Z-99 MODELING OF SINGLE REFLECTION BY TIP WAVE SUPERPOSITION METHOD USING EFFECTIVE COEFFICIENT

A.M. AIZENBERG[1], M.A. AIZENBERG[1], H.B. HELLE[2],
K.D. KLEM-MUSATOV[1] and J. PAJCHEL[2]
[1]*Institute of Geophysics, Russian Academy of Science, Ac. Koptuyg pr. 3, Novosibirsk 630090, Russia*
[2]*O&E Research Centre, Norsk Hydro ASA, Bergen, Norway*

Summary. Here we summarize the results of numerical tests of an improved algorithm for computing synthetic seismograms in 3-D models based on the Tip Wave Superposition Method (TWSM). Compared with earlier versions of TWSM the improvements comprise a generalization of the background theory that also accounts for wave phenomena at critical reflections (head waves) and the replacement of the plane-wave reflection coefficient with the effective coefficient that eliminates diffraction artefacts at critical angles.

Introduction. Seismic wave field simulation algorithms are usually developed so that they allow for inversion with respect to the model parameters. The purpose of such algorithms is to represent the wave field in the form of isolated wave events, which can be directly identified with elements of the geophysical model. To provide this possibility, the TWSM was introduced for simulation of the 3-D wave field scattered by curvilinear interfaces in terms of the Helmholtz integral [2, 3, 6]. However, in the initial version of TWSM, which is the high-frequency realization of the Helmholtz integral, the boundary values of wave field and its normal derivative were described by using the plane-wave reflection/transmission coefficients which are discontinuous at critical angles. This implies that instead of creating head waves at critical angles the Helmholtz integral produces artefacts in the form of edge waves diffracted from the contours formed by the critical points on the interface. To develop a theoretical foundation of the simulation methods based on the Helmholtz integral, we introduced in [5] the exact description of its boundary values in terms of integral reflection and transmission operators.

Exact description of the single reflection. The single reflected from an interface S wave field $s^{(1)}(\mathrm{R})$ can be expressed by the Helmholtz integral [5]

$$s^{(1)}(\mathrm{R}) = \iint_S \left[\partial g(\mathrm{R},\mathrm{r}_0)/\partial n \; r^{(1)}(\mathrm{r}_0) - g(\mathrm{R},\mathrm{r}_0)\, \partial r^{(1)}(\mathrm{r}_0)/\partial n \right] dS \ .$$

The boundary values of integral $r^{(1)}(\mathrm{r}_0)$ and $\partial r^{(1)}(\mathrm{r}_0)/\partial n$ describe the wave field locally reflected at the point r_0 of the interface S and its normal derivative respectively. The scattered wave field $s^{(1)}(\mathrm{R})$ represents superposition of the wave fields coming from the different reflection points. The limiting value of integral $s^{(1)}(\mathrm{r}_0)$ does not in general coincide with the corresponding boundary value $r^{(1)}(\mathrm{r}_0)$.

It was shown in [5] that the boundary values $r^{(1)}(\mathrm{r}_0)$ and $\partial r^{(1)}(\mathrm{r}_0)/\partial n$ at any chosen point r_0 of the interface can be exactly expressed through the traces of the source wave field $f^{(0)}(\mathrm{r})$ and its normal derivative $\partial f^{(0)}(\mathrm{r})/\partial n$ as

$$r^{(1)}(\mathrm{r}_0) = K_{11}(\mathrm{r}_0, \mathrm{r})\, f^{(0)}(\mathrm{r}), \quad \partial r^{(1)}(\mathrm{r}_0)/\partial n = -K_{11}(\mathrm{r}_0, \mathrm{r})\, \partial f^{(0)}(\mathrm{r})/\partial n,$$

using unknown earlier reflection operator

$$K_{11}(\mathrm{r}_0, \mathrm{r}) = \mathrm{F}^{-1}(\mathrm{r}_0, \zeta)\, k_{11}(\zeta)\, \mathrm{F}(\zeta, \mathrm{r}).$$

The space Fourier operator $\mathrm{F}(\zeta, \mathrm{r})$ transforms the traces $f^{(0)}(\mathrm{r})$ and $\partial f^{(0)}(\mathrm{r})/\partial n$ from the curvilinear interface coordinates $\mathrm{r} = (x_1, x_2)$ into the wave number domain $\zeta = (\zeta_1, \zeta_2)$. The kernel $k_{11}(\zeta)$ of the operator $K_{11}(\mathrm{r}_0, \mathrm{r})$ coincides with the plane-wave reflection coefficient.

Effective reflection coefficient. In the case of a plane interface the reflection and transmission operators can be exactly converted to the form of effective reflection and transmission coefficients [1]. It can be shown in the general case of curvilinear interface that the effective reflection coefficient $\chi_{11}^{EC}(\mathrm{r}_0)$ is the high-frequency approximation of the reflection operator $K_{11}(\mathrm{r}_0, \mathrm{r})$. Therefore the boundary values can be presented as

$$r^{(1)}(\mathrm{r}_0) \cong \chi_{11}^{EC}(\mathrm{r}_0)\, f^{(0)}(\mathrm{r}_0), \quad \partial r^{(1)}(\mathrm{r}_0)/\partial n \cong -\chi_{11}^{EC}(\mathrm{r}_0)\, \partial f^{(0)}(\mathrm{r}_0)/\partial n.$$

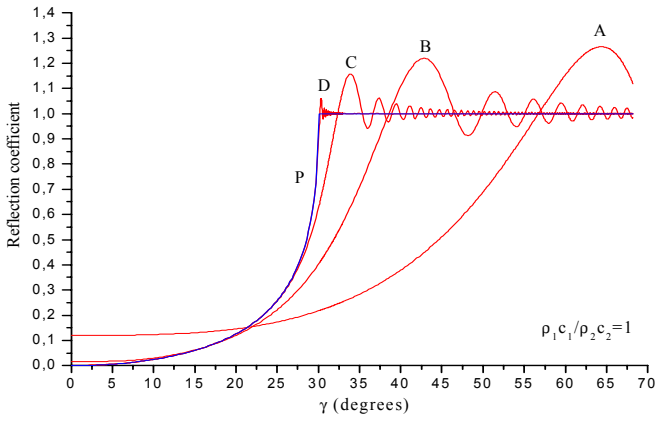


Figure 1 displays $\chi_{11}^{EC}(\mathrm{r}_0)$ as a function of reflection angle γ for a range of values of kh (curves A, B, C, D), where h is the distance between the source and the tangent plane and $k = \omega\sqrt{c_1^{-2} + c_2^{-2}}$. The plane-wave reflection coefficient (curve P) is shown for comparison. We observe that the modulus of effective reflection coefficient approaches that of the plane-wave when $kh \to \infty$.

Figure 1: *Comparison of plane-wave reflection coefficient (P) with the effective reflection coefficient for various values of kh (A: kh=1, B: kh=10, C: kh=100, D: kh=10⁶).*

Simulation of single reflection by the modified TWSM. Simulation is produced by the TWSM algorithm modified by the effective reflection coefficient [2]. To demonstrate the advantage of using the effective coefficients, we give the synthetic seismograms obtained with help of the initial version of TWSM (with the plane-wave coefficient) and the modified version of TWSM (with the effective coefficient). First test is devoted to reflection from an anticline type curvilinear interface. The model, source and data window are shown in Figure 2a. The corresponding seismograms (Figure 2b,c) contain only one event of a pure reflection

wave. The seismogram obtained by using the plane-wave reflection coefficient (b) is severely distorted by the artificial diffractions, while the seismograms obtained using the effective reflection coefficient (c) can easily be interpreted in terms of reflections and head waves. Two head waves are well defined. The ordinary head-wave arrival is generated in the overcritical zone, confined to the plane part of the interface. The head wave with low apparent velocity arises from the overcritical strip $0.185 < x < 0.29$ km and forms a small caustic loop. Second test is chosen to enable the caustic response to be properly recorded by the receivers. A tanh-like interface, source and data window are identical to those used in [4] and shown on Figure 2d. The seismograms (Figure 2e,f) contain pure reflections including a caustic formed by the reflected waves from the concave interface between the two points of normal incidence $x_1 = 0.593$ km and $x_1 = 0.832$ km. Although the critical angles are present, the associated events arrive beyond the present receiver array (d), except for the corresponding artificial diffraction in (e) approaching from far-offset due to the discontinuity of the plane-wave reflection coefficient at the critical reflection point (1.06 km). The features of the reflection triplication of the P-wave obtained by complex ray tracing and finite difference modeling in [4] are well reproduced in (f).

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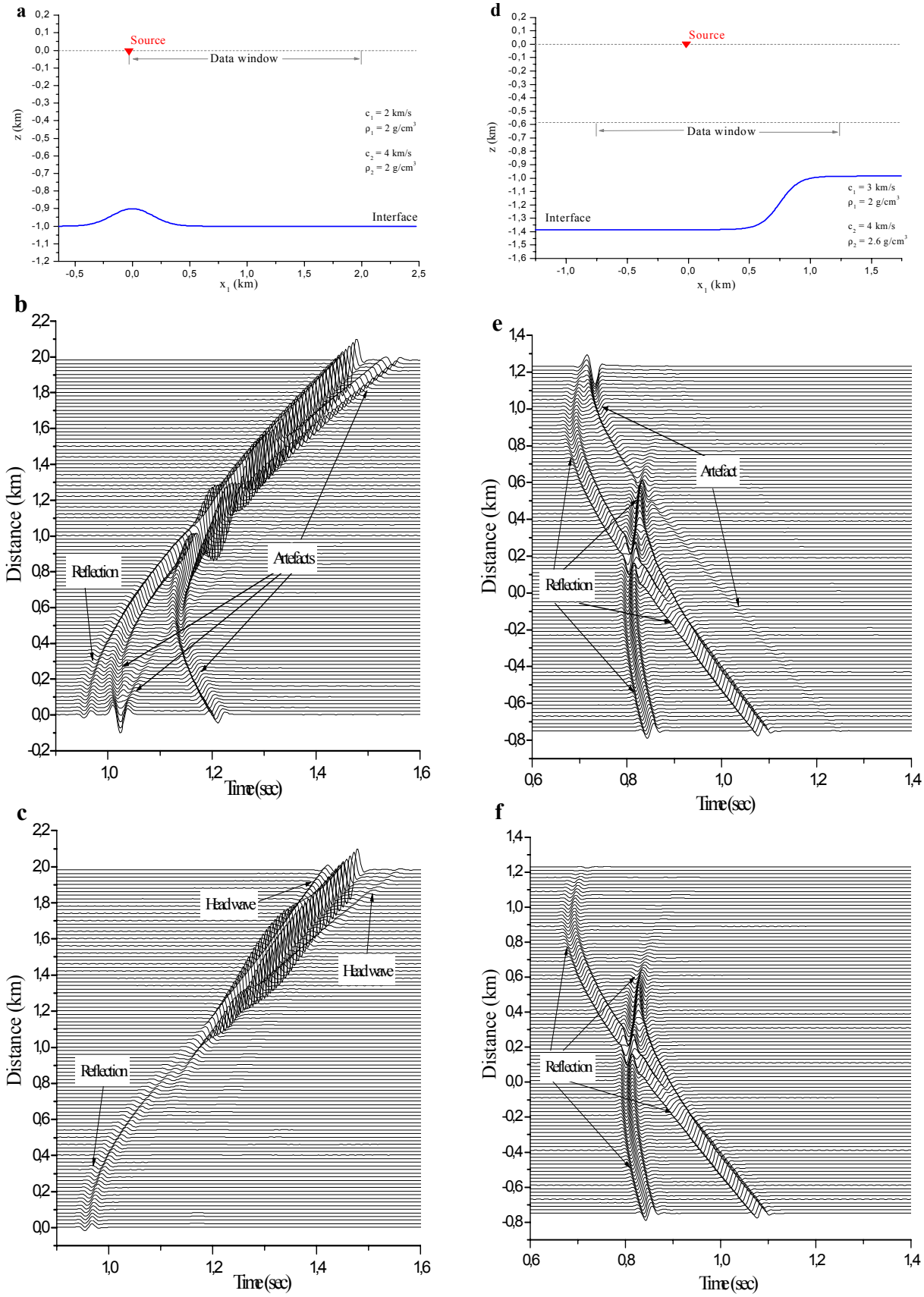


Figure 2: *Single reflection from curvilinear interface.*