo Object) and WIMPs (Weakly Interacting Massive Particle) at non-relativistic velocities and account for $\sim 70\%$ of the total dark matter [5], while massive neutrinos could make up the hot matter fraction [18]. The neutrino is an excellent candidate for hot dark matter, because it does not interact electromagnetically or with the strong nuclear force, which would explain why it is "dark". Although the mass of the neutrino is extremely small, the large number of neutrinos compensates for this and could represent a significant contribution to dark matter.

But as it turns out, recent observations during the 90s would suggest that even the idea of dark matter isn't the solution to the problem of missing mass. They state that visible mass accounts for $\sim 5\%$ of the necessary mass density to make the universe flat, while dark matter cannot represent more than 25% of this density. Dark energy should then be responsible for up to at least 70% of the mass density parameter Ω_m (recall that energy is equivalent to mass by $E = mc^2$). The remarkable thing is that supernova observations predicts exactly this number, and thus supports the idea of dark energy as the main contributor to the density of the universe. [11, 19].

2.5 Reactor and accelerator neutrinos

The probability that a neutrino should oscillate, *i.e* switch from one flavor to another, is dependent on the baseline length L , which is the total distance traveled by the neutrino up to the point where the probability is evaluated. The formal expression for this can be found in section 3.1.1 and 3.1.2, for respectively two-flavor and three-flavor oscillations. Because of this dependence, it is possible to study the behaviour of neutrinos created in reactors and

accelerators with varying baselines.

Over the last decade a number of reactor-based neutrino oscillations experiments have been conducted, such as CHOOZ, Palo Verde, and KamLAND. The baseline of these vary from 1 km to more than 100 km, but they share the objective of detecting neutrino oscillations. Antineutrinos $\bar{\nu}_e$ are produced in neutron decay, and then participate in the inverse β -decay reaction $p + \bar{\nu}_e \rightarrow n + e^+$, that has a relatively large cross section. The positrons yield an energy spectrum that can be predicted by the total effect produced by the reactor, and is directly linked to the flux of antineutrinos. By comparing the no-oscillation scenario spectrum with the detected positron distribution, it is possible to estimate neutrino oscillation probabilities and parameters. In reactor-based experiments, it is only necessary to consider $\bar{\nu}_e$ oscillations, since neutral current reactions (see table 3.1) of $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ have very small cross sections. [20, 21]

To illustrate the difference of a no-oscillation scenario and oscillating neutrinos with respect to positron spectrum, consider figure 2.4 taken from [22].

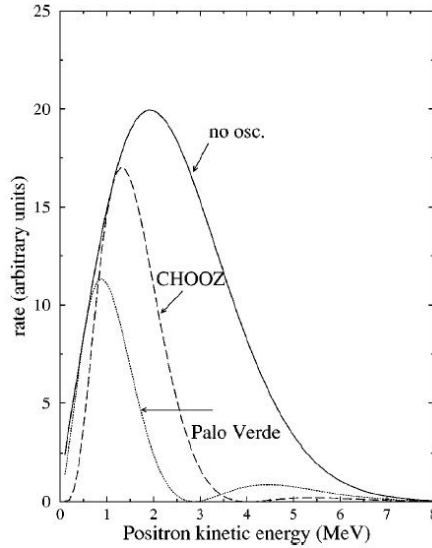


Figure 2.4: Positron energy spectra for no oscillations (solid line) and oscillations with $\Delta m^2 = 7.2 \times 10^{-3} \text{ eV}^{-2}$ and $\sin^2 2\theta = 1$ at CHOOZ and Palo Verde (dotted lines) [22].

The estimated neutrino flux is calculated by first determining the thermal power of the reactor, which will decide the fission-rate of nuclei such as ^{235}U , ^{238}U , ^{239}Pu , and ^{241}Pu . Typical values of thermal power generated by reactors today are of magnitude $\sim 5 - 10 \text{ GW}$. The fission rate then leads to the expected normalized neutrino spectrum as given in [23],

$$P(E_{\bar{\nu}_e}, E_0, Z)_{\bar{\nu}_e} = k(E_0, Z) F(E_0 - E_{\bar{\nu}_e}, Z) \times E_{\bar{\nu}_e}^2 (E_0 - E_{\bar{\nu}_e}) [(E_0 - E_{\bar{\nu}_e})^2 - m_e^2]^{1/2},$$

where $k(E, Z)$ is a spectrum normalization constant, $F(E, Z)$ is the Fermi function for the electron, Z is the nuclei proton number, $E_{\bar{\nu}_e}$ is the neutrino energy, and E_0 is the electron energy. At last, the positron energy distribution is obtained by considering the $\bar{\nu}_e$ spectrum at the reactor, the cross section for the inverse β -decay, the number of target protons, and

the distance between detector and reactor. All together, this method of deriving the positron spectrum is quite accurate. The uncertainties in L , number of target protons, and cross section are all less than 1%, while the neutrino flux is determined within an error margine of $\sim 3\%$ [20].

Neutrino beams originating from accelerators can also be used to study oscillations. These neutrino-observation sites are based on high-energy protons colliding with a target, preferably metal, which will produce pions. The further decay chain is $\pi^+ \rightarrow \mu^+ + \nu_\mu$, where the muon then participates in $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$. The flux of muon-neutrinos is predictable from the proton beam intensity, and any oscillation from $\bar{\nu}_\mu$ to $\bar{\nu}_e$ can then be registred in the detector by the inverse β -decay reaction. Strictly speaking, there is also a very small contribution to the $\bar{\nu}_e$ flux from decay of μ^- , but this only makes up a neglible fraction of the observed electron anti-neutrinos. The π^+ beam is focused towards the detector to achieve the largest neutrino flux possible, and typical baseline lengths of experiments today are less than 1 km. [5, 24]

Parameters for the two major accelerator experiments LSND (Liquid Scintillation Neutrino Detector) and KARMEN (Karlsruhe-Rutherford Medium Energy Neutrino) are presented in table 2.4 with data found in [25, 26].

Table 2.4: LSND and KARMEN experiment parameters [25, 26].

Event number	LSND	KARMEN
Proton kinetic energy	800 MeV	800 MeV
Baseline length	29 m	17 m
Data collection period	6 years	5 years
Observed excess events of $\bar{\nu}_e$	~ 89	8
Probability $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$\sim 2.6 \times 10^{-3}$	$< 1.03 \times 10^{-3}$

So far, LSND is the only accelerator-based experiment that has produced a significant neutrino oscillation signal for $\bar{\nu}_\mu$ to $\bar{\nu}_e$. The results from LSND are unfortunately not in good agreement with Super-Kamiokande and SNO data, which has caused the suggestion of a sterile neutrino flavor in order to explain the anomaly. This is further commented on in section 3.3. The MiniBOONE experiment at Fermilab started to run in november 2002, and was specifically designed to check the LSND results with increased sensitivity. If the results from LSND turns out to be consistent, the standard theory of three neutrino flavors would have to be revised in order to obtain a satisfactory explanation to both Super-Kamiokande, SNO and LSND data. [27]

Accelerators with long baselines (several hundred kilometers) are more useful when it comes to studying the atmospheric neutrino deficiency. Models that aim to provide more insight into this phenomena need to operate with long baselines in order to mimick the conditions for atmospheric neutrinos and oscillations of ν_μ into ν_e and ν_τ . An example of such a long baseline accelerator is the CNGS (Cern to Gran Sasso) construction that was commenced in late 2000. The experimental site will be active in early 2006, and provide a baseline of 732 km with a high-energy neutrino beam. The CNGS beam is particularly interesting due to its capability of producing τ particles, and thus offers the possibility of investigating $\nu_\mu \rightarrow \nu_\tau$

oscillations [28].

Chapter 3

Neutrino oscillation theory

There is little doubt today concerning the validity of the neutrino oscillation theory. Results from the neutrino detectors around the globe are conclusive.

3.1 Neutrino mixing theory

As mentioned earlier, the neutrinos have different mass eigenstates and also come in different flavors. This gives rise to oscillations and a probability amplitude assigned to each state. The flavor states are results of weak decays and interact with their corresponding leptons. Now, a mass eigenstate has contributions from each flavor and the oscillations occur due to the time-dependent phase of each flavor. This would not be happening if there were no eigenstates with different masses, because then they would have equal energies and there would be no time dependence. But as shown in the subsequent sections, the probability amplitude of a specific neutrino flavor is a function of parameters such as energy, distance traveled and time.

3.1.1 Two-flavor analysis

Let us consider the ν_μ and ν_τ neutrinos. The mass eigenstates ν_1 and ν_2 are given as linear combinations of the two flavors, with a mixing angle θ as the parameter that determines the contribution from each neutrino flavor.

$$\begin{pmatrix} |\nu_1(x, t)\rangle \\ |\nu_2(x, t)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\nu_\mu(x, t)\rangle \\ |\nu_\tau(x, t)\rangle \end{pmatrix}.$$

One can also invert these equations and express the flavors as linear combinations of the mass eigenstates,

$$\begin{pmatrix} |\nu_\mu(x, t)\rangle \\ |\nu_\tau(x, t)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} |\nu_1(x, t)\rangle \\ |\nu_2(x, t)\rangle \end{pmatrix}.$$

The neutrinos are now assigned a wave-function, that describes their propagation through space. The eigenstates are described by

$$\begin{pmatrix} |\nu_1(x, t)\rangle \\ |\nu_2(x, t)\rangle \end{pmatrix} = e^{i\vec{p}\cdot\vec{x}} \begin{pmatrix} |\nu_1(x, 0)\rangle e^{-iE_1 t} \\ |\nu_2(x, 0)\rangle e^{-iE_2 t} \end{pmatrix}$$

By assumption, the neutrino momentum $\vec{p}$ is considered to be equal for both mass eigenstates. The energy is different, however, since the eigenstates have different masses. Although this assumption is standard procedure in most derivations of oscillation probabilities, it has been criticized on basis of relativistic arguments in [29, 30]. Nevertheless, it seems reasonable that the mass eigenstates propagate with the same velocity, i.e. momentum, due to localization in space, and as pointed out in [31] the phase factor determining the time dependence of the oscillation probability is invariant whether you assume that all eigenstates have equal momentum or equal energy, as long as neutrinos are ultrarelativistic, which they most certainly are.

By combining these equations we achieve the following expression for the flavor-states, including initial conditions

$$\begin{pmatrix} |\nu_\mu(x, t)\rangle \\ |\nu_\tau(x, t)\rangle \end{pmatrix} = e^{i\vec{p}\cdot\vec{x}} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-iE_1t} & 0 \\ 0 & e^{-iE_2t} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_\mu(x, 0)\rangle \\ |\nu_\tau(x, 0)\rangle \end{pmatrix}.$$

To find the probability that the ν_μ changes into a ν_τ , we set the initial conditions to $|\nu_\mu(x, 0)\rangle = 1$ and $|\nu_\tau(x, 0)\rangle = 0$ and obtain [32]

$$P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) = |\langle\nu_\tau|\nu_\tau\rangle|^2 = \sin^2(2\theta) \sin^2 \frac{(E_2 - E_1)t}{2}, \quad (3.1)$$

where the energy is given by the relativistic expression

$$E_i = \sqrt{m_i^2 c^4 + p_i^2 c^2}. \quad (3.2)$$

Equation 3.1 poses some difficulties since we cannot know the original energy of the neutrino, E_1 , but only determine its final energy E_2 . Therefore, it is more instructive to express this probability with the justified assumption that the neutrino kinetic energy is much larger than its rest mass energy, $E_2 \gg m_2 c^2, E_1 \gg m_1 c^2$. Taylor-expanding equation 3.2 yields

$$E_2 - E_1 = \sqrt{m_2^2 c^4 + p_2^2 c^2} - \sqrt{m_1^2 c^4 + p_1^2 c^2} \approx \frac{m_2^2 c^4 - m_1^2 c^4}{2E},$$

where E is the kinetic energy of the neutrinos, assuming it to be the same for both flavors. This means that $p_1 = p_2$, and that a change in neutrino energy is solely due to change in mass, as stated earlier in this section. It is necessary to make the approximation $ct \approx L$, which is acceptable since a particle with nearly no mass travels with a velocity close to the speed of light. Making these substitutions and using natural units, $\hbar = c = 1$, we get

$$P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) = \sin^2(2\theta) \sin^2 \frac{L\Delta m^2}{4E}, \quad (3.3)$$

where $\Delta m^2 \equiv |m_2^2 - m_1^2|$ and all units are in GeV. The frequently used expression for probability amplitude of two-flavor oscillations in vacuum (see for instance [33]) is obtained by extracting proper numerical constants, leaving us with

$$P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) = \sin^2(2\theta) \sin^2 \frac{1.27 L(\text{km}) \Delta m^2 (\text{eV}^2)}{E(\text{GeV})},$$

Here, L is the distance traveled by the neutrino before it is detected, Δm^2 is the mass square difference between the two eigenstates and E is the neutrino energy.

3.1.2 Three-flavor analysis

The three different neutrino flavors are ν_e , ν_μ and ν_τ . To obtain a complete description of the neutrino, one also has to consider the mass eigenstates that causes the oscillations. A common model is to operate with a neutrino mixing matrix U given as

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix},$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ and θ_{ij} are the mixing angles between the neutrino states. The δ_{CP} is the CP -violating phase with allowed values $\in [0, 2\pi]$, and will receive further attention in section 3.3.

A similar analysis as in the two-flavor case gives us the oscillation probability. The flavor states are described by $|\nu_\alpha\rangle = \sum_{i=1}^3 U_{\alpha i}^* |\nu_i\rangle$, where $|\nu_\alpha\rangle$ is the flavor state produced in weak interactions, U is the leptonic mixing matrix, and $|\nu_i\rangle$ is the mass eigenstate with mass m_i . Correspondingly, we have that $|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$, where the sum over α goes over e, μ , and τ . Since the time dependence of the flavor states is given as $|\nu_\alpha\rangle = e^{-iE_\alpha t} |\nu_\alpha\rangle$, we can combine these relations to

$$|\nu_\alpha\rangle = \sum_{i=1}^3 \sum_\beta U_{\alpha i}^* U_{\beta i} e^{-iE_i t} |\nu_\beta\rangle. \quad (3.4)$$

Just as with two-flavor oscillations, we now square the amplitude factor in front of $|\nu_\beta\rangle$ to obtain the probability

$$P(|\nu_\alpha\rangle \rightarrow |\nu_\beta\rangle) = \left| \sum_{i=1}^3 U_{\alpha i}^* U_{\beta i} e^{-iE_i t} \right|^2$$

Using the same approximation as in the previous section, assuming equal momentum for all neutrinos and different masses, $E_i = \sqrt{p^2 + m_i^2} \simeq E + \frac{m_i^2}{2E}$, this treatment yields the probability

$$P(|\nu_\alpha\rangle \rightarrow |\nu_\beta\rangle) = \sum_{\substack{i \in 1,2,3 \\ j \in 1,2,3}} J_{\alpha\beta ij} e^{-i\Delta m_{ij}^2 L/2E} \quad (3.5)$$

with $J_{\alpha\beta ij} = U_{\beta i} U_{\beta j}^* U_{\alpha i}^* U_{\alpha j}$. As before, Δm_{ij}^2 is the mass square difference, L is the distance traveled and E is the neutrino energy. A similar derivation of the expression for three-flavor oscillations can be found in [29, 34, 35].

In general, *i.e.* in both vacuum and matter, the probability amplitude $P(|\nu_\alpha\rangle \rightarrow |\nu_\beta\rangle)$ depends on 8 parameters,

$$P_{\alpha\beta} = P_{\alpha\beta}(\Delta m_{21}^2, \Delta m_{31}^2, \theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, E, L, V(x)),$$

where $V(x)$ is the matter-induced potential, equal to zero for vacuum oscillations [36].

As an example, the analytical expression for the three-flavor oscillation probability in vacuum will be calculated for the transition $|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle$. Using equation 3.5, we get

$$\begin{aligned} P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) &= \sum_{i=1}^3 (U_{3i}U_{31}^*U_{2i}^*U_{21}e^{-i\Gamma_{i1}} + U_{3i}U_{32}^*U_{2i}^*U_{22}e^{-i\Gamma_{i2}} + U_{3i}U_{31}^*U_{2i}^*U_{21}e^{-i\Gamma_{i3}}) \\ &= (U_{31}U_{31}^*U_{21}^*U_{21}e^{-i\Gamma_{11}}) + (U_{32}U_{31}^*U_{22}^*U_{21}e^{-i\Gamma_{21}}) + (U_{33}U_{31}^*U_{23}^*U_{21}e^{-i\Gamma_{31}}) \\ &\quad + (U_{31}U_{32}^*U_{21}^*U_{22}e^{-i\Gamma_{12}}) + (U_{32}U_{32}^*U_{22}^*U_{22}e^{-i\Gamma_{22}}) + (U_{33}U_{32}^*U_{23}^*U_{22}e^{-i\Gamma_{32}}) \\ &\quad + (U_{31}U_{33}^*U_{21}^*U_{23}e^{-i\Gamma_{13}}) + (U_{32}U_{33}^*U_{22}^*U_{23}e^{-i\Gamma_{23}}) + (U_{33}U_{33}^*U_{23}^*U_{23}e^{-i\Gamma_{33}}) \end{aligned}$$

where $\Gamma_{ij} = \Delta m_{ij}^2 L/2E$. Inserting the appropriate values for U_{ij} , the final result yields a monster formula. However, approximative expressions are much more adequate in order to obtain numerical results. One method of obtaining such expressions is series expansion in one or more sufficiently small parameters. A sensible choice of expansion parameter would be the mass hierarchy parameter $\lambda = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$ [37]. From table 5.2, the value of λ is seen to be $\simeq 0.028$. It would also be considered reasonable to use $\sin \theta_{13} = s_{13}$ for this purpose, since its best-fit value is $\simeq 0.070$. It is also perfectly possible to expand the expressions for oscillation probabilities in both of these parameters. This is done in [38], with the result

$$\begin{aligned} P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) &= 2\lambda s_{13} \sin 2\theta_{12} \sin 2\theta_{23} (\sin \delta_{\text{CP}} \sin \omega - \cos 2\theta_{23} \cos \delta_{\text{CP}} \cos \omega) \omega \sin \omega \\ &\quad + \sin^2 2\theta_{23} \sin^2 \omega + \lambda^2 \sin^2 2\theta_{23} \omega^2 (c_{12}^4 \cos 2\omega - \frac{1}{2} \sin^2 \theta_{12} \sin^2 \omega) \\ &\quad - 2s_{13}^2 \sin^2 2\theta_{23} \sin^2 \omega - \lambda c_{12}^2 \sin^2 2\theta_{23} \omega \sin 2\omega \end{aligned}$$

up to second order in λ and s_{13} , where we have introduced $\omega_{31} = \frac{\Gamma_{31}}{2} = \frac{\Delta m_{31}^2 L}{4E}$ for the sake of convenience. Such analytical expressions are essential when it comes to comparing oscillation probabilities found in experiments with the theory itself.

3.2 MSW-effect

The analytical tool for considering three-flavor oscillations is modified when the neutrinos interact with matter by the MSW-effect (Mikheyev-Smirnov-Wolfenstein). So far the treatment has been solely in vacuum. Neutrinos can participate in mainly three types of reactions as illustrated in table 3.1.

Table 3.1: Neutrino reactions.

Neutrino reaction type	Reaction
Charged Current Reaction (CC)	$\nu_e + n \rightarrow p + e^-$ ($\bar{\nu}_e + p \rightarrow n + e^+$)
Neutral Current Reaction (NC), $x \in \{e, \mu, \tau\}$	$\nu_x(\bar{\nu}_x) + d \rightarrow n + p + \nu_x(\bar{\nu}_x)$
Elastic Scattering Reaction (ES), $\{x, y\} \in \{e, \mu, \tau\}$	$\nu_x(\bar{\nu}_x) + y \rightarrow \nu_x(\bar{\nu}_x) + y$

The neutrinos interact by these reactions in both vacuum and matter, but the probability for a neutrino to participate in a certain reaction is modified when it travels through matter. Earth contains electrons and has an absence of τ and μ particles, which in effect will increase the probability for the ES reaction with electrons, also with a contribution from the CC reaction [39].

To consider neutrino oscillation probabilities as they pass through Earth, the mixing matrix U must be modified by the transformation $U \rightarrow \tilde{U}$, where $\tilde{U}$ is the leptonic mixing matrix in matter. In order to find the oscillation probabilities in matter, one must first identify the evolution T -matrix given by $|\nu(x, t)\rangle = T(t)|\nu(x, 0)\rangle$. Then, as in section 3.1.1, the probability amplitude is given by $P(|\nu_\alpha\rangle \rightarrow |\nu_\beta\rangle) = |T_{\alpha\beta}(t)|^2$. The evolution matrix in matter is also derived from equation 3.4

$$T_{\beta\alpha}(t) = \sum_{i=1}^3 \tilde{U}_{\alpha i}^* \tilde{U}_{\beta i} e^{-iE_i L},$$

where (α, β) are the three neutrino flavors e, μ and τ . The eigenvalues for the Hamiltonian, E_i , differ in matter compared to vacuum. This is due to the matter-induced potential $V(x)$ that must be considered when the neutrinos travel through an object such as Earth. The oscillation probabilities in constant-density matter can be determined exactly, see for instance [40]. As mentioned in section 3.1.2, an approximation is more useful when it comes to obtaining numerical results. Expansion series for matter-induced oscillations are performed in [38], and give surprisingly accurate results. For the $\nu_\mu \rightarrow \nu_\tau$ scenario presented in section 3.1.2, the error is less than 1% for a double expansion of λ and s_{13} .

It was Wolfenstein who first calculated the increased oscillation probability of neutrinos interacting with matter [41] (later followed by Smirnov and Mikheyev). The expression for MSW-induced two-flavor oscillation probability for $\nu_e \rightarrow \nu_\mu$, which is most relevant for the scenario of cosmic neutrinos interacting with Earth, can be derived (see [42]) to

$$P_{\text{MSW}}(|\nu_e\rangle \rightarrow |\nu_\mu\rangle) = \frac{\sin^2(2\theta)}{W^2} \sin^2 \frac{x\pi W}{\lambda}, \quad (3.6)$$

where $W^2 = \sin^2 2\theta + (\sqrt{2}G_F N_e \frac{2E}{\Delta m^2} - \cos 2\theta)^2$ and $\lambda = \frac{4\pi E}{\Delta m^2}$. G_F is the Fermi constant, N_e is the density of electrons, and all other variables are as defined earlier. Natural units $\hbar = c = 1$ have been used. Note that when $N_e = 0$, *i.e.* vacuum, this expression reduces to equation 3.3 as one would expect.

As is seen, $W^2 < 1$ corresponds to an increased probability of oscillation. In order to calculate W^2 for solar neutrinos hitting Earth, one needs the mixing angle θ , the neutrino energy E , the square mass difference Δm^2 and the electron density N_e . Considering ^8Be neutrinos with an energy of $\simeq 10$ MeV and the best-fit parameters $\Delta m^2 = 7.1 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta = 0.44$ presented in chapter 5, we finally need to estimate the electron density of Earth. The electron density as a function of radius is not trivial, as illustrated in figure 3.1 found in [43].

A crude average estimate for the sake of argument would be $\simeq 3 \text{ mol/cm}^3$, which yields a W^2 value of 0.9539. This is a very approximate average of W^2 , which in reality takes values in the range [0.9162, 0.9758], as is seen when the maximum and minimum electron density of figure 3.1 is substituted in equation 3.6.

3.3 *CPT*-violation

The *CPT*-theorem is a cornerstone of the standard model in particle physics, and has proven to be the valid symmetry after the downfall of *CP* and *T* invariance. There has been no ex-

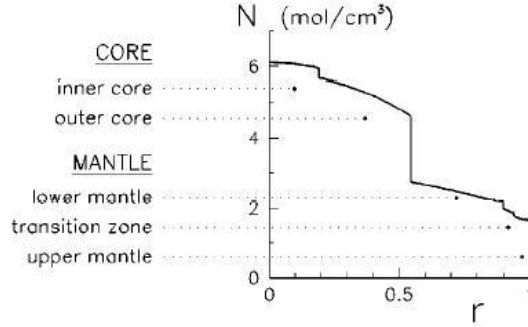


Figure 3.1: Electron density in Earth as a function of radius r [43].

perimental verification of CPT -violation so far [44]. The concept of CPT -invariance actually consists of three separate symmetries. C is charge conjugation and turns a particle into its antiparticle, P performs the transformation $\vec{r} \rightarrow -\vec{r}$, and T changes the direction of time by the transformation $t \rightarrow -t$. In short, CPT -invariance means that the physical properties of a system does not change when acted upon by a CPT -operator, such as mass and lifetime [45, 46].

The CPT -violation problem basically consists of neutrinos having different mass square differences than antineutrinos. A similar problem has been considered for the neutral kaons, which resulted in very convincing limits on CPT -invariance, $m_K - m_{\bar{K}} < 0.44 \times 10^{-18}$ GeV. The downfall of this invariance would be a serious offence towards the standard model of particle physics. If CPT -violation is a real phenomena, it would mean abandonment of the standard model and venturing into more exotic physics such as string theory.

In order to test if the CPT -theorem holds for neutrino oscillations, a consequence of this would be that $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$, where α and β designate one of the neutrino flavors [47]. If there is any breach of CPT -invariance, the effects are expected to be very small, and a magnifying resonance would be necessary to obtain a measurable amplitude for these violating effects. It is shown in [44] that CPT -invariance could actually be measured in neutrino oscillations.

The so-called LSND anomaly, mentioned in section 2.5, has yielded experimental support for worrying about whether CPT is a true symmetry of nature. The results of LSND are not consistent with the atmospheric [8, 48] and solar neutrino [49, 50] square mass differences, and seem to indicate a value of $\approx 1 \text{ eV}^2$ for Δm^2 . This is a large deviation from all other neutrino detection experiments, and call for two new mass square differences for the antineutrinos. This is not consistent with CPT -invariance, since different masses for a neutrino and its antiparticle would produce different oscillation probabilities and thus change the physical properties of the particle.

An explanation to this could simply be that the experiment or interpretation of data is wrong, and the MiniBOONE experiment in USA is currently cross-checking the results from LSND in order to accept or discard this possibility. The results from LSND would also be satisfactory if one introduces a sterile neutrino flavor, since the presence of such a ν_s would account for

the asymmetry of neutrino versus antineutrino events. The problem with this explanation, however, is that the existence of such a sterile neutrino flavor has been pretty much ruled out after the SNO experiment (see section 4.2.7) [47]. As a result of this, the possibility that *CPT* is broken cannot be ignored. [52]

Let us consider this scenario quantitatively. As mentioned earlier, the equality sign holds in $P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$ if the *CPT*-theorem is valid. Consider the probability difference

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha).$$

The $\Delta P_{\alpha\beta}$ would then be a measure of any *CPT*-violation, should it occur. The probability amplitudes are derived from the evolution operator T , which in general is given as

$$T(t) = e^{-i \int_0^t H(t') dt'},$$

where H is the Hamiltonian of the neutrino. The T -matrix describes the time evolution for a neutrino state by $|\nu_k(t)\rangle = T(t)_{k \rightarrow k} |\nu_k(0)\rangle$, and can be used to describe the probability for a neutrino oscillation as explained in section 3.1.2,

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^3 U_{\alpha i}^* T(t)_{i \rightarrow i} |\nu_i(0)\rangle.$$

For antineutrinos, one must make the substitution $V(x, t) \rightarrow -V(x, t)$ in the Hamiltonian $H = H_0 + V$, where V is the matter potential, which correspondingly has an effect on the probability amplitudes of oscillation. From this point, it is necessary to distinguish between intrinsic (vacuum) and extrinsic (matter-induced) *CPT*-violation. In vacuum, the matter potential $V(x, t)$ is obviously equal to zero, which means that the Hamiltonian is equal for neutrinos and antiparticles, and thus also the T -matrices. Therefore, *CPT*-violation is impossible in vacuum.

In matter, the potential is not equal to zero, and so one can take different approaches in order to approximate Earth's matter-induced potential, by using for instance constant density matter profiles or step-functions. A full analytical treatment of constant density matter is given in [36], and the results are quoted here. Figure 3.2 taken from [36] shows the *CPT* probability difference $\Delta P_{\alpha\beta}$ for the specific transition $\nu_\mu \rightarrow \nu_e$ with given baseline length L , neutrino energy E_ν and matter density ρ . The *CP*-violating phase δ_{CP} has been chosen to 0, which together with π corresponds to minimal *CP*-violation, since $e^0 = -e^{i\pi} = 1$. Maximum *CP*-violation comes from δ_{CP} values $\in \{\pi/2, 3\pi/2\}$, which gives a completely non-real contribution to the mixing matrix U .

The dependence on the baseline length L and energy E_ν is easily seen: the probability for *CPT*-violation increases with L , and decreases with growing E_ν . It is very interesting to consider the KamLAND experimental data for more insight in this matter. Figure 3.3 taken from [36] shows that the probability difference ΔP_{ee} for extrinsic *CPT*-violation could be as high as 5%, a significant contribution that would not go unnoticed if searched for in experimental data. It is worth mentioning that KamLAND offered the highest value of $\Delta P_{\alpha\beta}$, and that other experiments had negligible contributions from *CPT*-violation [36].

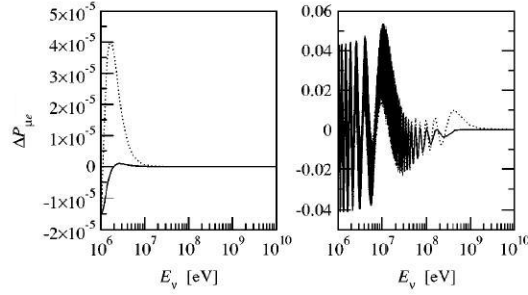


Figure 3.2: $\Delta P_{\mu e}$ as a function of neutrino energy E_ν . The left graph corresponds to a baseline $L = 1$ km, while the right has $L = 250$ km. A constant density of $\rho = 3$ g/cm³ has been used in this model, and neutrino parameter values are found in table 5.2, except for θ_{13} , which has been set to 9.2° . Dotted and solid curves are numerical (using the evolution operator method and Cayley-Hamilton formalism) and analytical results, respectively [36].

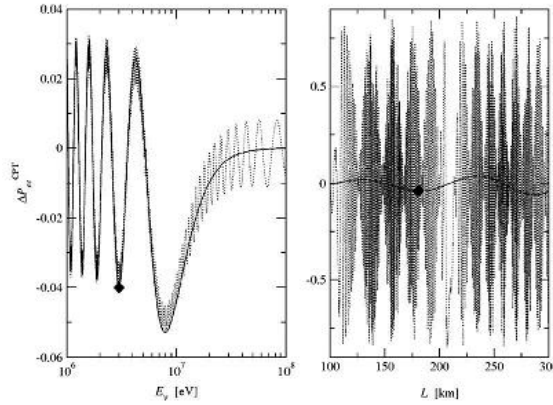


Figure 3.3: ΔP_{ee} for the KamLAND experiment, as a function of neutrino energy E_ν in the left graph and L in the right. Numerical results are presented in the dotted curves, while the analytical treatment is shown with the solid curves. Central values for KamLAND are indicated by the black diamonds [36].

3.4 Parity flip due to magnetic moment

Although the neutrino oscillation theory seems to be the solution to the missing neutrinos, other ideas have been presented over the years as possible explanations. The key issue in the solar neutrino problem is that the flux of electron-neutrinos we observe here on Earth is not consistent with the flux predicted by the SSM. Experiments strongly indicate that neutrinos actually have mass, in contradiction to what the standard model of particle physics dictates, which in turn leads to a non-vanishing magnetic moment $\vec{\mu}_\nu$. This magnetic moment can be used to classify neutrinos as "left-handed" or "right-handed", in terms of whether $\vec{\mu}_\nu$ points in the same direction as its momentum $\vec{p}$. This is illustrated in figure 3.4.

Now, the parity of the neutrino can be affected by an external magnetic field $\vec{B}$, thus flipping a left-handed neutrino to a right-handed one, or vice versa. The design of the current detector sites does not permit observation of right-handed neutrinos, which could be the reason to the

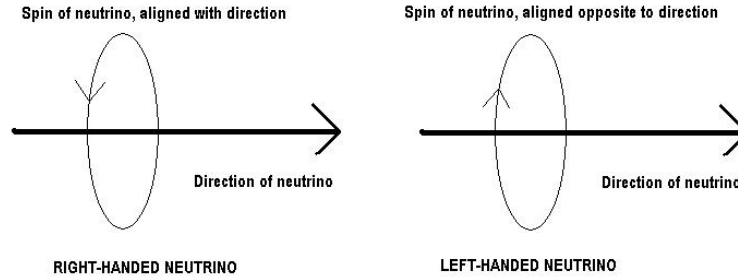


Figure 3.4: Definition of left- and right-handed neutrinos.

problematic missing flux [53]. Without the interference of a magnetic field, all neutrinos are born left-handed. Lee and Yang were able to show that parity was broken in the mid-1950s, and that the fixed handedness of neutrinos was a direct consequence of this experiment [1]. Should it turn out that a major part of the neutrinos emanating from the sun are turned into right-handed ones by the solar magnetic field, this explanation would be an alternative to the traditional oscillation theory.

The magnetic field of the sun is not constant, experiencing changes in both magnitude and direction every 11th and 22nd year, correspondingly [54]. If the neutrinos have a significant magnetic moment, this varying magnetic field should yield a time-dependent flux of the neutrino flux on Earth, since the number of neutrinos flipped will depend on $\vec{B}$. In order to investigate this, one must direct the attention to solar neutrino experiments that has been running for a period that is comparable to the suns magnetic field cycle. Unfortunately, a time variation of the neutrino flux in conjunction with the solar cycle has not been observed in neither the Homestake experiment running over 22 years, or the Kamiokande observation site collecting data for more than 8 years. Certainly, the flux has not been constant, as can be viewed in for instance figure 4.4, but the variation pattern does not fit the cycle of the solar $\vec{B}$ field.

There have been voices in favor of the magnetic moment explanation to the solar neutrino deficit [54], claiming that the lack of synchronized time variation of the neutrino flux and solar magnetic activity in the Homestake, GALLEX, and SAGE experiments was because the neutrinos most strongly affected by this parity flip were out of range for the detectors of the mentioned observation sites. However, the latest solar neutrino experiments also exclude the possibility of parity flip, in addition to specific searches for the neutrino magnetic moment $\vec{\mu}_\nu$ that impose a upper limit of $3.6 \times 10^{-10} \mu_B$ at 90% confidence level, where $|\vec{\mu}_B| = \frac{e\hbar}{2m_e}$ has the numerical value $9.24 \times 10^{-24} \text{ J T}^{-1}$ [55]. The magnetic moment of the neutrino is in other words tiny, and might very well be far too small to contribute significantly to any parity flip process, as the results seem to imply.

Chapter 4

Neutrino experimental data

There are only a handful of locations in the world where you can detect neutrinos [56]. As mentioned earlier, the tiny neutrinos possess no electric charge and seldom interacts with other particles. In order to construct a neutrino detector, size is an important keyword. The solar neutrino flux is very large and corresponds to about $6.5 \times 10^{10} \text{ cm}^{-2} \text{ s}^{-1}$ hitting Earth. But despite of this large flux, the number of interactions is quite small. Most neutrinos just pass right through Earth without interacting, so in order for detection to be possible one needs detectors with many reactants, so that at least a few of the incoming neutrinos will interact [57]. It is also necessary to construct efficient shielding around the detector, to ensure that all registered interactions are due to neutrinos instead of other cosmic particles. For instance, even at 4000 m below sea level, a cosmic high-energy muon will trigger a so-called Cerenkov detector (see section 4.1) 10^6 times more often than a neutrino [58].

It is common to use the unit SNU (Solar Neutrino Unit) in order to specify the interaction rate of an incoming neutrino flux. 1 SNU equals one interaction per second per 10^{36} atoms. Detection of neutrinos usually yield a few SNUs, so in order to have experimental data within a reasonable period of time, it is clear that one needs huge detectors with many capturing atoms. One also has to consider the fact that all incoming solar neutrinos are not equally energetic. Depending on the energy of the neutrino, a detector might or might not be able to interact with it. The most energetic neutrinos come from the decay of boron, as illustrated in table 2.1.

The square mass differences for solar and atmospheric neutrino detection experiments are often referred to as $\Delta m_{\text{sol}}^2 = \Delta m_{21}^2$ and $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$ in much literature, when using two-flavor analyses to account for results, and should therefore be mentioned. This notation will not be used in the following, but the context should be decisive in determining which of these square mass differences that is being discussed.

The fluxes detected at the probably most significant neutrino detectors are given in the table 4.1, with data found in [58, 59, 60, 61, 62, 63, 64, 67, 68, 69]. The first column gives the fluxes, the second column gives the ratio of the measured flux and the prediction of the SSM. Each experiment will be given individual attention in the following sections.

Table 4.1: Neutrino flux overview for major neutrino detection experiments [58, 59, 60, 61, 62, 63, 64, 67, 68, 69]. These experiments will be given individual attention in section 4.2.1-4.2.7.

Experiment	Measure of Flux	Ratio Flux/SSM (%)
SNO	$(9.24 \pm 0.740 \pm 0.660) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$	61 ± 6.0
Super-Kamiokande	$(2.35 \pm 0.02 \pm 0.08) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$	$46.5 \pm 0.50 \begin{smallmatrix} +1.60 \\ -1.50 \end{smallmatrix}$
Kamiokande	$(2.80 \pm 0.19 \pm 0.33) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$	$49.6 \begin{smallmatrix} +4.4 \\ -4.2 \end{smallmatrix} \pm 4.8$
Reines and Cowan	$1.2 \times 10^{13} \text{ cm}^{-2} \text{ s}^{-1}$	-
KamLAND	$1.3 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$	$61 \begin{smallmatrix} +9.0 \\ -8.0 \end{smallmatrix}$
Homestake	$2.32 \pm 0.22 \text{ SNU}$	$27 \pm 2.5 \pm 4.0$
GALLEX	$77.5 \pm 6.2 \begin{smallmatrix} +4.3 \\ -4.7 \end{smallmatrix} \text{ SNU}$	$58 \pm 9.0 \pm 5.0$

4.1 Types of detectors

There are mainly two types of neutrino detectors used today [70], namely Cerenkov detectors and radiochemical detectors. The principles of these two types are explained in the following sections.

4.1.1 Cerenkov detectors

This type of detector exploits the fact that charged leptons produced from weak neutrino interactions produce light, which can be recorded by photomultiplier tubes (PMTs). These, in turn, produce an electrical signal which can be measured. Consider figure 4.1 taken from [71] for an impression of the schematics.

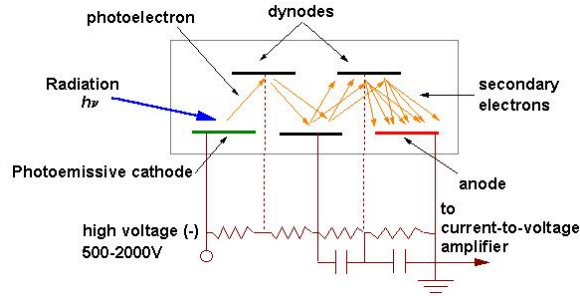


Figure 4.1: Schematic build-up of a PMT [71].

There are many advantages gained by using a Cerenkov detector. Not only does the direction of the incoming light allow for a reconstruction of the neutrino track, but the energy distribution of the incoming neutrinos can also be extracted from the recoiling electron. Another practical feature is that the detector gives the results in real time. The problem, however, with a Cerenkov detector lies mainly in the electronics used to register the light. At low energies, the signal-to-noise ratio is small for this detector, which in effect means that it can be hard to differentiate between a neutrino-hit and noise. Due to this fact, a Cerenkov detector is obliged to have a rather high threshold energy. The Kamiokande detector operated with a threshold of 7.5 MeV, only counting the most energetic ^8B neutrinos. The Super-Kamiokande

has been able to reduce the threshold to 5 MeV. [70, 72]

4.1.2 Radiochemical detectors

The radiochemical detector makes use of the incoming neutrinos by allowing them to trigger a reaction of type

$${}^A\text{Z} + \nu_e \rightarrow {}^A(\text{Z}+1) + e^-,$$

where A and Z are the nucleon and proton number, respectively. When a number of these reactions are triggered over time, one is able to count the electron capture decays of the product nuclei. This kind of detection has the advantage of much lower threshold energies required, compared to Cerenkov detection. For instance, the reaction

$${}^{37}\text{Cl} + \nu_e \rightarrow {}^{37}\text{Ar} + e^- \quad (4.1)$$

has a threshold energy of $E_\nu = 0.814$ MeV and can be detected by radiochemical techniques, such as using a proportional counter [3]. Due to the lower energy threshold, more neutrinos interact and a larger set of data is acquired. [70]

4.2 The experiments

This section will give a brief overview of the largest and most significant neutrino-detection experiments carried out so far, and the results they have yielded.

4.2.1 Reines and Cowan

Most of the data presented in this section is found in [61].

The first neutrino detection device became operational in 1953 and was designed to capture neutrinos originating from β -decay in a nearby nuclear reactor. Since Pauli proposed that the missing energy in the β -decay (see chapter 1) was carried off by the neutrino, one would expect that a nuclear reactor would provide an intense source of neutrino flux. The strategy in order to obtain solid proof of the neutrinos existence was laid out as follows. The detector itself consisted of a total ~ 400 l of water mixed with cadmium chloride. Two large scintillation detectors were placed at the edges of this detector, and would serve as photon registrators. Reines and Cowan looked for the reaction

$$\bar{\nu}_e + p \rightarrow n + e^+$$

to occur. When the positron was created, it would be almost instantly annihilated together with an electron from the water, emitting two 0.511 MeV photons in the process. The scintillator detectors would detect these photons and evidence would be at hand for this neutrino-induced reaction. This is illustrated in figure 4.2 found in [61].

This experiment was small in magnitude compared to today's neutrino detectors. Only a few events were recorded each hour, and the purpose was never to determine parameters such as the mass of the neutrino, but simply to provide substantial proof of the neutrinos existence. It certainly did, and Reines received the Nobel Prize in 1995.

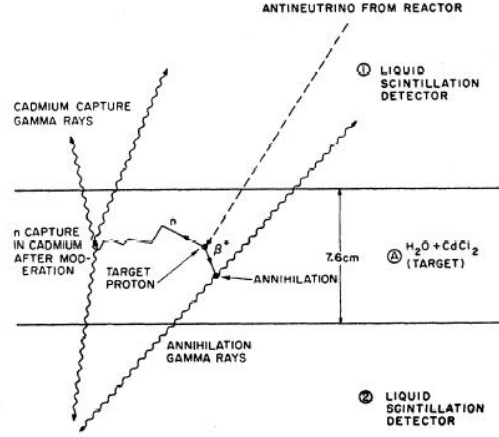


Figure 4.2: The neutrino detection reaction in the Reines-Cowan experiment [61].

4.2.2 Homestake

Most of the data presented in this section is found in [59].

The Homestake experiment is the neutrino detection experiment with the longest consecutive running-time. Built over the years 1965 to 1967, this observation site was active over the period 1970-1992. It was specifically designed to look for neutrinos emanating from the decay of solar ${}^7\text{Be}$ and ${}^8\text{B}$, which produce the most energetic neutrinos in the sun. Consider figure 4.3 taken from [59] for an overview of the detector.

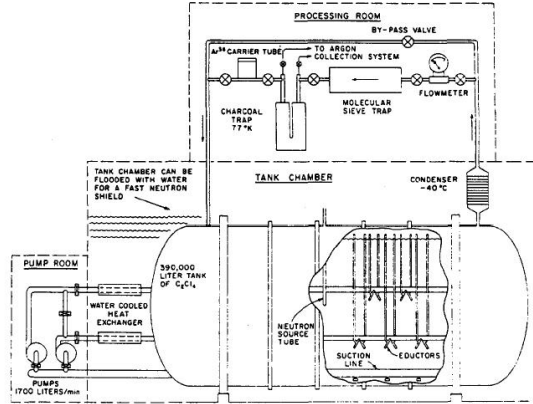


Figure 4.3: Schematics of the Homestake experiment [59].

The detector itself consisted of a tank containing 615 tons of perchloroethylene C_2Cl_4 , which was supposed to capture neutrinos from the inverse β -decay reaction



with an energy threshold of 0.814 MeV. The ${}^{37}\text{Ar}$ was chemically separated from the liquid

by making use of the ^3He gas collecting ability, and was further analyzed with a proportional counter in order to measure the amount of ^{37}Ar produced by electron capture. The Homestake detector was located 1480 m underground in an old mine in South Dakota, USA.

A benefit gained from placing detectors at such depths is the sheltering of rock against cosmic ray muons, that can mimick neutrino reactions by interacting with the ^{37}Ar nuclei. By using test volumes of perchloroethylene to measure the muon flux at various depths in the mine, it was possible to estimate the rate of this background level when considering the theoretical models of muon energy as a function of distance traveled, *i.e.* depth. The production rate of ^{37}Ar due to muon interaction was found to be 0.065 ± 0.005 atoms per day. This number was later adjusted to 0.052 ± 0.012 atoms produced per day, by using potassium hydroxide solutions as test volumes [73]. Other significant background levels include α -emitters from the steel walls surrounding the detector, and fast neutrons from the cave walls interacting with the liquid. The total background for the Homestake experiment, including all sources, was estimated to 0.08 ± 0.03 atoms per day. Figure 4.4 found in [59] shows the total capture rate for the detector over the entire period of 1970-92.

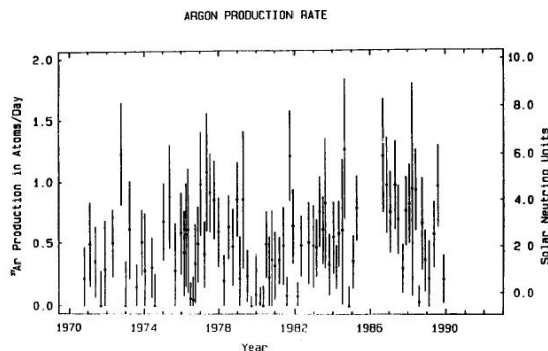


Figure 4.4: Total capture rate for Homestake experiment during 1970-92 [59].

The final results, subtracting estimated background levels, yielded an average of 0.437 ± 0.042 atoms per day, or 2.32 ± 0.22 SNUs, a number roughly one third of what the SSM of the time being predicted. This flux is in reasonable agreement with the observations at the Kamiokande detector during the period 1987-1990, when both detectors were measuring the solar neutrino flux.

4.2.3 Kamiokande

Most of the data presented in this section is found in [69].

The Kamiokande detector, consisting of 3000 ton of pure water, was located 1000 m underground within a mine in Kamioka, Japan. Started in 1983, the original purpose of this observation site was to look for proton decay. However, the focus soon shifted towards detecting neutrinos from supernovas and the sun [74]. The key idea of Kamiokande was to observe electron scattering when the neutrinos hit the water tank, producing Cerenkov light which in

turn can be detected by PMTs surrounding the detector. The reaction to look for is

$$\nu_e + e \rightarrow \nu_e + e, \quad (4.3)$$

and the neutrino energy threshold for this reaction to occur is 7.5 MeV. This high threshold excludes all solar neutrinos from interacting except the energetic ^8B neutrinos. Supernova neutrinos typically have energies of ~ 20 MeV, so these would also participate in 4.3. Another feature of reaction 4.3 is that both neutrinos and anti-neutrinos of all flavors can contribute to scattering electrons in 4.3.

The results of Kamiokande showed an inconsistency with theory. The absolute flux detected was $(2.80 \pm 0.19 \pm 0.33) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$, around 50% of what theory predicted. The data collected to calculate this flux spanned over a period of roughly 2000 days, comparable to a solar cycle period. The Kamiokande experiment also provided relevant information concerning correlation factors to the solar neutrino flux, such as the variation of sunspot numbers and the daytime/nighttime asymmetry.

The variations in the flux related to solar activity was measured during eight years with ~ 200 days of data each year. A positive correlation between the number of sunspots and neutrino flux was indicated, but not statistically significant. Figure 4.5 from [69] shows the data sampled.

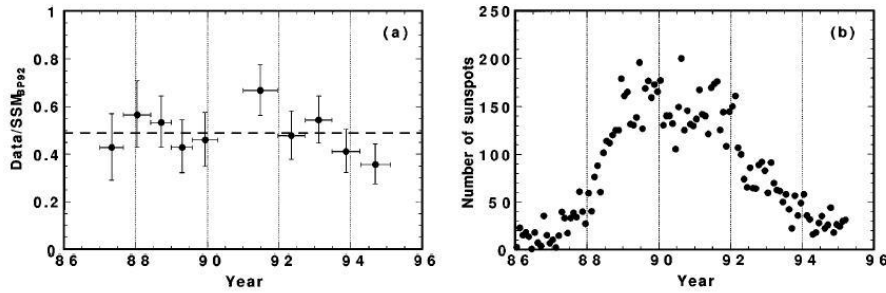


Figure 4.5: a) Ratio between Kamiokande data and SSM prediction for flux ratio. b) Sunspot numbers during the same period of time as a) [69].

Data from Kamiokande also revealed an asymmetry in the flux distribution during day- and nighttime. The flux was slightly larger during the night, but again there was no significant difference. A probable cause for this difference is that the neutrinos must travel through part of the Earth during night, which affects neutrino oscillation probabilities due to the presence of matter (see section 3.2). The data are displayed in figure 4.6 from [69].

The Kamiokande detector was another step towards understanding the nature of neutrinos. It was nevertheless necessary to upgrade the detector further with regard to sensitivity for neutrino reactions, in order to achieve improved statistical precision and obtain estimates of parameters such as neutrino mass and mixing angles. This motivated the construction of Super-Kamiokande based on the framework of the original Kamiokande detector.

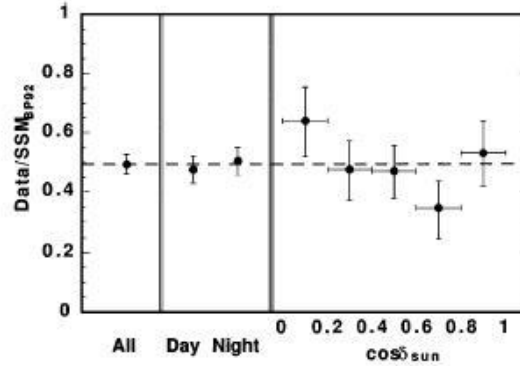


Figure 4.6: Ration between measured neutrino flux and SSM prediction during day- and nighttime, averaged out. The right part of the graph divides the nighttime data into five subsamples dependent on the angular direction of the Sun relative to the Kamiokande detector. The δ_{Sun} angle is the same as the angle θ in figure 2.1. [69]

4.2.4 Super-Kamiokande

Most of the data presented in this section is found in [62].

As the name implies, Super-Kamiokande is the big brother of the Kamiokande detector and became operational in 1996. This Cerenkov detector contains 50,000 ton of water, located 1 km beneath the surface at Kamioka Observatory, Japan. The detector is shaped cylindrically, and is optically divided into two regions, each serving a different purpose. The outer part is meant for holding back cosmic ray muons, while the inner part is the main neutrino catcher. Both atmospheric and solar neutrinos are detected, and one is able to distinguish the background events (such as atmospheric neutrinos) from the solar neutrinos by observing the peak in the solar direction. To get a visual impression of the detector, consider figure 4.7 found in [62].

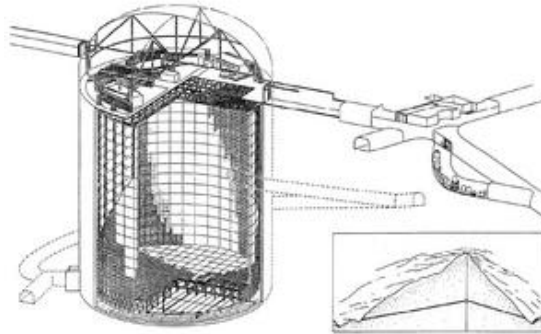


Figure 4.7: A schematic diagram of the detector. [75]

In order to obtain parameter estimates for the atmospheric neutrinos, one considered the zenith angle distribution of the neutrino events registred in the detector. The ν_e events agreed with prediction, but the ν_τ and ν_μ -events differed. Assuming that $\tau \rightarrow \mu$ oscillation

dominates for atmospheric neutrinos, the comparison of observed data with the oscillation probability (see section 3.1.1)

$$P(|\nu_\mu\rangle \rightarrow |\nu_\tau\rangle) = \sin^2 2\theta \sin^2 \frac{1.27L(\text{km})\Delta m^2(\text{eV}^2)}{E(\text{GeV})},$$

would provide best-fit estimates for the parameters $\sin^2 2\theta$ and Δm^2 . By varying these unknowns to fit the experimental data, the best result values yielded $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 2.0 \times 10^{-3} \text{ eV}^2$. This is illustrated in figure 4.8 found in [62].

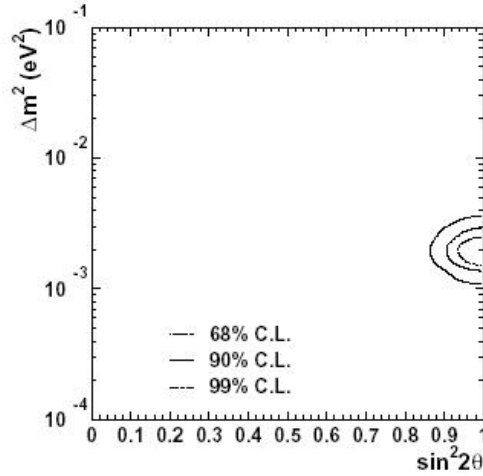


Figure 4.8: Allowed values for $(\sin^2 2\theta, \Delta m^2)$ from zenith-angle analysis. The inner, middle and outer regions correspond to 68%, 90% and 99% confidence levels [62].

The Super-Kamiokande data was also used for a L/E -analysis based on a 1489-days exposure of atmospheric neutrinos. Monte Carlo simulations of the observed energy determined the neutrino energy E , while the flight length L was estimated from the direction of momentum of the observed particles. This direction would reveal the incoming neutrino zenith angle, and thus one could calculate total path length from the atmosphere. The results were quite consistent with the $(\sin^2 2\theta, \Delta m^2)$ -scan, and the best-fit from this analysis is $\sin^2 2\theta = 1.02$ and $\Delta m^2 = 2.4 \times 10^{-3} \text{ eV}^2$. Note that the 1.02 value is unphysical since $\sin^2 2\theta \leq 1$, so it is more sensible to consider the 90% confidence level results $1.9 \times 10^{-3} \text{ eV}^2 < \Delta m^2 < 3.0 \times 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta > 0.90$.

Up to Super-Kamiokande, all solar neutrino detection experiments had produced a much smaller flux than the SSM would predict. During a 1496-days period of recording solar neutrino data, Super-Kamiokande turned out to be no exception. The observed flux was $(2.35 \pm 0.02 \pm 0.08) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$, roughly 46% of the current standard solar model BP2000 (see table 2.2) of that time.

To estimate the square mass difference and mixing angle, a maximum likelihood fit was performed by considering time variations and energy spectrum of the neutrino flux signal. Effects such as day and night asymmetry and seasonal variation were included, although the day- and nighttime asymmetry turned out to be statistically consistent with zero, just as

Kamiokande results. The search for energetic ^8Be solar neutrinos yielded best-fit values of $\Delta m^2 = 6.9 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta = 0.38$, combined with the neutrino detection experiments at Homestake, SNO, GALLEX and SAGE (see section 4.2). The measured day-night asymmetry was $-2.1 \pm 2.0 \text{ }^{+1.3}_{-1.2} \%$. These original values were given immediately after the end of the experiment in 2002 [49, 50].

A later analysis in 2004, [76], has produced new values for the oscillation parameters with increased statistical precision. The square mass difference solution from Super-Kamiokande is in synthesis with data from KamLAND, in addition to the earlier mentioned experiments, found to be $\Delta m^2 = 7.1 \times 10^{-5} \text{ }^{+0.6}_{-0.5} \text{ eV}^2$, while the neutrino flux from the sun is shown to exhibit a small day-night asymmetry of $-1.8 \pm 1.6 \text{ }^{+1.3}_{-1.2} \%$. The solar data is adequately accounted for using two mass eigenstates, which these parameters refer to.

4.2.5 KamLAND

The KamLAND detector is a liquid scintillator detector situated in a mine cavern in Japan, just as the Kamiokande and Super-Kamiokande. The detector itself is made of a 13 m in diameter weather balloon filled with this liquid scintillator, which works by the same principle used in the Reines and Cowan experiment (see section 4.2.1). Light is emitted when the positron from the reaction $\bar{\nu}_e + p \rightarrow n + e^+$ is annihilated with an electron, and is almost immediately registered in one of the 1879 PMTs surrounding the scintillator fluid. Consult figure 4.9 taken from [77] for an overview. The main goal of KamLAND was to measure the flux of anti-neutrinos from a set of reactors. Due to its distance of 100-200 km to the reactors, the KamLAND experiment gained the upper edge on previous reactor-based detectors such as CHOOZ and Palo Verde, which were located at the immediate vicinity ($\sim 1 \text{ km}$) of the reactor. The substantially longer baseline of KamLAND yielded a larger sensitivity to the oscillation effect, thus proving the location relative to the reactors to be of great value. A secondary purpose of this detector is to detect solar neutrinos originating from the decay of ^7Be and ^8B . One would suspect the large antineutrino flux from the reactors and background sources would make it hard to distinguish which source the detected antineutrinos come from. However, the photon energy signature from the solar neutrinos make the task of separating them from reactor neutrinos quite easy [63, 64].

The KamLAND detector is surrounded by 53 reactors in Japan and Korea, and makes use of the principles described in section 2.5 to detect the antineutrino fluxes from these power stations. A problem with KamLAND was the relatively large background signal from natural radioactivity and, as always, the atmospheric muon decay. Occurrence of solids such as ^{238}U , ^{232}Th and ^{40}K in the surrounding rock can lead to false neutrino-events due to the decaying nature of these elements. Even materials used in the fabrication of the detector can lead to background contamination, such as ^{60}Co . The neutron-capture in the scintillator fluid due to this type of decay can be mistaken for a neutrino-event, thus providing a false alias for the $\bar{\nu}_e$. These countings can be hard to distinguish from true neutrino-events. The cosmic muon events, however, can in general be discarded by time-mechanisms and energy-thresholds in the signal-processing units connected to the PMTs. Also, the outer water tank serves as a detector for Cerenkov radiation caused by incoming muons, thus providing a way of identifying background events. [64, 65]

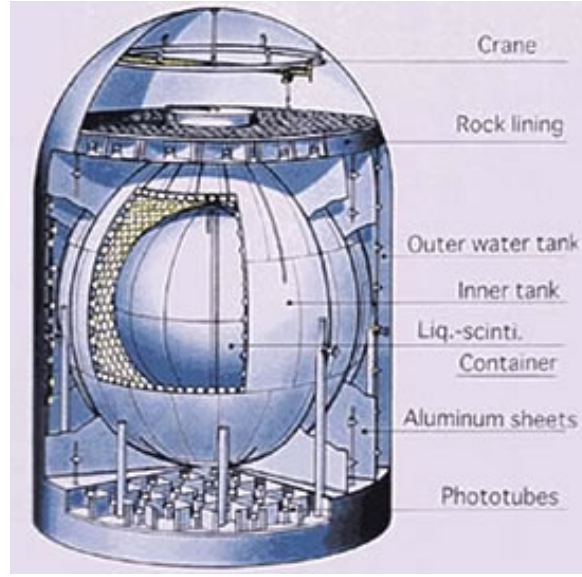


Figure 4.9: Illustration of the KamLAND anti-neutrino detector [77].

The neutrino oscillation parameters in the KamLAND experiment were determined by performing a χ^2 statistical analysis on the measured flux signal. The first results exclusively from KamLAND yielded parameter values of $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 6.9 \times 10^{-5} \text{ eV}^2$ [66]. Later corrections to these values produced the best determination of parameters up-to-date from KamLAND and combined solar neutrino data, namely $\tan^2 \theta = 0.40^{+0.10}_{-0.07}$ and $\Delta m^2 = 7.9 \times 10^{-5}^{+0.6}_{-0.5} \text{ eV}^2$ [65]. Figure 4.10 found in [65] shows the allowed regions for $(\tan^2 \theta, \Delta m^2)$.

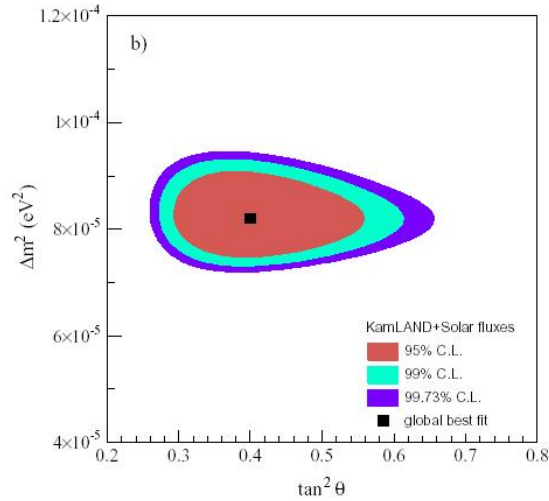


Figure 4.10: Global two-neutrino oscillation fit between data from KamLAND and other solar fluxes. Best-fit point at $\tan^2 \theta = 0.40^{+0.10}_{-0.07}$ and $\Delta m^2 = 7.9 \times 10^{-5}^{+0.6}_{-0.5} \text{ eV}^2$ [65].

4.2.6 GALLEX/GNO

The GALLEX experiment used ~ 30 tons of gallium in a $\text{GaCl}_3\text{-HCl}$ solution to capture solar neutrinos. The key idea was that the neutrinos would induce the reaction ${}^{71}\text{Ga} + \nu_e \rightarrow {}^{71}\text{Ge} + e^-$ with an energy threshold of roughly 233 keV. This low threshold would lead to a large detection rate, since the large majority of neutrinos produced in the sun would be energetic enough to interact with the gallium solution. This is illustrated in table 4.2 found in [78], where the SSM predicted capture rate for the Homestake experiment (energy threshold 0.814 MeV) is compared to GALLEX. The detector is located in the Gran Sasso Underground Laboratory, Italy, and is shielded by rock equivalent to 3300 m of water.

The neutrino events are counted as follows. The produced ${}^{71}\text{Ge}$ is extracted from the tank using nitrogen gas, leading it to a container of water which it allows the formation of GeH_4 . This compound can now be detected with a proportional counter, thus using a radiochemical technique. There was also conducted a test experiment with a ${}^{51}\text{Cr}$ neutrino source [79]. Consider figure 4.11 taken from [80] for a visualization of the system.

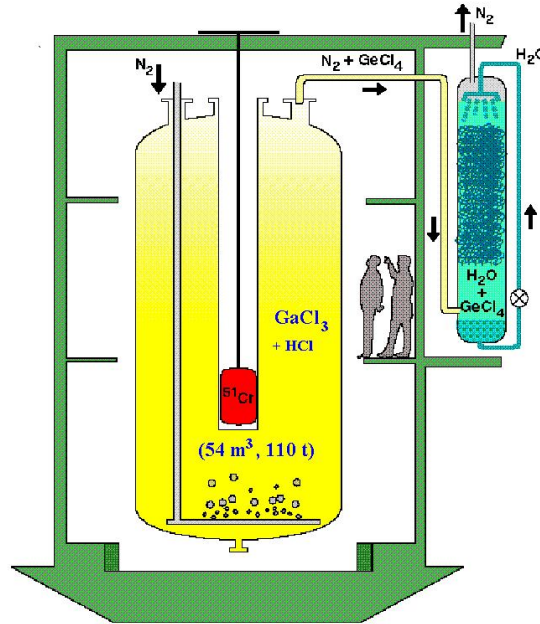


Figure 4.11: GALLEX detector schematically drawn. The detector contains a central reentrant tube which holds a ${}^{51}\text{Cr}$ neutrino source [80].

The GALLEX experiment was concluded in 1997 with a resulting capture rate of $77.5 \pm 6.2^{+4.3}_{-4.7}$ SNU [60], while SSM BP98 says 132 SNU. As a successor of GALLEX, the GNO (Gallium Neutrino Observatory) was initiated, with an objective to detect solar ν_e mainly from the pp chain. In order to eliminate as many of the statistical uncertainties as possible, the old GALLEX detector was renovated with respect to data acquisition equipment and electronics. The same amount of gallium was used, and the results from measurements during the period

1998-2002 yielded an improved capture rate of $65.2 \pm 6.4 \pm 3.0$ SNU for the GNO experiment. [81]

Table 4.2: Solar neutrino energy for possible reactions and corresponding fluxes and capture rates for Homestake and GALLEX according to the SSM [78].

Source reaction	Energy (MeV)	Flux at Earth ($10^{10} \text{ cm}^{-2} \text{ s}^{-1}$)	Homestake vs. GALLEX Production rate (SNU)
pp	< 0.42	6.0	0.0 vs. 70.8
pep	1.40	0.014	0.2 vs. 3.00
${}^7\text{Be}$	0.38, 0.86	0.49	1.2 vs. 35.9
${}^8\text{B}$	< 15.0	0.00057	6.2 vs. 13.8
${}^3\text{Hep}$	< 18.8	1.20×10^{-7}	0.005 vs. 0.01
${}^{13}\text{N}$	< 1.20	0.049	0.10 vs. 3.0
${}^{15}\text{O}$	< 1.73	0.043	0.030 vs. 5.0
${}^{17}\text{F}$	< 1.74	0.00054	0.0040 vs. 0.060
Total	-	6.60	8.0 vs. 132

4.2.7 SNO

The SNO detector (Sudbury Neutrino Observatory) is located in a underground mine in Sudbury, Ontario, shielded by 2 km of solid rock. The main detector is a tank consisting of 1 kton of heavy water (containing deuterium) serving to detect mainly solar electron-neutrinos, but is also capable of capturing the other two flavors. Background levels are held at bay by 7 kton of ordinary water surround the detector, and $\sim 10\,000$ PMTs standby to register any neutrino events in the inner tank. For an overview, consult figure 4.12 taken from [67]. SNO is able to detect all three types of neutrino reactions (see table 3.1). The water in the inner and outer tank is extremely pure in order to prevent any neutron-capture in the detector from radioactive decay in contaminants. Such reactions are, as previously mentioned, hard to separate from true neutrino-events. [67, 82]

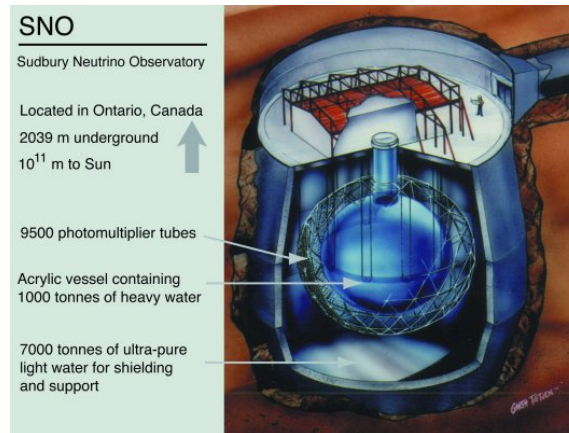


Figure 4.12: Illustration of the Sudbury Neutrino Observatory [67].

The SNO experiment is actually divided into three continuous phases. The first phase consists of detecting neutrons from the NC reaction $\nu_x + d \rightarrow n + p + \nu_x$ with only heavy water in the detector. The neutrons are captured by D_2O with a probability of 25%, a reaction producing Cerenkov light of 6.3 MeV. Then, in the second phase, this capture probability is greatly enhanced by adding 2 tons of NaCl to the heavy water. The capture rate is estimated to 83% during this phase, producing gamma rays of 8.6 MeV. Finally, the salt is removed in the third phase and the neutrons are detected at 45% efficiency by installed proportional counters filled with ^3He . The Cerenkov light from the electron in the CC reaction $\nu_e + d \rightarrow p + p + e^-$ will during the entire experiment be detected by the PMTs, though with a different photon characteristic energy than the neutron-induced light. [68]

A great triumph for the neutrino oscillation theory presented itself after roughly one year of observations at the SNO. Although the detected flux of electron-neutrinos measured roughly 34% of the SSM, the combined flux of all three neutrino flavors was in excellent agreement with the prediction of ν_e -flux from the SSM [83, 84]. In other words,

$$\Phi_e^{Sun} = \Phi_e^{SNO} + \Phi_\mu^{SNO} + \Phi_\tau^{SNO}.$$

This was a strong indication that the SSM correctly predicts the electron-neutrino flux at the sun's surface, and that neutrino oscillations take place while in transit to Earth.

The salt phase of SNO has recently produced firm data, and considered in conjunction with the KamLAND experiment best-fit parameters are $\tan^2 \theta = 0.41$ and $\Delta m^2 = 7.1 \times 10^{-5} \text{ eV}^2$. This is in good agreement with solar neutrino fits of Super-Kamiokande (section 4.2.4). The allowed parameter regions are displayed in figure 4.13 found in [85].

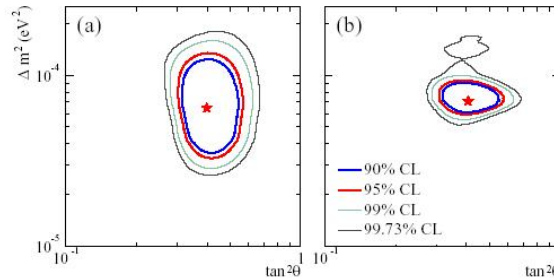


Figure 4.13: *a)* Global solar parameter-fits from D_2O and salt phase. Best-fit is $\tan^2 \theta = 0.40$ and $\Delta m^2 = 6.5 \times 10^{-5} \text{ eV}^2$, marked with the red star. *b)* *a)* + KamLAND data. Best-fit is $\tan^2 \theta = 0.41$ and $\Delta m^2 = 7.1 \times 10^{-5} \text{ eV}^2$ [85].

Chapter 5

Conclusion

The sources that provide us with neutrinos today are given in table 5.1, with a categorizing as found in [51]. The neutrino fluxes from experiments conducted with these sources have in a merciless manner turned out to be smaller than expected in all cases, and this has called upon the need for an explanation ever since the Homestake experiment (see section 4.2.2). The prevailing theory is the one of neutrino oscillations, which states that a neutrino can change identity between its flavors $\{e, \mu, \tau\}$, dependent on a number of parameters.

Table 5.1: Neutrino sources and their classification [51].

Type	Classification
Reactor	Artificial
Accelerator	Artificial
Solar	Natural
Atmospheric	Natural
Supernovae	Natural
Dark matter/energy (Big Bang)	Natural

This project thesis has presented several indications to that neutrino oscillations are actually happening, such as the atmospheric neutrino anomaly and solar flux deficit, and referred to experiments that provide quite substantial evidence, especially Super-Kamiokande, KamLAND, and SNO. The detection principles are in general based on neutrinos participating in reactions producing Cerenkov light, or radiochemical techniques that count the number of product nuclei in neutrino reactions. Taking all the major neutrino detection experiments into account, the best-fit parameter values for the mass square difference and mixing angles are given in table 5.2 with data from [36, 86]. The reader may consult section 3.1.2 to recapitulate the role of Δm_{ij}^2 and θ_{ij} in the oscillation theory. In short, the physical significance of these parameters are related to the mass of the neutrino and the oscillation frequency between different flavors.

The square mass difference involved in solar neutrino experiments is named $\Delta m_{\text{sol}}^2 = \Delta m_{21}^2$, and the mass square difference associated with atmospheric neutrino experiments is called $\Delta m_{\text{atm}}^2 = \Delta m_{31}^2$, and applies to two-flavor analyses of experimental data. Δm_{32}^2 is not listed, since it is given by $\Delta m_{31}^2 - \Delta m_{21}^2$, assuming normal mass hierarchy $m_3 \geq m_2 \geq m_1$. The determination of the absolute values of neutrino eigenstate masses has not been completed,

and is dependent on improvement of experimental techniques and sensitivity.

Table 5.2: Today's best-fit values for neutrino parameters [36, 86].

Parameter	Best-fit value	Confidence interval 3σ
Δm_{21}^2	$7.1 \times 10^{-5} \text{ eV}^2$	$(5.4 - 9.5) \times 10^{-5} \text{ eV}^2$
Δm_{31}^2	$2.5 \times 10^{-3} \text{ eV}^2$	$(1.5 - 3.7) \times 10^{-3} \text{ eV}^2$
θ_{12}	34°	$29^\circ - 39^\circ$
θ_{13}	4°	$0^\circ - 13^\circ$
θ_{23}	45°	$34^\circ - 58^\circ$

Data from numerous experimental sites point in the direction of oscillations occurring, for instance solar ν_e changing into ν_μ and ν_τ , with results from SNO having provided the firmest evidence of oscillations so far (see section 4.2.7). The total flux registered at SNO, including all three neutrino flavors, is in perfect agreement with the solar ν_e flux as predicted by the SSM. This strongly supports the idea of ν_e oscillating to other flavors while in transit to Earth. Furthermore, Super-Kamiokande data indicates oscillations between atmospheric ν_μ and ν_τ (see section 4.2.4). There is a significant up/down asymmetry displayed in the atmospheric neutrino flux pattern (see section 2.1), due to difference in baseline length for the neutrinos, which affects the oscillation probability. Also, Earth itself has an impact on the neutrino behaviour, as a result of the MSW-effect (see section 3.2). The CNGS beam (see section 2.5) will give ample opportunity for extensive studying of oscillations between ν_μ and ν_τ , which seem to dominate the atmospheric neutrino scenario. Reactor sites have reported oscillations between $\bar{\nu}_\mu$ and $\bar{\nu}_e$, but the results from LSND have yet to be fully confirmed and could seem problematic without introducing a fourth neutrino flavor or abandoning *CPT*-invariance (see section 2.5).

I would also like to comment on the combined data analysis of KamLAND and other major neutrino detection experiments in section 4.2.5 that seems to stand in opposition to the best-fit value of table 5.2. The reason is simply that KamLAND spectrum provides a higher sensitivity for Δm^2 in the combined analysis, and thus dominates the contribution to the square mass difference. However, the rest of the solar neutrino experiments provide a stricter constraint on the mixing angle, thus adjusting $\tan^2 \theta$ from 0.46 to 0.40 in the combined analysis [65]. This means that although KamLAND suggests an increase in best-fit value for Δm^2 of roughly 10% compared to the other experiments, the latter are more emphasized in the total estimate.

The future of the neutrino oscillation theory in its present form is not certain. Should the MiniBOONE experiment (see section 2.5) turn out to yield consistent results with the LSND anomaly, one would have to work on a new theory, probably including more than three flavors of neutrinos. MiniBOONE has been running for nearly two years as this is being written, and the conclusion of this experiment is of great interest to the further development of neutrino oscillation theory.

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